

Математика

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ZH. ZH. BAIGUNCHEKOV¹, N. ZH. BAIGUNCHEKOV²

¹Kazakh-British technical university, Almaty, Kazakhstan,
²Institute of mechanics and mechanical engineering, Almaty, Kazakhstan)

DIFFERENTIAL EQUATIONS OF MOTION OF PLANAR EXECUTIVE MECHANISMS OF PARALLEL MANIPULATING ROBOTS

Annotation. In this paper the differential equations of motion of planar executive mechanisms of parallel manipulating robots (PEM PMR) based on Lagrange equations of the second type are made up. Differential equations of motion of PEM PMR have a universal character. They are written in matrix form and applied to study the dynamics of PEM PMR with an arbitrary number of degrees of freedom.

Keywords: parallel manipulating robot, differential equations, dynamics.

Тірек сөздер: параллель манипуляциялық робот, дифференциалдық тәндеулер, динамика.

Ключевые слова: параллельный манипуляционный робот, дифференциальные уравнения, динамика.

Different methods are used for dynamic analysis of robots depending on their structures [1-15]. In addition, the separate equations of dynamics are made up for each type of robot. In order to universalization of study of robots dynamics it is necessary to make up the generalized differential equations of motion. In this paper the universal differential equations of motion of planar executive mechanisms of parallel manipulating robots (PEM PMR) with an arbitrary number of degrees of freedom are made up.

For make up the differential equations of motion of PEM PMR we use the Lagrange equations of the second kind [16]:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_k} \right) - \frac{\partial T}{\partial q_k} = Q_k, \quad (k = n-m+1, \dots, n), \quad (1)$$

where n and m – number of mobile and input links of PEM PMR respectively; T – kinetic energy of PEM PMR; q_k, \dot{q}_k and Q_k – the generalized coordinates, velocities and forces respectively having the following forms:

$$q_k, \dot{q}_k, Q_k = \begin{cases} \varphi_k, \omega_k, M_k & \text{– for PEM PMR with rotation input links,} \\ s_k, v_k, F_k & \text{– for PEM PMR with translation input link.} \end{cases}$$

Let enter a matrix \mathbf{M}_i of masses m_i and moments of inertia I_i of the i -th mobile link of PEM PMR concerning an axis passing through the center of masses, and a vector \mathbf{v}_i of velocities of the i -th link:

$$\mathbf{M}_i = \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & I_i \end{bmatrix}, \quad \mathbf{v}_i = \begin{bmatrix} v_i^x \\ v_i^y \\ \omega_i \end{bmatrix}. \quad (2)$$

Then a kinetic energy of the i -th link of PEM PMR is defined by expression:

$$T_i = \frac{1}{2} \mathbf{v}_i^T \cdot \mathbf{M}_i \cdot \mathbf{v}_i. \quad (3)$$

As the kinetic energy of PEM PMR is equal to the sum of kinetic energies of the mobile links, we will enter a matrix \mathbf{M} of masses and moments of inertia and a vector \mathbf{v} of velocities of links of PEM PMR:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_1 & & & 0 \\ & \mathbf{M}_2 & & \\ & & \ddots & \\ & & & \mathbf{M}_i \\ & & & & \ddots \\ 0 & & & & & \mathbf{M}_n \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \vdots \\ \mathbf{v}_i \\ \vdots \\ \mathbf{v}_n \end{bmatrix}. \quad (4)$$

By means of the matrix \mathbf{M} and the vector \mathbf{v} kinetic energy of PEM PMR can be expressed in the following view:

$$T = \frac{1}{2} \mathbf{v}^T \cdot \mathbf{M} \cdot \mathbf{v}. \quad (5)$$

The vector of velocities \mathbf{v} of the i -th link of PEM PMR is defined by expression:

$$\mathbf{v}_i = \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{u}_{ij}, \quad (6)$$

where \mathbf{u}_{ij} – is a vector of analogues of velocities of the i -th link on the j -th generalized coordinate having a view:

$$\mathbf{u}_{ij} = [u_{ij}^x, u_{ij}^y, \varphi'_{ij}]^T. \quad (7)$$

Then the vector of velocities \mathbf{v} of links of PEM PMR is defined by expression:

$$\mathbf{v} = \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{u}_j, \quad (8)$$

where \mathbf{u}_j – is a vector of analogues of velocities of the links having a view:

$$\mathbf{u}_j = [\mathbf{u}_{1j}, \mathbf{u}_{2j}, \dots, \mathbf{u}_{ij}, \dots, \mathbf{u}_{nj}]^T. \quad (9)$$

Let differentiate kinetic energy T on the generalized coordinate q_k :

$$\frac{\partial T}{\partial q_k} = \frac{1}{2} \frac{\partial}{\partial q_k} (\mathbf{v}^T \cdot \mathbf{M} \cdot \mathbf{v}) = \frac{1}{2} \frac{\partial \mathbf{v}^T}{\partial q_k} \mathbf{M} \cdot \mathbf{v} + \frac{1}{2} \mathbf{v}^T \cdot \mathbf{M} \cdot \frac{\partial \mathbf{v}}{\partial q_k}, \quad (10)$$

where:

$$\frac{\partial \mathbf{v}}{\partial q_k} = \frac{\partial}{\partial q_k} \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{u}_j = \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{w}_{jk}. \quad (11)$$

