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**PROBLEM ON THE DISTRIBUTION  
OF THE HARMONIC TYPE RELAY WAVE**

**Abstract.** In this paper, we study the class of flat problems on the effect of moving loads on the surface of a laminated plate. The problems of this class are of great practical interest and in addition, can serve as a benchmark for the development of certain numerical algorithms for solving dynamic problems.

Among various periodic and non-periodic motions of deformable medium, plane waves of simple harmonic type, distributed along the surface of a body or half-plane, whose influence is limited by the vicinity of this surface, are of great importance. Therefore, we consider the problem of the distribution of the relay wave.

**Key words:** stratified plates, live-load, waves of Relay, wave equalization.

The equation of motion of a half-plane material in potentials  $\varphi$ ,  $\psi$  is described by wave equations.

$$\begin{aligned} \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} &= \frac{1}{a^2} \frac{\partial^2 \varphi}{\partial t^2}; \\ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} &= \frac{1}{b^2} \frac{\partial^2 \psi}{\partial t^2}, \end{aligned} \quad (1)$$

where a and b are the distribution speed of the longitudinal and transverse wave, respectively.

We assume that the boundary of the half-plane  $z=0$  is stress-free, i.e.

$$\sigma_{zz} = \sigma_{xz} = 0 \quad (z=0) \quad (2)$$

Let there be an elastic half-plane  $z \leq 0$ .

Suppose that a flat harmonic wave propagates in the medium, i.e. potentials  $\varphi$  и  $\psi$  will be given in the form of [1]

$$\varphi(x, z, t) = \Phi_0(z) \exp[i(pt - qx)] \quad \psi(x, z, t) = \Psi_0(z) \exp[i(pt - qx)] \quad (3)$$

$\Phi_0$  and  $\Psi_0$  satisfies the equations

$$\Phi_0'' - \left( q^2 - \frac{p^2}{a^2} \right) \Phi_0 = 0; \quad \Psi_0'' - \left( q^2 - \frac{p^2}{b^2} \right) \Psi_0 = 0. \quad (4)$$

Considering oscillations decaying with depth  $z \rightarrow -\infty$ , there must be met condition

$$q^2 - \frac{p^2}{a^2} > 0; \quad q^2 - \frac{p^2}{b^2} > 0; \quad (5)$$

But since the speeds  $a$  and  $b$  satisfy the inequality  $a > b$ , it suffices to fulfill one condition instead of conditions (5)

$$\frac{p}{q} < b \quad (6)$$

Therefore, solutions of equations (4), decayed at infinity  $z \rightarrow -\infty$ , have the form

$$\Phi_0(z) = A \exp\left(\sqrt{q^2 - \frac{p^2}{a^2}} \cdot z\right); \quad \Psi_0(z) = B \exp\left(\sqrt{q^2 - \frac{p^2}{b^2}} \cdot z\right), \quad (7)$$

and for potentials  $\varphi$  и  $\psi$  we get expressions

$$\varphi = A \exp\left[i(pt - qx) + \sqrt{q^2 - \frac{p^2}{a^2}} z\right]; \quad \psi = B \exp\left[i(pt - qx) + \sqrt{q^2 - \frac{p^2}{b^2}} z\right], \quad (8)$$

where A and B are arbitrary constants of integration.

Putting solutions (7) into the boundary conditions (2), we obtain

$$A\left[2 - \left(\frac{p}{qb}\right)^2\right] + 2iB\sqrt{1 - \left(\frac{p}{qb}\right)^2} = 0; \quad -2iA\sqrt{1 - \left(\frac{p}{qa}\right)^2} + B\left[2 - \left(\frac{p}{qb}\right)^2\right] = 0. \quad (9)$$

In order for the solution of the problem to be non-zero, it is necessary that the determinant of system (9) be non-zero, i.e. to make the relation [2]

$$\left[2 - \left(\frac{p}{qb}\right)^2\right]^2 - 4\sqrt{1 - \left(\frac{p}{qb}\right)^2} \sqrt{1 - \left(\frac{p}{qa}\right)^2} = 0. \quad (10)$$

The ratio  $(p/q)$  is called the propagation velocity of the relay surface wave.

Denoting  $\xi = \left(\frac{p}{qb}\right)^2$  and introducing the Poisson's ratio  $\nu$ , from relation (10) we obtain the equation for the dimensionless velocity of the relay surface wave  $\sqrt{\xi}$ :

$$\xi^3 - 8\xi^2 + 8\xi \frac{2-\nu}{1-\nu} - 8 \frac{1}{1-\nu} = 0. \quad (11)$$

Equation (11) has a single real positive root [1, 3,4].

