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DESTROYING A NEAR-EXTREMAL KERR BLACK HOLE WITH A CHARGED PARTICLE

Abstract. We investigate the process of destroying a near-extremal Kerr black hole event horizon by throwing charged particles. In this paper, we show that charged particle having angular momentum and energy can turn Kerr black hole into near Kerr-Newmann naked singularity in the sense of neglecting back reaction effects.
Keywords: Near-extremal black hole; naked singularity.
Ключевые слова: Около экстремальная черная дыра; голая сингулярность.

Introduction. Recently, Jacobsan and Sotirio (JS) have shown that it is possible to destroy the rotating black holes by the infalling of non-spinning test particles across the event horizon and rotating black holes could be spun up past the extremal limit in the sense of neglecting the backreaction effects [1, 2]. Authors of the paper [3] have shown that radiation reaction effects can prevent the formation of naked singularities only for some of the orbits for non-spinning particles around almost extremal Kerr balck holes.

Black holes are important objects in the Theory of General Relativity. Among the black hole types, those with axial symmetry are considered more realistic. The most important solution with this symmetry is the Kerr metric [4], which is characterized by its mass M and angular momentum J. For angular momentum greater than the "extremal" limit GM^2/c, the Kerr metric has no horizon and has a naked singularity. In general relativity, a naked singularity is a gravitational singularity without an event horizon although occurrence of naked singularities is an unanswered question in the reality. The theoretical existence of naked singularities is important because their existence would mean that it would be possible to observe the collapse of an object to infinite density. It would also cause foundational problems for general relativity, because general relativity cannot make predictions about the future evolution of space-time near a singularity. Thus, the investigation of naked singularity formation in collapse models has been attracted much attention in the recent years [5–8].

The possibility of destroying the event horizon by pushing test particles towards an extremal Kerr black hole is elegantly described by Jacobson and Sotiriou [1]. Here we briefly reproduce and try to approach this issue from the slightly different perspective. We investigate the near-extremal Kerr black hole by throwing charged test bodies across the black hole event horizon. Here we show that charged test body having angular momentum and energy can turn Kerr black hole into near Kerr-Newmann naked singularity in the sense of neglecting backreaction effects.

Throughout this paper we use a system of units in which $G = c = 1$.

NEAR-EXTREMAL KERR BLACK HOLE

We consider a charged particle with a appropriate values of energy $\delta E << M$ and angular momentum $\delta J << J$ around a near extremal Kerr black hole. Based on the laws of black hole thermodynamics the infalling object changes the black hole's mass and intrinsic angular momentum. Assuming that the energy emitted and the radiation reaction are negligible, the charged particle moves along a test particle geodesic. From being captured the charged particle the final parameters of black hole remain with total mass $M + \delta E$, total angular momentum $J + \delta J$ and charge e. We consider the following expression in order to form a near Kerr-Newman naked singularity

$$(\mathrm{M} + \delta\mathrm{E}) < \left(\frac{\mathrm{J} + \delta\mathrm{J}}{\mathrm{M} + \delta\mathrm{E}}\right)^2 + \mathrm{e}^2, \tag{1}$$

hence, we obtain the lower bound of angular momentum, aligned with the spin of the black hole, of the charged particle

$$\delta J > \delta J\min = (M^2 - J) + 2M\delta E + \delta E^2 - \frac{e^2}{2}. \quad (2)$$

Note that the black hole must start out very close to extremal if the small perturbation caused by the body is to have any chance of pushing it over the extremal limit. For a near extremal rotating black hole the upper bound of angular momentum implies that

$$\delta J < \delta J_{max} = \frac{2Mr_h}{a}\delta E. \quad (3)$$

It is clear that expressions of (2) and (3) can not have the same values simultaneously for the extremal case ($r_h = 1$). In this respect naked singularity can never be produced due to the reason of impossibility of overspinning the black hole. According to expression (3) the charged coming from some point outside object can fall across the horizon.

Since we are considering near extremal case $J/M2 = a/M = 1 - 2□2$ with small dimensionless quantity □<< 1, the minimum and maximum values of the δE and δJ will have the following form by using units with $M = 1$

$$\delta J_{min} = 2\varepsilon^2 + 2\delta E + \delta E^2 - \frac{e^2}{2}, \quad (4)$$

$$\delta J_{max} = (2 + \delta\varepsilon)\delta E. \quad (5)$$

The allowed range of δE is defined as

$$\left(2 - \sqrt{2}\sqrt{1+\left(\frac{e}{2\varepsilon}\right)^2}\right)\varepsilon < \delta E < \left(2 + \sqrt{2}\sqrt{1+\left(\frac{e}{2\varepsilon}\right)^2}\right)\varepsilon. \quad (6)$$

In particular, δE must be of order □, which is consistent with the requirements $\delta E << M$ and $\delta J << J$ that the body make only a small perturbation. For a given δE, the allowed values of δJ are near $2\delta E$, so we must have $\delta J \sim \delta E$.

