UDC 530.12:531.51

F. ATAMUROTOV [1], U. PAPNOI [2]

([1] Institute of Nuclear Physics, Ulughbek, Tashkent 100214, Uzbekistan,
[2]Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi 110025, India)

SILHOUETTE OF ROTATING BLACK HOLE IN MYERS-PERRY GRAVITY

Abstract. The geodesic equation of motion of photons is presented in the five-dimensional (5D) rotating black hole spacetime with rotation parameter a. We study the radial motion of the photons which depends on the parameters of the spacetime metric to visualize the shape of the shadow for five-dimensional Myers-Perry black hole defining observable radius R_s. We have found that the shape of the shadow depends on the rotation parameter as well as on the angle of inclination of observer.

Keywords: photon motion, silhouette of black hole, shadow of black hole, Myers-Perry spacetime.

Ключевые слова: движение фотона, силуэт черной дыры, тень черной дыры, пространство-время Майерс-Перри.

I. Introduction. Black holes are very interesting gravitational, as well as geometric, objects which may exist in multidimensional spacetimes. Other interesting axisymmetric object is the five dimensional(5D) supergravity black hole [1,2], which is an impotant solution of supergravity Einstein-Maxwell equation. Recently, a charged black hole solution in the limit of slow rotation was constructed in [3]. Also, charged rotating black hole solutions have been discussed in the context of supergravity and string theory [4]. The solution obtained by Chong et. al. [1] of minimal gauged supergravity theory comes closest to Kerr-Newman analogue. Energetics of a rotating charged black hole in 5-dimensional supergravity spacetime has been studied in [5] where energy extraction even for axial fall has been predicted.

Despite the fact that a black hole is not visible, it may be observable nonetheless since it may create a shadow if it is in front of a bright background. The apparent shape of an extremely rotating black hole was first studied by Bardeen [6]. The closed photon orbits in general Kerr-Newman spacetimes were analytically studied in Ref. [7], even in cases where he so-called cosmic censorship is violated. It is strongly believed that the observability of black hole shadows in the near future is very realistic [8]. Recently, great interest emerged especially for the observability of the black hole in the center of Milky Way, Sgr A* [9]. The shadow of Schwarzschild [10], rotating non-Kerr [11], rotating black hole with gravitomagnetic charge[12], rotating Horava-Lifshitz black hole[13] and other spherically symmetric black holes [14] have been intensively studied. The shadow cast by a rotating braneworld black hole was studied in Ref. [15] in the Randall-Sundrum scenario.

The paper is organized as follows. In Sec. II, we review the basic aspects of the geometry and the geodesics around the 5D black hole. In Sec. III, we obtain the shadows of black holes with different values of the black hole's spin parameter *a*. Finally, in Sec. IV we discuss the results obtained.

II. PARTICLE MOTION AROUND 5D BLACK HOLE

The five-dimensional rotating black hole is described by the Myers-Perry metric [2]:

$$ds^2 = -dt^2 + \frac{\rho^2}{4\Delta}dr^2 + \rho^2 d\theta^2 + (r^2+a^2)\sin^2\theta d\phi^2 + (r^2+b^2)\cos^2\theta d\psi^2$$
$$+\frac{4M^2}{\rho^2}(dt + a\sin^2\theta d\phi + b\cos^2\theta d\psi)^2, \tag{1}$$

$$\Delta = \frac{(r^2+a^2)^2}{r^2} - 2Mr \ , \ \rho^2 = r^2 + a^2\cos^2\theta + b^2\sin^2\theta \ . \tag{2}$$

Here a, b are angular momentum per total mass of black hole M. In the case $a = b$.

Consider a black hole placed between a source of light and an observer. Then the light reaches the observer after being deflected by the black hole's gravitational field. But some part of the deflected light with small impact parameters can be emitted by the source falling into the black hole. This phenomena result a dark figure in the map of the space called the shadow. The boundary of this shadow defines the shape of a black hole (see e.g.[11]). The study of the geodesic structure around black hole is very important to obtain the apparent shape. The Hamilton-Jacobi equation determines the geodesics for a given space-time geometry:

$$\frac{\partial S}{\partial\sigma} = -\frac{1}{2}g^{\alpha\beta}\frac{\partial S}{\partial x^\alpha}\frac{\partial S}{\partial x^\beta} \tag{3}$$

with σ is an affine parameter along geodesics, the action can be written as following:

$$S = \frac{1}{2}m^2\sigma - Et + L_\varphi\varphi + L_\psi\psi + S_\theta + S_r, \tag{4}$$