If through  $z_1$  и  $z_2$  and designate the depth of penetration at which the amplitude of the voltage drops in  $e$  times due to the longitudinal and transverse wave, respectively, then for them we get the expression

$$z_1 = -\frac{l}{2\pi\sqrt{1-a^{-2}b^2\xi}}; \quad z_2 = -\frac{l}{2\pi\sqrt{1-\xi}},$$

at the same time  $l = \frac{1}{q}$  - the wavelength. For example, with  $\nu = 0,5$  we have

$$z_1 = -\frac{l}{2\pi}; \quad z_2 \approx -\frac{l\sqrt{10}}{2\pi}.$$

Let the normal and tangential load intensity  $-F_1(x+Dt)$  и  $-F_2(x+Dt)$  be distributed on the surface  $z=0$  with constant speed  $D$  i.e. when  $z=0$  we have boundary conditions

$$\sigma_{zz} = -F_1(x+Dt), \quad \sigma_{xz} = -F_2(x+Dt) \quad (12)$$

Initial conditions for such a problems are absent. [2, 47-50].

We introduce moving coordinates

$$x' = x + Dt; \quad y' = y,$$

and the strokes in the future for simplicity will be omitted. Then for potentials  $\varphi$  and  $\psi$  we get the equations

$$\begin{aligned} \alpha^2 \frac{\partial^2 \varphi}{\partial x^2} - \frac{\partial^2 \varphi}{\partial z^2} &= 0; \\ \beta^2 \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2} &= 0; \\ \alpha^2 = (D/a)^2 - 1; \quad \beta^2 = (D/b)^2 - 1. \end{aligned} \quad (13)$$

General solutions of equations (13) are d'Alembert method and have the form

$$\begin{aligned} \varphi(x, z) &= \varphi_1(x + \alpha z) + \varphi_2(x - \alpha z); \\ \psi(x, z) &= \psi_1(x + \beta z) + \psi_2(x - \beta z). \end{aligned} \quad (14)$$

By virtue of the absence of reflected waves from the lower infinitely distant boundary of the function  $\varphi_2$  and  $\psi_2$  should go to zero and for  $\varphi_1$  and  $\psi_1$  from the boundary conditions (12) we obtain the functional relations [5]

$$\begin{aligned} (\beta^2 - 1)\varphi_1''(x) - 2\beta\psi_1''(x) &= -\frac{F_1(x)}{\rho D^2}(\beta^2 + 1)H(x); \\ 2\alpha\varphi_1''(x) + (\beta^2 - 1)\psi_1''(x) &= -\frac{F_2(x)}{\rho D^2}(\beta^2 + 1)H(x). \end{aligned} \quad (15)$$

From relations (15) we get

$$\begin{aligned} \varphi_1''(x) &= \frac{\beta^2 + 1}{\rho D^2} [(\beta^2 - 1)F_1(x) + 2\beta F_2(x)]H(x)\Delta^{-1}; \\ \psi_1''(x) &= \frac{\beta^2 + 1}{\rho D^2} [2\alpha F_1(x) - (\beta^2 - 1)F_2(x)]H(x)\Delta^{-1}; \\ \Delta &= 4\alpha\beta + (\beta^2 - 1)^2. \end{aligned} \quad (16)$$

Using dependencies (16) for stress values, we obtain the expression

$$\begin{aligned}
\Delta \cdot \sigma_{xx} &= -(\beta^2 - 2\alpha^2 + 1)[(\beta^2 - 1)F_1(x + \alpha z) + 2\beta F_2(x + \alpha z)] \times \\
&\times H(x + \alpha z) + 2\beta [2\alpha F_1(x + \beta z) - (\beta^2 - 1)F_2(x + \beta z)]H(x + \beta z), \\
\Delta \cdot \sigma_{zz} &= -(\beta^2 - 1)[(\beta^2 - 1)F_1(x + \alpha z) + 2\beta F_2(x + \alpha z)]H(x + \alpha z) - \\
&- 2\beta [2\alpha F_1(x + \beta z) - (\beta^2 - 1)F_2(x + \beta z)]H(x + \beta z); \\
\Delta \cdot \sigma_{xz} &= -2\alpha [(\beta^2 - 1)F_1(x + \alpha z) + 2\beta F_2(x + \alpha z)]H(x + \alpha z) + \\
&+ (\beta^2 - 1)[2\alpha F_1(x + \beta z) - (\beta^2 - 1)F_2(x + \beta z)]H(x + \beta z),
\end{aligned} \tag{17}$$

$$H(\varsigma) = \begin{cases} 1, & \varsigma \geq 0 \\ 0, & \varsigma < 0 \end{cases},$$

and for shift  $u$  and  $w$  accordingly

$$\begin{aligned}
u &= -\frac{\beta^2 + 1}{\rho D^2 \Delta} [(\beta^2 - 1)F_3(x + \alpha z) + 2\beta F_4(x + \alpha z)]H(x + \alpha z) + \\
&+ \beta \frac{\beta^2 + 1}{\rho D^2 \Delta} [2\alpha F_3(x + \beta z) - (\beta^2 - 1)F_4(x + \beta z)]H(x + \beta z), \\
w &= -\alpha \frac{\beta^2 + 1}{\rho D^2 \Delta} [(\beta^2 - 1)F_3(x + \alpha z) + 2\beta F_4(x + \alpha z)] \times \\
&\times H(x + \alpha z) - \frac{\beta^2 + 1}{\rho D^2 \Delta} [2\alpha F_3(x + \beta z) - (\beta^2 - 1)F_4(x + \beta z)]H(x + \beta z)
\end{aligned} \tag{18}$$

where  $F_3(x) = \int_0^x F_1(\xi) d\xi$ ;  $F_4(x) = \int_0^x F_2(\xi) d\xi$ .

