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# ENERGY EXTRACTION FROM D-DIMENSIONAL BLACK HOLE

**Abstract.** It is well known that a spinning black hole can extract enormous amount of energy due to the high value of the angular velocity. In order to detect this energy and estimate its magnitude several methods have been made. One of the way the efficiency of the energy extraction from the black hole is called the Penrose process. In current work the energy extraction from *d*-dimensional cylindrical black hole by the Penrose process has been studied. We have considered that the black hole is noncharged, rotating and isolated. The efficiency of the Penrose process for four dimensional black hole has been calculated. We have shown results by graph which describes the dependence of the effeciency of the Penrose process on cosmological parameter $l$ for fixed value of a rotation parameter $a$. Our results have shown that the maximum efficiency limit of the Penrose process can be rather high with about 70 % compared with than that in four dimensional Kerr black hole with 20,7 %. As well as it is worthy to emphasize that with increase of the value of the cosmological parameter $l$ efficiency of the Penrose process tends to zero asymptotically.

**Keywords:** *d*-dimensional black hole, Penrose process, ergosphere, event horizon.

**Introduction.** The observations show that there is a spinning supermassive black hole (SMBH) in the center of the galaxy. As a result of high values of the angular velocity of the black hole the enormous amount of energy of SMBH is extracted. Up to now, several processes related to the energy extraction from the black hole have been discussed such as Hawking radiation. If we are capable to detect this energy, it would be the proof of existence of these processes. So the energy extraction and its efficiency are one of the most important problems of the energetics of the black hole. In order to detect this energy and estimate its magnitude several methods have been made. One of the way the efficiency of the energy extraction from the black hole is called the Penrose process. According to this process as it has been stated in the papers [1], [2] and [3] there are negative energy orbits of the particle around rotating black hole in a region of ergosphere.

In the paper [4] the energy extraction by the Penrose process in *d*-dimensional spacetime was studied with $S^{d-2}$ topology and achieved higher value of the efficiency compared to the one in Kerr black hole by considering some rotation parameters is equal to zero.

As it has been mentioned in the paper [5] there are three type of solutions of the Einstein equation depending on the cosmological constant $\Lambda$. If the value of the cosmological constant $\Lambda$ is $\Lambda > 0$, $\Lambda = 0$ and $\Lambda < 0$ the asymptotic solutions of the Einstein equation are called asymptotically de Sitter (dS), flat and anti-de Sitter (AdS), respectively.

**The background metric.** In this paper we use the solution of the Einstein equation with negative cosmological constant ($\Lambda<0$) in cylindrical symmetry. In other words we consider asymptotically anti-de Sitter (AdS) case. In general the spacetime metric for d-dimensional AdS spacetime was generalized by Lemos with multiple rotation parameters and has been written as following [5]:

$$ds^2 = -f(r)\left(\Xi dt - \sum_{i=1}^{n} a_i d\varphi_i\right)^2 + \frac{r^2}{l^4}\sum_{i=1}^{n}(a_i dt - \Xi l^2 d\varphi_i)^2 + \frac{dr^2}{f(r)} - \frac{r^2}{l^2}\sum_{i<j}^{n}(a_i d\varphi_j - a_j d\varphi_i)^2 + r^2 d\Omega^2 \qquad (1)$$

Where $\Xi = \sqrt{1 + \sum \frac{a_i^2}{l^2}}$, $n = \left[\frac{d-1}{2}\right]$ is the integer part of the fraction $\frac{d-1}{2}$ and it represents number of rotation parameters ($a_i$). $d\Omega^2$ is Euclidean metric on $(d-n-2)$-dimensions with manifold $\omega_{d-2}$ and its form is as following $d\Omega^2 = dx^k dx^k$ with $k = 1, \ldots, d-n-2$ . Moreover, here $f(r)$ for the noncharged black hole is [5]

$$f(r) = \frac{r^2}{l^2} - \frac{M}{r^{d-3}}, \tag{2}$$

Here $a_i$ is rotation parameter and $M$ is the mass per unit length of $z$-line. The parameter $l$ is one with dimension of length which related to the cosmological constant $\Lambda$ and dimension $d$ as $\Lambda = -\frac{(d-1)(d-2)}{2l^2}$ [5],[6]. The range of coordinates $\varphi_i$ are assumed to be $\varphi_i \in [0, 2\pi]$.