where m is the mass of a test particle. E, L_φ and L_ψ are conservation energy and angular momentum of the particles. In the case of null geodesics, we have that $m = 0$, and from the Hamilton-Jacobi equation, the following equations of motion are obtained:

$$\rho^2\frac{\partial t}{\partial\sigma} = E\rho^2 + \frac{2(r^2+a^2)^2}{\Delta}\Sigma, \tag{5}$$

$$\rho^2\frac{\partial\varphi}{\partial\sigma} = \frac{L_\varphi}{\sin^2\theta} - \frac{2a(r^2+a^2)^2}{\Delta}\Sigma, \tag{6}$$

$$\rho^2\frac{\partial\psi}{\partial\sigma} = \frac{L_\varphi}{\cos^2\theta} - \frac{2a(r^2+a^2)^2}{\Delta}\Sigma, \tag{7}$$

$$\rho^2\frac{\partial r}{\partial\sigma} = 2\sqrt{R}, \tag{8}$$

$$\rho^2 \frac{\partial \theta}{\partial \sigma} = 2\sqrt{\Theta}, \tag{9}$$

where the functions $R(r)$ and $\Theta(\theta)$ are following as:

$$R(r) = \Delta(2a^2E^2 - K) + 2(r^2 + a^2)^2\Sigma^2, \tag{10}$$

$$\Theta(\theta) = E^2a^2 - \frac{\cos^2\theta}{\sin^2\theta}L_\varphi^2 - \frac{\sin^2\theta}{\cos^2\theta}L_\psi^2, \tag{11}$$

where K as a constant of separation.

Every orbit can be characterized by three impact parameters, which can be expressed in the terms of constant of motion E, L_φ, L_ψ and the Carter constant K. Combining these quantities, we define as usual $\xi = L_\varphi / E, \zeta = L_\psi / E$ and $\eta = K / E^2$, which are impact parameters for general orbits around the black hole. We use Eq. (10) to derive the orbits with constant r in order to obtain the boundary of the shadow of the black hole. These orbits satisfy the conditions $R(r) = 0 = dR(r)/dr$, which are fulfilled by the values of the impact parameters that determine the contour of the shadow, therefore,

$$\xi + \zeta = \frac{r^6 + 3a^2r^4 + 3r^2a^4 - 4r^4 + a^6}{a(r^4 - a^4)}, \tag{12}$$

$$\eta = \frac{8r^{10} + 18a^2r^8 + 8a^4r^6 - 16r^8 - 4a^6r^4 + 2a^{10}}{(r^4 - a^4)^2}. \tag{13}$$

III. SHADOW OF BLACK HOLE

Adopting the celestial coordinates is very convenient to describe the shadow [16]:

$$\alpha = \lim_{r_0\to\infty}(-r_0^2(\sin^2\theta_0 \frac{d\varphi}{dr} + \cos^2\theta_0 \frac{d\psi}{dr})), \tag{14}$$

$$\beta = \lim_{r_0\to\infty} r_0^2 \frac{d\theta}{dr}, \tag{15}$$

since here an observer far away from the black hole is considered $r_0 \to \infty$, θ_0 is the angular coordinate of the observer, i.e. the inclination angle between the rotation axis of the black hole and the line of sight of the observer.

Calculation $d\varphi / dr$, $d\psi / dr$ and $d\theta / dr$ using the spacetime metric given by expression (1) and taking the coordinate's limit of a far away observer, one can obtain celestial coordinates functions of the constants of motion in the form:

$$\alpha = -\left(\xi \frac{1}{\sin\theta_0} + \zeta \frac{1}{\cos\theta_0}\right), \tag{16}$$

$$\beta = \sqrt{\eta - \xi^2 \cot^2\theta_0 - \zeta^2 \tan^2\theta_0 + a^2}, \tag{17}$$

We show the shadow of a 5D rotating black hole using Eqs.(16)-(17). In fig. 1 we plot β Vs α to show the contour of the shadow of the BHs for different values of rotation parameters a ($a = 0.5(solid line)$, $a = 1(dashed line)$, $a = 1.2(dot-dashed line)$, $a = 1.4(dotted line)$), at different inclination angles θ_0 (θ_0 =0(left panel) θ_0 =45(right panel)).

Conclusion. In this paper we have studied the optical features of rotating Myers-Perry black hole and have analyzed how the shadow of the black hole is distorted by the presence of the a parameter. The shape of the shadow of the black hole was affected by the a parameter that is with increasing a parameter the radius of the shadow decrease.