Similarly we get:

$$\frac{\partial \mathbf{v}^T}{\partial q_k} = \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{w}_{jk}^T. \quad (12)$$

In expression (11) \mathbf{w}_{jk} is a vector of analogues of accelerations of mobile links which has a view:

$$\mathbf{w}_{jk} = \frac{\partial \mathbf{u}_j}{\partial q_k} = [\mathbf{w}_{1jk}, \mathbf{w}_{2jk}, \dots, \mathbf{w}_{ijk}, \dots, \mathbf{w}_{njk}]^T, \quad (13)$$

where:

$$\mathbf{w}_{ijk} = [\mathbf{w}_{ijk}^x, \mathbf{w}_{ijk}^y, \varphi''_{ijk}]^T. \quad (14)$$

Then the equation (10) becomes:

$$\frac{\partial T}{\partial q_k} = \frac{1}{2} \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{w}_{jk}^T \cdot \mathbf{M} \cdot \mathbf{v} + \frac{1}{2} \mathbf{v}^T \cdot \mathbf{M} \cdot \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{w}_{jk}. \quad (15)$$

As:

$$\sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{w}_{jk}^T \cdot \mathbf{M} \cdot \mathbf{v} = \mathbf{v}^T \cdot \mathbf{M} \cdot \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{w}_{jk},$$

then we have:

$$\frac{\partial T}{\partial \dot{q}_k} = \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{w}_{jk}^T \cdot \mathbf{M} \cdot \mathbf{v}. \quad (16)$$

Let differentiate kinetic energy T on the generalized velocity \dot{q}_k :

$$\frac{\partial T}{\partial \dot{q}_k} = \frac{1}{2} \cdot \frac{\partial \mathbf{v}^T}{\partial \dot{q}_k} \cdot \mathbf{M} \cdot \mathbf{v} + \frac{1}{2} \cdot \mathbf{v}^T \cdot \mathbf{M} \cdot \frac{\partial \mathbf{v}}{\partial \dot{q}_k}, \quad (17)$$

where:

$$\frac{\partial \mathbf{v}}{\partial \dot{q}_k} = \frac{\partial}{\partial \dot{q}_k} \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{u}_j = \mathbf{u}_k. \quad (18)$$

Similarly we receive:

$$\frac{\partial \mathbf{v}^T}{\partial \dot{q}_k} = \mathbf{u}_k^T. \quad (19)$$

Then the equation (17) has a view:

$$\frac{\partial T}{\partial \dot{q}_k} = \frac{1}{2} \cdot \mathbf{u}_k^T \cdot \mathbf{M} \cdot \mathbf{v} + \frac{1}{2} \cdot \mathbf{v}^T \cdot \mathbf{M} \cdot \mathbf{u}_k \quad (20)$$

or

$$\frac{\partial T}{\partial \dot{q}_k} = \mathbf{u}_k^T \cdot \mathbf{M} \cdot \mathbf{v}. \quad (21)$$

Let differentiate the equation (21) on time t :

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_k} \right) = \frac{d}{dt} (\mathbf{u}_k^T \cdot \mathbf{M} \cdot \mathbf{v}) = \frac{d\mathbf{u}_k^T}{dt} \cdot \mathbf{M} \cdot \mathbf{v} + \mathbf{u}_k^T \cdot \mathbf{M} \cdot \frac{d\mathbf{v}}{dt}, \quad (22)$$

where:

$$\frac{d\mathbf{u}_k^T}{dt} = \sum_{j=n-m+1}^n \frac{d\mathbf{u}_k^T}{dq_j} \cdot \frac{dq_j}{dt} = \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{w}_{kj} = \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{w}_{jk}, \quad (23)$$

$$\begin{aligned} \frac{d\mathbf{v}}{dt} &= \frac{d}{dt} \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{u}_j = \sum_{j=n-m+1}^n \left(\frac{dq_j}{dt} \cdot \mathbf{u}_j + \dot{q}_j \cdot \frac{d\mathbf{u}_j}{dt} \right) = \\ &= \sum_{j=n-m+1}^n \left(\frac{d\dot{q}_j}{dt} \cdot \mathbf{u}_j + \dot{q}_j \cdot \sum_{l=n-m+1}^n \dot{q}_l \cdot \mathbf{w}_{jl} \right). \end{aligned} \quad (24)$$

Then the equation (22) becomes:

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_k} \right) &= \sum_{j=n-m+1}^n \dot{q}_j \cdot \mathbf{w}_{jk}^T \cdot \mathbf{M} \cdot \mathbf{v} + \mathbf{u}_k^T \cdot \mathbf{M} \cdot \\ &\cdot \left(\sum_{j=n-m+1}^n \frac{d\dot{q}_j}{dt} \cdot \mathbf{u}_j + \sum_{j=n-m+1}^n \sum_{l=n-m+1}^n \dot{q}_j \cdot \dot{q}_l \cdot \mathbf{w}_{jl} \right). \end{aligned} \quad (25)$$

Let all external forces and moments acting on the i -th link of PEM PMR we brought into the center of mass of a link to the main vector \mathbf{f}_i and the main moment \mathbf{m}_i which components form a vector $\mathbf{f}_i = [F_i^x, F_i^y, M_i]^T$. Then the generalized force Q_k is defined by expression:

$$Q_k = \sum_{i=1}^n \mathbf{u}_{ik}^T \cdot \mathbf{f}_i = \mathbf{u}_k^T \cdot \mathbf{f}, \quad (26)$$

where \mathbf{f} is a vector of the main vectors and the main moments of the links of PEM PMR.