**Energy extraction by Penrose process.** It is known that in four dimension black hole has only one angular momentum with respect to rotation axis and only one rotation parameter corresponding to this momentum. In the case of higher dimension as it has been stated in [3] there are several angular momentums and each momentum corresponds its rotation parameter. Consequently, by the reason of multiple parameters and variables studying the black hole will be complicated. That is why we consider one of the simplest case which the black hole has only one angular momentum and only one rotation parameter corresponding to this angular momentum. According to this approximation AdS spacetime metric for the d-dimensional cylindrical black hole (1) takes a form

$$ds^2 = -f(r)(\Xi dt - a d\varphi)^2 + \frac{r^2}{l^4}(a dt - \Xi l^2 d\varphi)^2 + \frac{dr^2}{f(r)} - \frac{r^2}{l^2}dz^2 + r^2 d\Omega_{d-4}^2, \tag{3}$$

Where $\Xi = \sqrt{1 + \frac{a^2}{l^2}}$, the coordinates $\varphi$ and $z$ change in $0 \le \varphi \le 2\pi$ and $-\infty < z < \infty$, respectively. As well as $\Omega_{d-4} = \Phi_1, \Phi_2, \Phi_3, \dots, \Phi_{d-4}$ or

$$d\Omega_{d-4}^2 = d\Phi_1^2 + d\Phi_2^2 + d\Phi_3^2 + \dots + d\Phi_{d-4}^2, \tag{4}$$

We have already known from [2], [4] that the Penrose process is one theorized by Roger Penrose in 1969 wherein energy can be extracted from a rotating black hole. Energy extraction occurs not inside the event horizon of the black hole, it occurs in the region of ergosphere on account of rotational energy of the black hole. In this process massive particle enters into the ergosphere and splits into two pieces: one of them escapes from the black hole to infinity while the another one falls to the black hole. The escaping piece can possibly have greater energy than the infalling one, by the reason of the infalling piece has negative energy. As a result of this process black hole reduces its angular momentum and consequently black hole extracts energy. It derives from signature of energies of two pieces that the escaping particle has more energy than one which entered into ergosphere.

If particle enters into ergosphere of the black hole and splits into two: and pieces. We have considered that the first piece has more energy $E_{(2)}$ than the incident particle and the first piece of it and exits ergosphere while the first piece is falling into the black hole with negative energy $E_{(1)}$ [2], [4], i.e. according to the law of conservation of energy

$$E_{(0)} = E_{(1)} + E_{(2)} \tag{5}$$

Where $E_{(1)} < 0$, then $E_{(2)} > E_{(0)}$.

$$v = \frac{dr}{dt}, \qquad \Omega = \frac{d\varphi}{dt}, \tag{6}$$

Where $v$ and $\Omega$ are the radial and angular velocity of the particle with respect to asymptotic infinity observer.

We know that the penrose process extracts energy from the rotating black hole by the reason of decreasing black hole's angular momentum. From the conservation of energy and angular momentum [4]

$$E = -p^t A, \qquad L = p^t \Omega, \qquad A = g_{tt} + \Omega g_{t\varphi}. \tag{7}$$

From Hamilton-Jacobi equation for the timelike geodesics $p^\mu p_\mu = -m^2$, one can obtain

$$g_{tt}\dot{t}^2 + g_{rr}\dot{r}^2 + g_{\varphi\varphi}\dot{\varphi}^2 + 2g_{t\varphi}\dot{t}\dot{\varphi} = -m^2, \tag{8}$$

