KIYOSHI KATO[1], YUMA KIKUCHI[2], MASAYUKI AIKAWA[1],
MERUERT TAKIBAYEVA[3;4] NURGALI TAKIBAYEV[3]

[1]Nuclear Reaction Data Centre (JCPRG), Faculty of Science, Hokkaido University, Sapporo 060-0810,
[2]Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki 567-0047, Japan
[3]Central Asian Nuclear Reaction Data Base, Almaty 050-040, Kazakhstan
[4]Graduate School of Science, Hokkaido University, Sapporo 060-0810, Japan

RESONANCE STRUCTURES IN THE (α–α)+*n* SYSTEM

Abstract. We study the resonance structures in the low-energy (*α-α*)+*n* scattering. For this purpose, the (*α-α*)+*n* coupled-channel orthogonality condition model is explained. Reso-nance states are solved by applying the complex scaling method.

1. Introduction. Recently, it has been discussed that three-body resonances of the two-heavy-nuclei plus one-neutron system play important roles in high-density matter like neutron stars [1]. Such three-body resonances strongly depends on the structure of two heavy nuclei, and they are called as "the structural neutron resonances". In the ^{9}Be nuclei, a very interesting resonant state has been observed at the very low energy just above ^{8}Be+*n* threshold [2].

For this resonant state, many experimental and theoretical studies have been performed so far. However, a clear understanding of resonance mechanism and structure has not yet been obtained [3, 4].

Recently a new observation of the ^{9}Be(*γ, n*) reaction cross section has been reported by Arnold *et al.* [5]. This new data suggest a ^{8}Be+*n* resonance, but indicate inconsistent with old data [6]. It is desired to study the photo-dissociation of ^{9}Be theoretically. Using the reliable model which can explain the experimental data, it is interesting to investigate the resonance mechanism and the structural resonances because the ^{8}Be+*n* system is also considered as one example of the two-heavy-nuclei plus one-neutron systems.

To understand the low-energy resonances and the structural neutron resonance in the (*α-α*)+*n* system, we try to investigate the (*α-α*)+*n* coupled-channel model using the *α-α* and *α-n* interactions constructed microscopically. In the present model, we calculate neutron resonances coupled with vibration and rotation of the two-alpha system. To calculate the photo-disintegration cross section and resonant poles of the S-matrix, we apply the complex scaling method to the (*α-α*)+*n* coupled-channel model.

In section 2, the present method is brie y explained. Section 3 is devoted to some comments on numerical calculations. Summary is given in section 4.

2. The model (*α – α*)+*n* coupled-channel OCM. We assume an $\alpha + \alpha + n$ model for description of low-lying states in ^{9}Be, because the ^{8}Be nucleus has been understood to be well explained by the $\alpha + \alpha$ model. Experimentally, the ^{9}Be nucleus has been observed to have a small binding energy (1.57 MeV) from the $\alpha + \alpha + n$ threshold [2]. It is also known that every subsystem of $\alpha + \alpha$ and $\alpha + n$ in the $\alpha + \alpha + n$ system has no bound state. Therefore, we can call this three-body $\alpha + \alpha + n$ model as one of the Borromean systems.

In this model, we consider the *α* cluster as a point particle because excitations of the *α* cluster is not treated explicitly. However, the anti-symmetrization between nucleons in the *α* clusters and valence neutron is taken into account by the orthogonality condition model (OCM)[7]. In OCM, the relative motion between clusters is solved for the corresponding wave function so as to be orthogonal to the Pauli forbidden states, which are de ned the harmonic oscillator wave functions with $(N, L) = (0, 0)$, $(2, 0)$, and $(0, 2)$ and N=odd for the $\alpha + \alpha$ system, and with $(N, L) = (0, 0)$ for the $\alpha + n$ system.