Here we describe orbital motion of charged particle in the equatorial plane, playing an important role in understanding the essential properties of the dynamics of such body, in order to reveal that whether it falls into the black hole from some point of infinity for the given values of energy δE and angular momentum δJ. The radial coordinate of orbital motion of charged body in the equatorial plane is defined as

$$\frac{\dot{r}^2}{2} + Veff(r, \delta\tilde{E}, \delta\tilde{J}) = 0, \quad (7)$$

where $\delta\tilde{E} = \delta E / m$ and $\delta\tilde{J} = \delta J / m$, and m is the rest mass of the particle.

As already mentioned, the black hole must start out very nearly extremal, but now we can be somewhat more quantitative. Based on (6) we must have $\varepsilon << 1$, and $a\text{–}1 = 2\varepsilon^2$ is parametrically smaller. For example, if $\varepsilon = 10^{-2}$, then the initial black hole must have $a = 0.9998$. In this case, if the body can be treated as a point charged particle, the black hole can indeed be over-spun. Here, let us to make parametrization the range of the specific angular momentum as

$$\delta\tilde{J} = (2 + b\varepsilon)\delta\tilde{E} - (4 - b)\tilde{c}, \quad (8)$$

where $b \in [3, 4]$ and $\tilde{c} = e^2 / 2m$. It is vital to investigate the effective potential of orbital motion around extremal rotating gravitational object so as to seek out the different values of parameters □, m, and c for which $V_{eff} < 0$ everywhere outside the horizon, in which the charged body coming from some point of nfinity is provided with the opportunity to fall into the black hole without reservation. Based on the expression (8), we define effective potential of motion of the charged particle around a near extremal Kerr black hole as follows

$$V_{eff}=-\frac{\delta E^2}{2}[1-\frac{3+bc(b-4)+4b\varepsilon+(4+b^2)\varepsilon^2}{r^2}+\frac{2-2bc(b-4)+4b\varepsilon+4c(4-b)\varepsilon+2(4+b^2)\varepsilon^2}{r^3}+ \\ +\frac{32\lambda-8b\lambda-(4-b)^2\lambda^2}{4r^2}+\frac{4b\lambda-16\lambda+(4-b)^2\lambda^2}{2r^3}]=-\frac{\delta E^2}{2}V_{eff}[b], \qquad (9)$$

where $\lambda = c/\varepsilon$.

Investigating the features of effective potential given in Eq. (7) one can conclude that the shape of the effective potential as function of b can be only altered in the range $3.4641 < b < 4$ where $V_{eff} < 0$ is everywhere negative one outside the horizon, in which case the charged particle is captured all the way into the black hole. Consequently, after capture of the charged particle it is possible to achieve that the final parameters of the black hole are beyond extremal limit in which it can be able to destroy black hole horizon and turn extremal Kerr black hole into near Kerr-Newman naked singularity. From above expression of effective potential one can easily see that the potential is negative at $r = 0$ and $r = \infty$ and has a maximum at near extremal point $r_{\max}$, which is given by

$$r=\frac{3}{3+bc(b-4)}\left[1-bc(b-4)+2\left(\frac{2b[(bc(b-4)-1]}{3+bc(b-4)}+b-c(b-4)\right)\varepsilon+O(\varepsilon^2)\right], \qquad (10)$$

where effective potential is described by $V_{eff}=-\delta E^2\tilde{V}_{eff}(b)/2$. To fall freely across the horizon of black hole the expression *Veff* (b) must be positive at near extremal.

In Fig. 1. the plot of specific angular momentum and energy of the charged particle as a function of charged parameter c for the near extremal Kerr black hole with smaller values of □ has been shown. As can be seen in Fig. 1 (a) and (b) it is essential to investigate the properties of shadowed region corresponding the allowed range of angular momentum and energy of infalling particle. Applying the black hole thermodynamics one can easily see that the changes in the mass and angular momentum of the black hole can only alter in the allowed zone shaded in Fig 1 for the formation of naked singularity. Indeed the contribution of charged particle causing the change in parameters of black hole is limited in the allowed region, in which case it can be able to turn black hole into naked singularity, otherwise beyond such region no singularity is likely to form. In addition, the presence of the allowed region plays an important role to create over-spinning black hole by tossing a charged particle.

