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GEOMETRIC MODELING OF LAYING GEODETIC LINES ON RULED SURFACES

Abstract. Geodesic lines find interesting applications during solving most tasks of the fundamental sciences (mathematics, physics and other) and engineering practice. In differential geometry, the geodesic lines are typical lines for determination of the inner properties of the surface. However, the formation of the geodesic line on the surface envisages certain difficulties, and are solved generally by methods of the computing mathematics and descriptive geometry.

In this article, the development of simple and convenient algorithm of formation of the geodesic line on ruled surfaces are considered. In common case, the three-dimensional model of the geodetic line formation algorithm on the ruled surface is expressed in following: the ruled surface we replace by sided surface, and at any position of the consideredside, the intersection point of the geodesic line with the edge of fracture (curvature of the two-sided angle) will be identified as the cross point of the adjacent generatrix with the surface of the cone of revolution – congruence of geodesic line laying directions with the peak at initial point, rotation axis, incident to the considered generatrix, and angle at the cone peak, equal to the doubled angle between rotation axis and geodesic line laying direction. Next, as the initial parameters the adjacent with mentioned generatrix, upper determined point, laying on it, and direction of the geodesic line – angle between section of the obtained geodetic and adjacent generatrix, are accepted. Thus, multiply repeating the described cycle, we obtain mass of points, which composes the desired geodetic line. Hereby the mathematic description ofthis algorithm is provided.

Key words: descriptive geometry; ruler surface; generatrix; laying direction; direction congruence; geodetic line; point of intersection.

When solving many problems of fundamentalsciences (mathematics, physics, etc.) and of engineering practice, the creation of a mathematical model of an object or process is reduced to the construction of networks of special lines (curvature, asymptotic, geodesic, level, the maximum slope, etc.) belonging to curved surfaces of objects, surfaces describing the dependence of "composition-property", etc. Families of geodetic lines occupy a worthy place among them and have great theoretical and applied value.

In differential geometry, geodesic lines are characteristic lines for determining the internal properties of a surface, and by drawing geodesic lines and measuring their lengths, you can determine all the internal properties of a surface (for example, Gaussian curvature). This is explained by the fact that any sufficiently small arc of the geodesic line represents the shortest line on the surface among all the lines connecting the ends of this arc that can be drawn on the surface [4]. However, the construction of a geodesic line on the surface is a certain difficulty, which is solved mainly by methods of computational mathematics and descriptive geometry.

In differential geometry the geodesic line is specified by differential equation of the 2^{nd} order with ordinary derivative [4]:

$$\frac{d^2u}{dt^2}+\Gamma_{11}^{1}\left(\frac{du}{dt}\right)^2+\Gamma_{12}^{1}\left(\frac{du}{dt}\frac{dv}{dt}\right)+\Gamma_{22}^{1}\left(\frac{dv}{dt}\right)^2=0,$$

$$\frac{d^2v}{dt^2}+\Gamma_{11}^{2}\left(\frac{du}{dt}\right)^2+\Gamma_{12}^{2}\left(\frac{du}{dt}\frac{dv}{dt}\right)+\Gamma_{22}^{2}\left(\frac{dv}{dt}\right)^2=0, \qquad (1)$$

where Γ_{ij} - Cristoffer's symbols of the 2nd order. They expressed only through the coefficients of the first quadratic form and its' derivative.

From the properties of the differential equations, it is observed that through each point on each direction one and only that one geodesic line permeates. Therefore, geodesic lines form the two-parameter plurality.These parameters are determined from the initial conditions, which are specifiedas below:

- with point A(u_o, v_o) and direction t of curve($du/dt = p$) in this point A;
- with two points A($u_н$, $v_н$) and B($u_к$, $v_к$),incident to that curve.

Problems solved with these initial conditions are formulated as:

1) build on the surface the geodesic line through the point in specified direction;
2) connect by geodesic line two points of the surface.

Differential equation (1) of geodesic line in common cases is not possibleto be integrated, even in case of unarysurface, therefore it is solved mainly by numerical methods [4]. The technical surfaces in general cases are constituent, therefore it is necessary to elaborate a simple and convenient algorithm, designated for the geometric modelling of the geodesic lines on composite surfaces.