The subsystems of $\alpha + \alpha$ and $\alpha + n$ in the $\alpha + \alpha + n$ system have been well studied experimentally and theoretically so far. We can employ the reliable interactions $V_{\alpha\alpha}$ and $V_{\alpha n}$ constructed on the basis of microscopic studies for $\alpha + \alpha$ and $\alpha + n$ systems, respec-tively. Using this model, we study the (*α-α*)+*n* scattering problem. The Hamiltonian for the (*α-α*)+*n* model is expressed as

$$H = h(\vec{\xi}) - \frac{\hbar^2}{2\mu}\nabla_r{}^2 + V_{\alpha n}[\![(\vec{r})_1]\!]), \quad h(\vec{\xi}) = -\frac{\hbar^2}{4M}\nabla_e{}^2 + V_{\alpha\alpha}(\xi)$$

where $h(\vec{\xi})$ is the internal Hamiltonian between two α's and μ =8/9 M the reduced mass between 2α and a neutron. The vectors ξ, $\vec{r}_{\square}, \vec{r}_1$ *and* $\vec{r}_1$ present the coordinates of α-α, (2α)-n, α_1-n and α_2-n, respectively.

The Schrodinger equation $H = E$ is solved variationally by employing the appropriate basis functions. The wave function $\Psi^{J\pi}$ is expressed as

$$\Psi^{J\pi} = \sum_{c,\alpha,\beta} A^{J\pi}_{c,\alpha,\beta}\, \phi^{\alpha}_{\xi}(\xi)\phi^{\beta}_{c}(r)\phi^{\alpha}_{c}(\hat{\xi},\hat{r}), \qquad h_c(\hat{\xi},\hat{r}) = \left[Y_{l_c}(\hat{\xi}), Y_{l_c}(\hat{r})\right]^J$$

$\phi^{\alpha}_{\xi}(\xi), \phi^{\beta}_{c}(r)$ and $h_c(\hat{\xi},\hat{r})$ are relative wave functions of α-α and (2α)-n and the channel wave function, respectively. The subsystem of α+α is described by the following Schrodinger equation:

$$h(\vec{\xi})\left[\phi^{\alpha}_{\xi}(\xi)Y_{l_c}(\hat{\xi})\right] = \epsilon^{c}_{\alpha}\left[\phi^{\alpha}_{\xi}(\xi)Y_{l_c}(\hat{\xi})\right]$$

To investigate resonances and photo-disintegration cross sections, we apply the complex scaling method [8], which is defined by the transformations of coordinates and conjugate momenta

$$U(\theta); \qquad \vec{r}_i \to \vec{r}_i e^{i\theta} \text{ and } \vec{k}_i \to \vec{k}_i e^{i\theta} \text{ for } i=1,2.$$

The degrees of freedom, i = 1 and 2, of the system correspond to the relative motions of α-α and (2α)-n, respectively. The scaling parameter θ is a real number of $0 \le \theta < \theta_{max}$, where θ_{max} is determined from analyticity of the interactions. The complex scaled Schr•odinger equation is solved by diagonalization of the complex scaled Hamiltonian. The wave function of this system is described as an expansion of a basis set of L^2 functions.

The eigenvalues and eigenstates of the complex scaled Schr•odinger equation are classied as

$$(E_\alpha, \Psi^\alpha) = \begin{cases} (E_B, \Psi^B) & B = 1,\dots,N_B \\ (E_R, \Psi^R) & R = 1,\dots,N_R \\ (E_c(\theta), \Psi^c) & c = 1,\dots,N - N_B - N_R(\theta) \end{cases}$$

where N_B and $N_R(\theta)$ are the number of bound states and the θ-dependent number of resonant states, respectively. The energies of resonant states are independent of $\theta > \tan^{-1}(^R/2E_r^R)$ because of the intrinsic quantities of the system; $E_R = E_r^R - i\,^R$. But the discretized energies $E_c(\theta)$ of continuum states are θ-dependent and expressed as E_c = ى$_r$ − iى$_i$. These eigenstates have been shown to construct an extended completeness relation [15].