Dividing both sides of (8) by $t^2$ and using (6) and (7)

$$g_{tt} + g_{rr}v^2 + g_{\varphi\varphi}\Omega^2 + 2g_{t\varphi}\Omega = -m^2\left(-\frac{A}{E}\right)^2, \tag{9}$$

As you see from the expression (9) right hand side of it is negative or zero and the second term in left hand side is positive. This is why we can write (9) in the following form:

$$g_{\varphi\varphi}\Omega^2 + 2g_{t\varphi}\Omega + g_{tt} = -m^2\left(-\frac{A}{E}\right)^2 - g_{rr}v^2 \le 0\,. \qquad (10)$$

From the inequality (10) one can find the value of $\Omega$ is in the range $\Omega^- \le \Omega \le \Omega^+$. Here $\Omega^\pm$ is

$$\Omega^\pm = -\frac{g_{t\varphi}}{g_{\varphi\varphi}} \pm \sqrt{\frac{g_{t\varphi}^2}{g_{\varphi\varphi}^2} - \frac{g_{tt}}{g_{\varphi\varphi}}} = \frac{Mar^{1-d}\sqrt{1+\frac{a^2}{l^2}} \pm \sqrt{\frac{1}{l^2} - Mr^{1-d}}}{1+Ma^2r^{1-d}}\,, \qquad (11)$$

Using the expression (7) equations of conservation of the energy (5) can be written as [3]

$$p^t_{(0)}A_{(0)} = p^t_{(1)}A_{(1)} + p^t_{(2)}A_{(2)}\,, \qquad (12)$$

$$p^t_{(0)}\Omega_{(0)} = p^t_{(1)}\Omega_{(1)} + p^t_{(2)}\Omega_{(2)}\,. \qquad (13)$$

In order to find the efficiency of the Penrose process one uses expression (5) and taking into account $E_{(1)} < 0$

$$\eta = \frac{E_{(1)}}{E_{(0)}} = \frac{E_{(2)} - E_{(0)}}{E_{(0)}} = \chi - 1\,, \qquad (14)$$

with $\chi = \frac{E_{(2)}}{E_{(0)}}$, which $\chi > 1$. With the help (7), (12) and (13)

$$\chi = \frac{E_{(2)}}{E_{(0)}} = \frac{(\Omega_{(0)} - \Omega_{(1)})A_{(2)}}{(\Omega_{(2)} - \Omega_{(1)})A_{(0)}}\,. \qquad (15)$$

Here let us consider that incident particle has $E_{(0)} = 1$ energy and it splits into two photons, namely their momentums are equal to zero ($p_{(1)} = p_{(2)} = 0$). In this case as you see from the expression (15) the maximum value of the efficiency of the Penrose process corresponds to the maximum one of $\Omega_{(2)}$ and minimum one of $\Omega_{(1)}$ at the same time. At this time radial velocity of both pieces will be zero ($v_{(1)} = v_{(2)} = 0$), namely

$$\Omega_{(1)} = \Omega^- = \frac{Mar^{1-d}\sqrt{1+\frac{a^2}{l^2}} - \sqrt{\frac{1}{l^2} - Mr^{1-d}}}{1+Ma^2r^{1-d}}\,, \qquad (16)$$

$$\Omega_{(2)} = \Omega^+ = \frac{Mar^{1-d}\sqrt{1+\frac{a^2}{l^2}} + \sqrt{\frac{1}{l^2} - Mr^{1-d}}}{1+Ma^2r^{1-d}} \qquad (17)$$

Due to the radial velocity $v = 0$, the equation (9) takes the form

$$\left(g_{\varphi\varphi} + g_{t\varphi}^2\right)\Omega^2 + 2g_{t\varphi}(1+g_{tt})\Omega + g_{tt}(1+g_{tt}) = 0, \qquad (18)$$

From the equation (18) one can find the angular velocity of the incident particle as

$$\Omega_{(0)} = \frac{-g_{t\varphi}(1+g_{tt}) + \sqrt{(1+g_{tt})\left(g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}\right)}}{g_{\varphi\varphi} + g_{t\varphi}^2}\,, \qquad (19)$$