In the area of the applied geometry the rank of authors elaborated the graphic ways of solving the problems of building the geodesic line on surface with usage of the descriptive geometry. These works are G.E. Pavlenko [9], V.V. Vanin [2], IvanovaL.S. [6], Elkin L.M. [11],Kovalev V.N., Harchenko A.I. [7], Scheffers [12], Glagolev N.A. [5], Ryzhov N.N. [10] and others.

Known methods [2, 5-7, 9, 10-12] of building a geodetic line on the surface do not provide thefulfillment of the condition of equality to zero of the geodetic curvature of the obtained line, and as a consequence, the accuracy of laying a geodetic line in a given direction is not provided. Increasing accuracy by reducing the iteration step results in a large amount of computing operations. Also, insufficiently developed issues that take into account the features arising when overcoming the geodetic line of points and lines of the junction of the compartments of composite surfaces. Therefore, one of the components of the tasks of mathematical support of automation of some technological processes is the development of algorithms that provide high accuracy of laying a geodetic line on the technical (composite) surface.

In order to eliminate these shortcomings as much as possible, the author [13] proposed a method of constructing a geodetic line on a technical surface based on the following property of the geodetic [4]: the g line, described by the middle M of arc AB, is geodetic for a given surface F, if an infinitely small arc AB of the curve l on this surface always moves "straight", i.e. the trajectories of movement of points A and B have equal length and are perpendicular to the arc AB.

In the paper [13] it is shown that the proposed method of constructing a geodetic line has a significant advantage. The main advantage of the developed method is a strict adherence to the specified direction of laying geodetic, as well as the exclusion of unnecessary operations - construction of the involute. The error of the method is accumulated only by the error of approximation of the arcs of geodesic triangle by the corresponding line segments. Thus, the relative error of the method at the iteration step of 2 mm amounted to 0.11%. With an increase of the iteration step to 3 mm, which for modern high-speed computers is a relatively high value, the error of the method increased to 0.823%.

This article discusses the development of a simple and convenient algorithm for a particular case, the construction of a geodetic line on ruled surfaces.

The linear surfaces, formed by move of the straight line through the specified law, make a largeclass of surfaces of zero and non-zero Gausse's curvature, which are unrolling and not unrolling. Solving of the problem of construction of the geodesic line on unrolling surfaces comes to the identification of one-to-one concordance between surface and its' involute.Then the straight line, connecting two specified points, is reflected in geodesic line on the surface. In case of non-developablesurface such method is inapplicable. Therefore the elaboration of simple and convenient algorithm of construction of geodesic line on the ruler surfaces is an important task. The developed method can be used not only for ruler surfaces. The curvilinear surfaces can be approximated ruled lines for simplification of construction the geodesic lines on them.

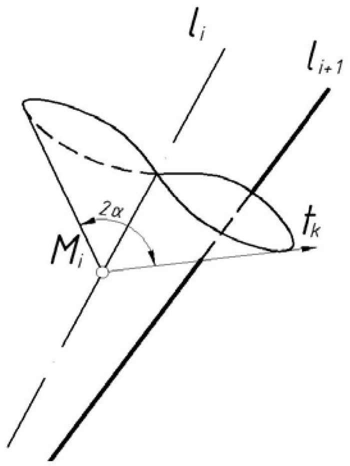
During the elaboration of the method the ruler surface was considered as approximated by sided surfaces (exposed to triangulation), where the rectilinear generatrixes of the ruler surface – are the essence of curvature of the sided surface.

Due to this, the method of construction of the geodesic line on the ruler surface was found on the next property of the geodesic line [5]: in case of compound surface the respective angles of the adjacent arcs of geodesic line to the junction line of composing ones are equal.

The proposed algorithm, based on the given property of geodesic line, consists of the following operations:

We have ruler surface *Ф*, formed by move of the straightforward generatrix$l$ per the advanced specified law. Let specify on this surface *Ф* the initial point M_0,belonging to some generatrixl_0.We know also the initial direction t_0 of laying the geodesic line g, coming from the initial point M_0.Generatrix$l$ on the surface *Ф* in each its' position is uniquely defined. Therefore, the choice or identification of the point M location, belonging to that line, is not difficult.