The photo-disintegration cross section of the $E1$ transitions is calculated as

$$\frac{\frac{d\sigma}{dE} = \frac{16\pi^3}{9\hbar c} N_E(E_\gamma)\big(dB(E)\big)}{dE}$$

where $N_{E1}(E_\gamma)$ is the virtual photon number and

$$\frac{dB(E)}{dE} = \frac{1}{2J_{gr}+1}\sum_{\nu}\langle\tilde{\Psi}_{gr}|\hat{E}_1|\Psi_\nu\rangle\langle\tilde{\Psi}_\nu|\hat{E}_1|\Psi_i\rangle\delta(E-E_\nu)$$

$$= \frac{1}{\pi}\mathrm{Im}\left[\frac{1}{2J_{gr}+1}\sum_{\nu}\frac{\langle\tilde{\Psi}_{gr}|\hat{E}_1(\theta)|\Psi_\nu\rangle\langle\tilde{\Psi}_\nu|\hat{E}_1(\theta)|\Psi_i\rangle}{E-E_\nu}\right]$$

The last equation is calculated with CSM, where $^{\theta}{}_{\nu}$ are solutions belonging to the eigen-energy E_ν^θ and satisfy the outgoing boundary condition. It should be noted that the result is independent of the parameter θ [16, 17].

3. Summary. We explained our approach of the (α-α)+n coupled-channel orthogonality condition model. The aim of study is to understand the resonance structure of the (α-α)+n system. The resonances of the (α-α)+n sys-tem are expected to play important roles in the equation of states of neutron stars and the nucleosynthesis in stars.

In this article, a brief explanation of the model and some needs for numerical calcu-lations are given. Resonances and photo-disintegration cross sections are calculated by applying the complex scaling method [13, 14].

REFERENCES

1 Takibayev N. and Kato K., Takibayeva M., Sarsembayeva A. and Nasirova D., Few-Body Systems, 54, 1587 (2013).
2 TUNL Nuclear Data Evaluation, http://www.tunl.duke.edu/nucldata/
3 Efros V.D. and Bang J.M., Eur. Phys. J. A 4, 33 (1999).
4 Garrido E., Fedorov D.V. and Jensen A.S., Phys. Letters B 684, 132 (2010).
5 Arnold C.W., Clegg T.B., Iliadis C., Karwowski H.J., Rich G.C. and Tompkins J.R., Phys. Rev. C 85, 044605 (2012).
6 Utsunomiya H., Yonezawa Y., Akimune H., Yamagata T., Ohta M., Fujishiro M., Toyokawa H., 420 and H. Ohgaki, Phys. Rev. C 63, 018801 (2000).
7 Saito S., Prog. Theor. Phys. 40, 893 (1968); 41, 705 (1969); Prog. Theor. Phys. Suppl. 62, 11 (1977).
8 Aguilar J. and Combes J.M., Commun. Math. Phys. 22, 269 (1971); Balslev E. and Combes J.M., Commun. Math. Phys. 22, 280 (1971).
9 Kukulin V.I., Krasnopol'sky V.M., Voronchev V.T. and Sazonov P.B., Nucl. Phys. A417, 128 (1984).
10 Hiyama E., Kino Y. and Kamimura M., Prog. Part. Nucl. Phys. 51, 223 (2003).
11 Kurokawa C. and Kato K., Nucl. Phys. A 792 (2007), 87.
12 Kanada H., Kaneko T., Nagata S. and Nomoto M., Prog. Theor. Phys. 61, 1327 (1979).
13 Aoyama S., Myo T., Kato K. and Ikeda K., Prog. Theor. Phys. 116, 1 (2006).
14 Horiuchi H., Ikeda K. and Kato K., Prog. Theor. Phys. Suppl. 192, 1, (2012).
15 Myo T., Ohnishi A. and Kato K., Prog. Theor. Phys. 99, 801 (1998).
16 Myo T., Kato K., Aoyama S. and Ikeda K., Phys. Rev. C 63, 054313 (2001).
17 Kikuchi Y., Myo T. and Kato K. and Ikeda K., Phys. Rev. C 87, 034606 (2013).