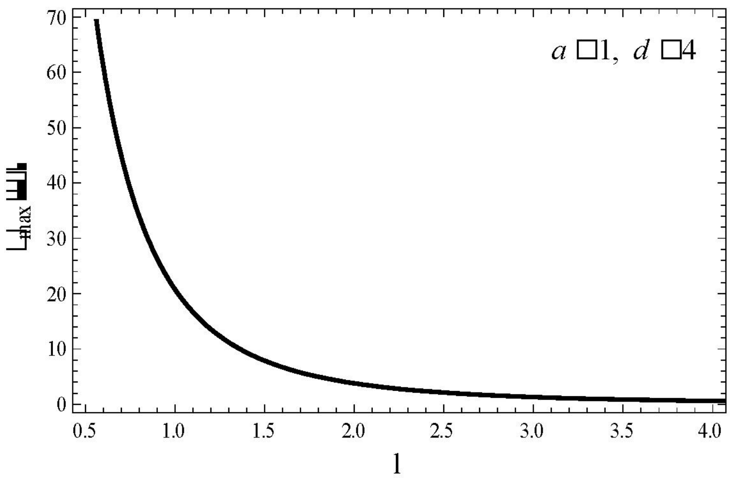


Fig. 1 – The dependence of maximum value of the efficiency of the extracted energy from the black hole by the Penrose process $\eta_{max}$ □ on cosmological parameter $l$

As it has been shown in [2], [4] efficiency of the Penrose process can achieve its maximum value when particle splits into pieces at the event horizon of the black hole. Let us consider the black hole is four dimensional (*d* = 4). Inserting the expressions (16), (17) and (19) into (15) and taking into account above statement we will have

$$\eta_{max} = \left[\frac{\sqrt{1+g_{tt}}-1}{2}\right]_{r=r_+} = \frac{\sqrt{1+a^2 l^{-\frac{8}{3}}}-1}{2}\,. \qquad \textbf{(20)}$$

Where $r_+$ is the radius of the event horizon of the black hole. One can find $r_+$ by solving the equation $f(r) = 0$.

**Conclusion.** In this paper we have studied the efficiency of the extracted energy from the cylindrical AdS black hole by the Penrose process. As a special case the maximum efficiency of the Penrose process for the four dimensional (*d* = 4) black hole has been calculated. It is well known from several papers such as [2], [3] and [6] that the maximum value of the efficiency of the Penrose process for the Kerr black hole is equal to 20,7 % . As you see from the Fig 1. In the current black hole one can achieve the efficiency of the Penrose process up to 70 % by decreasing the value of the cosmological parameter *l* up to 0,5. With increase of the value of the *l* efficiency tends to zero asymptotically.

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### Аннотация


Хорошо известно, что огромное количество энергии можно извлечь из вращающейся черной дыры за счет высокого значения угловой скорости. Для того чтобы обнаружить эту энергию и оценить его величину, были предложены несколько механизмов. Один из эффективных способов извлечения энергии из черной дыры называется процессом Пенроуза. В настоящей работе изучено извлечение энергии из *d*-мерной цилиндрической черной дыры за счет процесса Пенроуза. Предполагается, что черная дыра- незаряженная, вращающаяся и изолированная. Рассчитана эффективность процесса Пенроуза для четырехмерной черной дыры и результаты представлены в виде графика, который описывает зависимость эффективности процесса Пенроуза от космологического параметра *l* для фиксированного значения параметра вращения *a*. Наши результаты показали, что максимальный предел эффективности процесса Пенроуза может быть достаточно высокой около 70 %, тогда как для четырехмерной черной дыры Керра – 20,7 %. Нужно подчеркнуть, что с увеличением значения космологического параметра *l* эффективность процесса Пенроуза асимптотически стремится к нулю.


**Ключевые слова:** *d*-мерная черная дыра, процесс Пенроуза, эргосфера, горизонт событий.