In common case, the spatial model of the algorithm of the geodesic line construction on the ruler surface, is expressed in following: when replacing the ruler surface by sided surface, at any disposition of the considered side, the point ofintersection of geodesic line with the edge of fracture (curvature of the two-sided angle)will be determined as the point of intersection of adjacentgeneratrix l_{i+1} with the surface of cone of revolution – congruence of directions t_i with the peak in point M_i, axis of rotation, incident to the generatrixl_i and to the angle at cone peak, equal to the doubled angle between axis of rotation l_i and direction t_k (figure).



Figure

Let imagine the mathematic description of the suggested algorithm.

Problem: it is required to determine the coordinates of points of intersection of the straight line with the surface of cone.

Equation of the straight line in space, passing through the two specified points, is defined by the formula:

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}, \quad (1)$$

where $a = x_2 - x_1, b = y_2 - y_1, c = z_2 - z_1$.

Equation of surface of the straight circular cone with peak at the origin of coordinates, while axis of rotation coincide with the z-axis, is expressed by formula:

$$k^2z^2 = x^2 + y^2 \quad (2)$$

where k = tgφ, andφ – half of the angle at peak of cone of revolution.

Let solve the system of given equations. For this we express x and y by z

$$x = x_1 + \frac{a}{c}(z - z_1), y = y_1 + \frac{b}{c}(z - z_1)$$

and put in (2)

$$k^2 z^2 = x_1^2 + 2\frac{a}{c}x_1(z - z_1) + \frac{a^2}{c^2}(z - z_1)^2 + y_1^2 + 2\frac{b}{c}y_1(z - z_1) + \frac{b^2}{c^2}(z - z_1)^2$$

grouping the terms of the same name, we obtain the expressions for solving the quadratic equation

$$\propto z^2 + 2\beta z + \gamma = 0$$

$$\propto = (a^2 + b^2) - c^2 k^2,$$

$$\beta = c\,(ax_1 + by_1) - z_1(a^2 + b^2),$$

$$\gamma = c^2(x_1^2 + y_1^2) - 2cz_1(ax_1 + by_1) + z_1^2(a^2 + b^2).$$

$$z_{1,2} = \frac{-\beta \pm \sqrt{\beta^2 - 4 \propto \gamma}}{2 \propto}$$

For preparation of the mathematic provision for compiling the program of calculation on computer the trajectory of geodesic line laying, we perform the analytic description of the proposed algorithm of solving the problems in space.

For simplification of the analytical expressions, we execute the conversion of rectangular coordinate system with transfer of origin of coordinates to the point $M_i$of the sought geodesic line and superposition of z-axis with generatrix direction $l_i \supset M_i$.

Formulas of conversion of the rectangular coordinates of the point in general case are expressed with the help of Euler's angles (ψ–angle of precession, θ–angle of saltation, φ–angle of pure rotation)

$$x = x_0 + (cos\psi cos\varphi - sin\psi cos\theta sin\varphi)x' + (-cos\psi sin\varphi - sin\psi cos\varphi cos\varphi)y' + sin\psi sin\theta * z',$$

$$y = y_0 + (sin\psi cos\varphi + cos\psi cos\theta sin\varphi)x' + (-cos\psi sin\theta + cos\psi cos\varphi cos\varphi)y' + cos\psi sin\theta * z',$$

$$z = z_0 + sin\theta sin\varphi * x' + sin\theta cos\varphi * y' + cos\theta * z'.$$

In our case the rotations are performed only twice, therefore all mathematic expressions are slightly simplified.

$$x = x_0 + (cos\psi)x' + (-sin\psi)y' + sin\psi sin\theta * z',$$

$$y = y_0 + (sin\psi)x' + (-cos\psi sin\theta + cos\psi cos\varphi cos\varphi)y' + cos\psi sin\theta * z',$$

$$z = z_0 + sin\theta * y' + cos\theta * z'.$$

Since, the position of each generatrixl_i is determined, then the Euler's angles can be reckoned as known.

$$\psi = arctg\frac{(YB - YA)}{(XB - XA)},$$

$$\theta = arctg\frac{(ZB - ZA)}{sqrt\{(XB - XA)^2 + (YB - YA)^2\}}$$

Thus, multiple repeating the described cycle, we obtain the plurality of points M_i, composing the geodesic line g.

Afterward, we assume the making of computer program and execution of calculations of geodesic line laying on different ruler surfaces. Based on calculation results we can perform the comparative analysis of degree of accuracy of geodesic line laying.