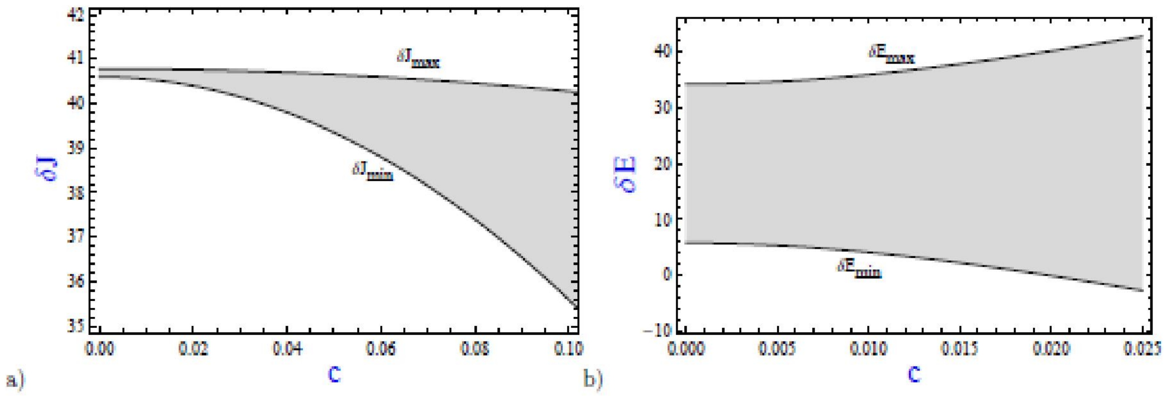

Fig. 1 – Plot of the allowed range of specific angular momentum and energy of the charged particle as a function of charged parameter c for the near extremal Kerr black hole with smaller values of ε

As can be seen in Fig. 2 (a) and (b), the presence of charged particle shifts the shape of the effective potential towards the maximal limit of parameter b, which is significantly different in comparison with the one considered by JS. In addition the point on the dashed curve of the effective potential in plot (a) corresponds to the non-charged body with value of $b = 3.4641$ as mentioned by JS while the line curve corresponds to the charged particle with value of charge parameter. From Fig. 2, one can notice that the shape of the effective potential as function of b can be only altered in the range $b_{\min} < b < b_{\max}$, where $V_{eff} < 0$ is everywhere negative one outside the horizon.

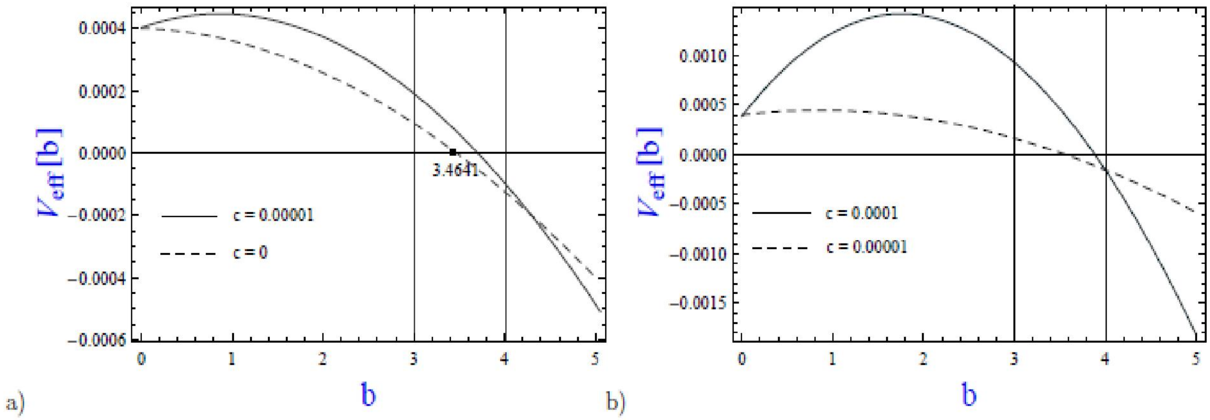

Fig. 2 – The dependence of the effective potential at near extremal point r_{max} on the parametrization parameter b for the different values of charge parameter c

Conclusion. For the near extremal Kerr black hole, we have obtained a general expression of the effective potential for the allowed region of the angular momentum and energy of the charged particle coming from far away black hole. Our results show that the effective potential as function of parametrization parameter b $V_{eff}(b) > 0$ can have its positive values in the range $3.4641 < b < 4$ for which $V_{eff} < 0$ is everywhere negative outside the horizon, in which case the charged particle is captured all the way into the black hole. Consequently, we have found that after capture of the charged particle it is possible to achieve that the final parameters of the black hole are beyond extremal limit, so it can be able to succeed in reaching the over-spinning black hole and turn extremal Kerr black hole into near Kerr-Newman naked singularity.

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Резюме

Исследован процесс разрушения горизонта событий около экстремальной черной дыры Керра при падении на нее заряженных частиц. Показано, что падающие заряженные частицы, имеющие ненулевой угловой момент и энергию, при определенных условиях могут превратить сильно вращающуюся черную дыру в голую сингулярность Керра-Ньюмана.

Ключевые слова: около экстремальная черная дыра; голая сингулярность.