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BOUNDEDNESS AND COMPACTNESS OF MATRIX
OPERATORS IN WEIGHTED SPACES OF SEQUENCES
AND THEIR APPLICATIONS

The abstract of dissertation presented for the scientific degree of the
PhD at specialty of 6D060100- “Mathematics”

Importance of research. One of the main problems in the theory of matrices is to find necessary and sufficient conditions for the elements of a matrix so that the corresponding matrix operator maps continuously one normed space of sequences into another normed space of sequences. Thus it is very important to find the norm of a matrix operator, or at least, an upper or lower bound for the norm. However, in several spaces, which are very important both theoretically and in the applications, such problems have not been solved yet in full generality for operators corresponding to arbitrary matrices. Therefore, in such spaces researchers have considered some specific classes of matrix operators and have established criteria of boundedness and compactness for operators of such classes.

For a summary of results on matrix operators acting in 11 spaces of sequences and on their norms, we refer to the paper of M. Stieglitz, H. Tietz. However, as pointed out in their paper general criteria for the action of a matrix operator from l_p to l_q with $p > 1$, $q > 1$ and for the corresponding norms are not known yet. Such operators have their own self interest and they are also a discrete analogue of integral operators, which play a very important role in functional analysis.

Not only the theory of matrix operators has an important significance, but also different and versatile applications. The present Thesis is dedicated to such important problems.

Object of the investigation. Discrete Hardy type inequalities, weighted spaces of sequences, matrix operators, boundedness, compactness, monotone sequences, summable matrices, composition of operators, additive inequalities.

The aim of the PhD thesis. The purposes of this PhD thesis are to obtain criteria of boundedness and compactness of matrix operators and to estimate their norms in weighted spaces of sequences.

Scientific novelty. In this PhD thesis a triangular infinite dimensional matrix operator is considered, where the elements of this matrix satisfy an even weaker condition, than studied before. We introduce a general class of matrices, and introduce their properties, and give examples. We show that such classes of matrices include well-known classical operators such as the operator of multiple summation, Hölder's operator, Cesàro operator and others. For these classes of matrix operators the following new results are obtained:

- Necessary and sufficient conditions of the boundedness and compactness in terms of the elements of a matrices and weighted sequences of the matrix operators in weighted Lebesgue spaces of sequences in different metrics;
- Necessary and sufficient conditions for the validity of Hardy type inequalities on the cone of non-negative and non-increasing sequences in different metrics;
- Criteria on boundedness and compactness of composition of matrix operators in weighted spaces of sequences;
- we obtain necessary and sufficient conditions for the validity of three-weighted Hardy type inequality in weighted spaces of sequences;
- two-sided estimates for summable matrices in weighted spaces of sequences and on the cone of non-negative and non-increasing sequences.

The general technique of research. The blocking technique method has been used to obtain the main results of the present PhD thesis, which allows to estimate sums on blocks. Various classical inequalities and weighted Hardy inequality have been used in the present PhD thesis.

The theoretical and practical importance. The results of this PhD thesis have theoretical character but several applications can be found both in other areas of mathematics and in applied sciences e.g. in the theory of functions, in the theory of embeddings of discrete weighted spaces such as Sobolev type spaces and in the theory of difference operators.

Publications. The main results of the present PhD thesis are published in 20 publications. Two of them are published in the journals with impact factor. Three of them are published in the publications, which are recommended by the Committee of the educational and scientifically system's control of MES RK. Three of them are published in the foreign publication.

Structure and volume of the dissertation. The present PhD thesis is consists of an introduction, four sections, a final summary and a bibliography including 119 items. The total volume of the PhD thesis is 136 pages.