ABSTRACT

to the doctoral thesis for scientific degree Doctor of Philosophy (PhD)

on the specialty 6D060100-Mathematics

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THE COMPARATIVE ANALYSIS OF METHODS, CONVERGENCE AND STABILITY IN IIL-POSED PROBLEMS FOR HYPERBOLIC EQUATIONS

Relevance of thesis theme research: The work is devoted to the construction and sudy of numerical methods for solving the Dirichlet problem for the wave equation as a one-dimensional and two-dimensional.

The Dirichlet problem for the wave equation is one of the most sophisticated models of mathematical physics. The wave equation describes almost all varieties of small vibrations in distributed mechanical systems, such as the longitudinal sound waves in gases, liquids, solids, transverse vibrations in the strings, etc.

One of the fundamental problems of mathematical physics on the plane - the study of vibrating string behavior - is ill-posed, when the boundary conditions are given on the entire boundary, the Dirichlet problem is not well-posed, not only for the wave equation , but also for the general hyperbolic equations. It gives rise to a strong instability with respect to data variations of the inverse problem and a big problem in the construction of approximate solutions, that is, numerical solutions, so far as in practice the input data are known only approximately. These problems have many practical applications, for example, the problem of the tsunami. The fact that modern satellite, above-water and underwater surveillance tool allow to obtain information about the shape of the tsunami waves at different times. These data, together with the appropriate boundary conditions lead to a two-dimensional Dirichlet problem for the wave equations in the shallow water.

The research conducted in the thesis are relevant and have important practical meaning, because in recent years the Dirichlet problem for the wave equation has the new application in the study of tsunami waves.

The object of research. In this work, the object of study is the Dirichlet problem for the wave equation, correctness questions of the problem formulation, the algorithm for constructing numerical solutions.

The subject of research. The subject of the research is ill-posed initial boundary problem for the wave equation, reduced to the inverse problem, optimization methods for solving ill-posed and inverse problems.

The purpose and task of research. The purpose and task of thesis research is to find the solution of the Dirichlet problem for the wave equation and the construction of numerical optimized solving methods; comparative analysis of these methods, their stability and convergence , the development of numerical algorithms of solution of the

considered problem in one-dimensional and two-dimensional cases by the steepest descent method and Landweber iteration method, and construction of functional gradient. Development of software packages for solving the Dirichlet problem for the wave equation. Carry out the comparative analysis of numerical algorithms for the problem.

Research methods. There are numerical methods for solving ill-posed problems, iterative optimization methods, gradient methods, Fourier method, finite-difference methods, Landweber iteration method, steepest descent method in this thesis.

Scientific novelty of research. The following new results were obtained in this thesis:

- the question of correctness of the Dirichlet problem for the wave equation was researched;
- the numerical optimization method was built for solving the Dirichlet problem for the wave equation ;
- numerical algorithm was built for solving the Dirichlet problem for the wave equation in one-dimensional and two-dimensional case by Landweber iteration;
- numerical algorithm was built for solving the Dirichlet problem for the wave equation in one-dimensional and two-dimensional case by the steepest descent method;
- software packages were developed for the solution of the Dirichlet problem for the wave equation;
- constructed the functional gradient for the Dirichlet problem for the wave equation;
- numerical calculations for solving Dirichlet problem for the wave equation in one-dimensional and two-dimensional case were carried for the first time;
- carried out a comparative analysis of the Landweber iteration method and the steepest descent method , the convergence of solutions of the Dirichlet problem for the wave equation.

The theoretical significance of the research. The results of the thesis make a significant contribution to the algorithms construction of numerical methods for solving inverse and ill-posed problems.

The practical significance of the research. The practical significance of this work is that research results can be applied to study the tsunamis phenomena. The developed algorithms allow for creation and improvement of prediction systems of tsunami distribution and forecast consequences in specified regions of the coastal zone.

The structure and scope of work. The thesis consists of an introduction, two chapters, conclusion, list of sources, applications. The work is on 106 pages of typescript, contains 97 figures and 13 tables.