Substituting the expressions (16), (25) and (26) in the equation (1) we get a matrix type of the differential equations of motion of PEM PMR with many degrees of freedom:

$$\mathbf{u}_k^T \cdot \mathbf{M} \left(\sum_{j=n-m+1}^n \frac{d\dot{q}_j}{dt} \cdot \mathbf{u}_j + \sum_{j=n-m+1}^n \sum_{l=n-m+1}^n \dot{q}_j \cdot \dot{q}_l \cdot \mathbf{w}_{jl} \right) = \mathbf{u}_k^T \cdot \mathbf{f}, \quad (k = n-m+1, \dots, n). \quad (27)$$

The differential equation of motion PEM PMR with one degree of freedom has been get from the equations (27) as a special case at $m=1$:

$$\mathbf{u}^T \cdot \mathbf{M} \left(\frac{d\dot{q}}{dt} \cdot \mathbf{u} + \dot{q}^2 \cdot \mathbf{w} \right) = \mathbf{u}^T \cdot \mathbf{f}. \quad (28)$$

The equations of motion PEM PMR (27) represent the ordinary differential equations of the first order which have a view:

$$\frac{d}{dt} \begin{bmatrix} q_k \\ \dot{q}_k \end{bmatrix} = \begin{bmatrix} q_k \\ \mathbf{A}^{-1} \cdot \mathbf{b} \end{bmatrix}, \quad (k = n-m+1, \dots, n), \quad (29)$$

where elements of a matrix \mathbf{A} and a vector \mathbf{b} are defined by expressions:

$$a_{kj} = \mathbf{u}_k^T \cdot \mathbf{M} \cdot \mathbf{u}_j, \quad (k = j = n-m+1, \dots, n), \quad (30)$$

$$\mathbf{b} = \mathbf{u}_k^T \cdot \mathbf{f} - \mathbf{u}_k^T \cdot \mathbf{M} \cdot \sum_{j=n-m+1}^n \sum_{l=n-m+1}^n \dot{q}_j \cdot \dot{q}_l \cdot \mathbf{w}_{jl}. \quad (31)$$

The system of the equations (27) is calculated by known numerical methods [17].

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Резюме

Ж. Ж. Байгүниеков¹, Н. Ж. Байгүниеков²

(¹Қазақ-Британ техникалық университеті, Алматы, Қазақстан,

²Академик Ө. А. Жолдасбеков атындағы Механика және машинатану институты, Алматы, Қазақстан)

ПАРАЛЛЕЛЬ МАНИПУЛЯЦИЯЛЫҚ РОБОТТАРДЫҢ ЖАЗЫҚ ОРЫНДАУШЫ МЕХАНИЗМДЕРІ ҚОЗҒАЛЫСЫНЫҢ ДИФФЕРЕНЦИАЛДЫҚ ТЕНДЕУЛЕРИ

Макалада екінші түрлі Лагранж тендеулері негізінде параллель манипуляциялық роботтардың жазық орындаушы механизмдері (ПМР ЖОМ) қозғалысының дифференциалдық тендеулері күрылған. ПМР ЖОМ-нің қозғалысының дифференциалдық тендеулері жалпы түрде келтірілген. Олар матрицалық түрде күрылып, кез келген еркіндік дәрежесі бар ПМР ЖОМ-дің динамикасын зерттеуге пайдалануға болады.

Тірек сөздер: параллель манипуляциялық робот, дифференциалдық тендеулер, динамика.

Резюме

Ж. Ж. Байгүнчеков¹, Н. Ж. Байгүнчеков²

(¹Казахстанско-Британский технический университет, Алматы, Казахстан,

²Институт механики и машиноведения имени академика У. А. Джолдасбекова, Алматы, Казахстан)

ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ ДВИЖЕНИЯ ПЛОСКИХ ИСПОЛНИТЕЛЬНЫХ МЕХАНИЗМОВ ПАРАЛЛЕЛЬНЫХ МАНИПУЛЯЦИОННЫХ РОБОТОВ

В работе составлены дифференциальные уравнения движения плоских исполнительных механизмов параллельных манипуляционных роботов (ПИМ ПМР) на основе уравнений Лагранжа второго рода. Дифференциальные уравнения движения ПИМ ПМР имеют универсальный характер. Они составлены в матричной форме и применимы для исследования динамики ПИМ ПМР с произвольным числом степеней свободы.

Ключевые слова: параллельный манипуляционный робот, дифференциальные уравнения, динамика.