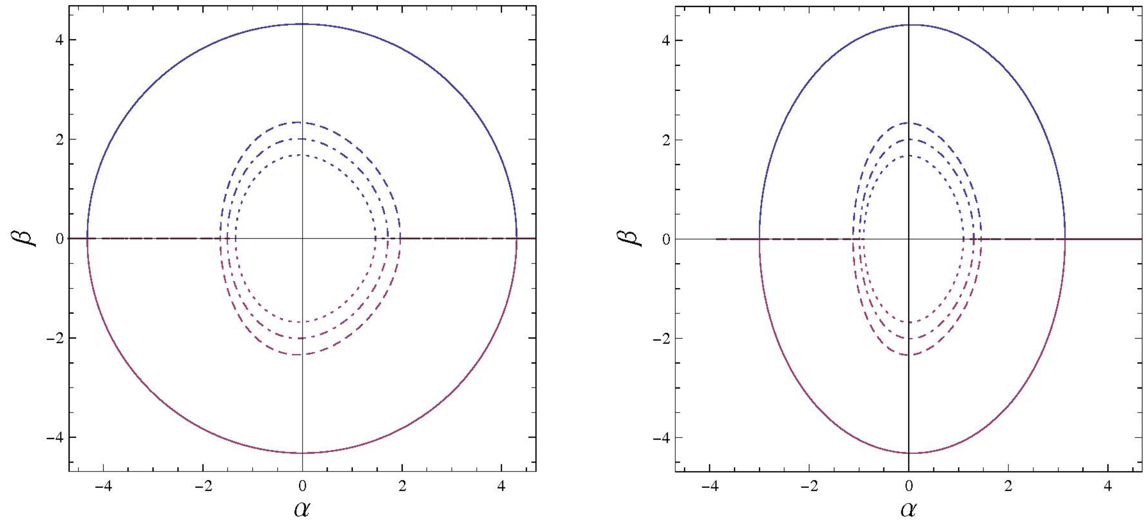

Fig. 1. Silhouette of rotating Myers-Perry black hole for different value of spin parameter and inclination angle

REFERENCES

1 Z. Chong, M. Cvetic, H. Lu and C. N. Pope, Phys. Rev. Lett. 95d, 161301 (2005).
2 R. C. Myers and M. J. Perry, Annals Phys. 172, 304 (1986).
3 A.N.Aliev, Phys. Rev. D 74, 024011 (2006).
4 M.Cvetic and D.Youm, Phys. Rev. D 54, 2612 (1996).
5 K. Prabhu and N. Dadhich, Phys. Rev. D 81, 024011 (2010).
6 J. M. Bardeen, in Black Holes, Proceedings of the Les Houches Summer School, Session 215239, edited by C. DeWitt and B. S. DeWitt (Gordon and Breach, New York, 1973).
7 A. de Vries, Classical Quantum Gravity 17, 123 (2000).
8 K. Hioki, and K. I. Maeda, Phys. Rev. D 80, 024042 (2009).
9 H. Falcke, F. Melia, and E. Agol, Astrophys. J. 528, L13 (2000)
10 C. Darwin, Proc. R. Soc. A 249, 180 (1959).
11 F. Atamurotov, A. Abdujabbarov, B. Ahmedov, Phys. Rev. D 88, 064004 (2013).
12 A. Abdujabbarov, F. Atamurotov, Y. Kucukakca, B. Ahmedov, and U. Camci, Astrophys. Space Sci. 344, 429 (2013).
13 F. Atamurotov, A. Abdujabbarov, B. Ahmedov, Astrophys. Space Sci. 348, 179 (2013).
14 V. Bozza, Phys. Rev. D 66, 103001 (2002).
15 L. Amarilla and E. Eiroa, Phys. Rev. D 85, 064019 (2012).
16 S. Vazquez and E. Esteban, Nuovo Cimento Soc. Ital. Fis. 119B, 489 (2004).

Резюме

Представлено геодезическое уравнение движения фотонов в пространстве-времени пятимерной (5D) вращающейся черной дыры с ненулевым параметром вращения. Изучено радиальное движение фотонов, которое зависит от параметров метрики пространстве-времени с целью получения отображения фигуры тени для пятимерной черной дыры Майерс-Перри, которое определяется через наблюдаемый радиус R_s. Показано, что форма тени зависит от параметра вращения, а также от угла наклона наблюдателя.

Ключевые слова: движение фотона, силуэт черной дыры, тень черной дыры, пространство-время Майерс-Перри.