Let it be  $F_2 = 0$  and consider the stress  $\sigma_{xx}$  on the boundary  $z = 0$ . We obtain

$$\sigma_{xx} = F(v, D_0)F_1(x), \quad D_0 = D/a,$$

$$\text{where } F(v, D_0) = \frac{A_1(v, D_0) - A_2(v, D_0)B(v, D_0)}{A_1(v, D_0) - A_2(v, D_0)},$$

$$A_1 = (1 - 2v)^{3/2} \sqrt{(D_0^2 - 1)(1 - v) - (1 - 2v)};$$

$$A_2 = [D_0^2(1 - v) - (1 - 2v)]$$

$$B = [D_0^2(1 - v) - (D_0^2 - 1)(1 - 2v)]$$

Let an elastic layer  $0 \leq z > -h$   $|x| < \infty$  lie on the half-space  $z \leq -h$ , over the surface of which the normal load is distributed, i.e. when  $z = 0$  we have boundary conditions

$$\sigma_{zz}^{(0)} = -F(x + Dt), \quad \sigma_{xz}^{(0)} = 0. \tag{19}$$

The sizes and parameters of the layer will be denoted by the index "0", and the half-space - by the index "1".

At the contact boundary  $z = -h$ , you can set the conditions: hard contact

$$\sigma_{zz}^{(0)} = \sigma_{zz}^{(1)}, \quad \sigma_{xz}^{(0)} = \sigma_{xz}^{(1)}, \quad u_0 = u_1; \quad w_0 = w_1; \quad (20)$$

perfect contact

$$\sigma_{zz}^{(0)} = \sigma_{zz}^{(1)}, \quad \sigma_{xz}^{(0)} = \sigma_{xz}^{(1)} = 0; \quad w_0 = w_1; \quad (21)$$

Can be set other conditions for  $z = -h$ .

In moving coordinates, solutions of equations for potentials in a layer and a half-plane have the form [3, 171-176].

$$\begin{aligned} \alpha_j^2 \frac{\partial^2 \varphi_j}{\partial x^2} - \frac{\partial^2 \varphi_j}{\partial z^2} &= 0; \quad \beta_j^2 \frac{\partial^2 \psi_j}{\partial x^2} - \frac{\partial^2 \psi_j}{\partial z^2} = 0; \\ \left( x' = \frac{x + Dt}{h}; \quad y' = \frac{y}{h}; \quad \varphi_0 = \frac{\varphi_0}{h^2}; \quad \psi_0 = \frac{\psi_0}{h^2} \right). \end{aligned} \quad (22)$$

Putting (22) into the boundary conditions (20), we obtain a system of functional equations which using in expressions for displacements  $u_j, w_j$  and stresses  $\sigma_{ij}$ , we obtain the solution of the problem.

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## ГАРМОНИКАЛЫҚ ТИПТЕГІ РЕЛЕЙ ТОЛҚЫНДАРЫНЫҢ ТАРАЛУЫ ЖАЙЛЫ ЕСЕП

**Аннотация.** Жұмыста қатпарлы пластинкалардың бетіне қозғалмалы жүктемелердің эсері туралы бірнеше жазық есептер класы зеріттеді. Осы типтес динамикалық есептер проблемалары жайлы смәселелерді шешүгө арналған белгілі сандық алгоритмдерді дамытудың негізгі бағыты бола алудымен қызығушылық тудырады. Деформацияланатын ортаның әртурлі периодты және преиодтты емес қозғалыстарының арасында шектелген дененің бетіне немесе жарты жазықтықта тарайтын қарапайым гармоникалық үлгідегі жазық толқындар эсер етеді. Соңдықтан да Релей толқынының таралуын зерттейтін боламыз.

**Түйін сөздер:** қатпарлы пластинкалар, қозғалмалы жүктеме, Релей толқындары, толқындар тендеуі.

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## ЗАДАЧА О РАСПРОСТРАНЕНИИ ВОЛНЫ РЕЛЕЯ ГАРМОНИЧЕСКОГО ТИПА

**Аннотация.** В работе исследуем класс плоских задач о воздействии подвижных нагрузок на поверхность слоистой пластиинки. Задачи данного класса представляют большой прикладной интерес и, кроме того, могут служить эталоном для разработки тех или иных численных алгоритмов для решения динамических задач. Среди различных периодических и непериодических движений деформируемых сред важное значение имеют плоские волны простого гармонического типа, распространяющиеся по поверхности тела или полу-плоскости, влияние которых ограничивается окрестностью этой поверхности. Поэтому рассмотрим задачу о распространении волн Релея.

**Ключевые слова:** слоистые пластиинки, подвижная нагрузка, волны Релея, волновые уравнение.

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