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ТҮЗУ СЫЗЫҚТЫ БЕТТЕРДЕ ГЕОДЕЗИЯЛЫҚ СЫЗЫҚТАРДЫ ЖҮРГУЗУДІҢ ГЕОМЕТРИЯЛЫҚ ҮЛГІСІ

Аннотация. Геодезиялық сызықтар түбегейлі ғылымдар (математика, физика және басқа) және инженерлік тәжірибенің көптеген есептерін шығаруда қызықты қолданыс табады.Дифференциалды геометрияда геодезиялық сызықтар беттің ішкі қасиеттерін анықтайтын ерекше сызықтар болып табылады. Алайда берілген дененің бетінде геодезиялық сызықты тұрғызу көптеген қиындыққа келіп тіреледі, сондықтан негізінде есептеу математикасы және сызба геометрия әдістерімен шешіледі.

Ұсынылып отырған мақалада геодезиялық сызықтың түзу сызықты беттерде тұрғызудың қарапайым және пайдалануға ыңғайлы алгоритмі қарастырылған. Жалпы жағдайда, геодезиялық сызықтың түзу сызықты беттерде тұрғызудың алгоритмінің кеңістіктік үлгісін келесідей тұжырымдауға болады: түзу сызықты беттi көп жақты бетпен алмастырамыз, қарастырылып жатқан жатың кез-келген жағдайында геодезиялық сызықтың сыну қырымен (екі жақты беттің ортақ қыры) қиылысу нүктесі көрші жасаушының айналу конус бетімен қиылысу нүктесі болып анықталады. Бұл жерде айналу конус беті - төбесі бастапқы нүктеде орналасқан, осі ағымдағы жасаушы түзумен беттескен, геодезиялық сызықтың бағыттарының тобы. Конус төбесінің бұрышы айналу осі мен геодезиялық сызықтың бағыты арасындағы бұрыштың екі еселенген шамасына тең. Ары қарай көрші жасаушы бастапқы жасаушы ретінде, анықталған нүкте бастапқы нүкте ретінде (келесі конус төбесі) қабылданады да, келесі нүктені табу үшін келтірілген алгоритм қайталанады. Осылайша, жазылған циклды көп рет қайталау арқылы ізделінген геодезиялық сызықты құрайтын көп нүктелер жиынын анықтаймыз. Осы алгоритмның математикалық жазбасы келтірілген.

Түйін сөздер: сызба геометрия; түзу сызықты бет; жасаушы; сызықтардың бағыты; бағыттардың конгруэнциясы; геодезиялық сызық; қиылысу нүктесі.

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ГЕОМЕТРИЧЕСКОЕ МОДЕЛИРОВАНИЕ ПРОКЛАДКИ ГЕОДЕЗИЧЕСКИХ ЛИНИИ НА ЛИНЕЙЧАТЫХ ПОВЕРХНОСТЯХ

Аннотация. Геодезические линии находят интересные приложения при решении многих задач фундаментальных наук (математики, физики и др.) и инженерной практики. В дифференциальной геометрии геодезические линии являются характерными линиями для определения внутренних свойств поверхности. Однако построение геодезической линии на поверхности представляет определенные сложности, решается в основном методами вычислительной математики и начертательной геометрии.

В статье рассматривается разработка простого и удобного алгоритма построения геодезической линии на линейчатых поверхностях. В общем случае, пространственная модель алгоритмапостроения геодезической линии на линейчатойповерхности, выражается в следующем: линейчатую поверхность заменяем гранной поверхностью, при любом расположений рассматриваемой грани, точка пересечения геодезической с ребром излома (линия изгиба двугранного угла) будет определяться как точка пересечения смежной образующей с поверхностью конуса вращения – конгруэнции направлений прокладки геодезической с вершиной в исходной точке, оси вращения, инцидентной рассматриваемой образующей, и углом при вершине конуса, равной удвоенному углу между осью вращения и направлением прокладки геодезической. Далее за исходный параметры принимаются смежная с рассмотренной образующая, определенная выше точка, лежащая на ней, и направление геодезической – угол между отрезком полученной геодезической и смежной образующей. Таким образом, многократно повторяя описанный цикл, получим множество точек, составляющее искомую геодезическую линию. Приводится математическое описание данного алгоритма.

Ключевые слова: начертательная геометрия; линейчатая поверхность; образующая; направление прокладки; конгруэнция направлении; геодезическая линия; точка пересечения.

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