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**SOME ASPECTS OF THE CONSTRUCTION OF
A MATHEMATICAL MODEL FOR CUTTING METALS**

Abstract. Creation of reliable high-performance machines specifies the use of new materials for manufacture of their parts. At the same time, manufacturers do not have time to introduce new processing technologies using durable cutting tools that require a lot of time and financial resources. Scientists and technologists are increasingly using modern information technologies and application packages to quickly solve these problems. To use existing software products, it is necessary to introduce a lot of initial data for which additional research or a lot of time to search for them is required. The developed software products are based on empirical data, as a result of which the spectrum of their use is narrowed.

The distinction of the program being created is that the models being developed will be based on the results of approximation dependencies on the parameters of cutting conditions, with corrected data obtained by solutions of probably static models.

This article discusses some aspects of the construction of a mathematical model for cutting an elastic metal strip in a non-classical formulation. The action of the cutter is modeled as a moving inclined concentrated force, at the point of application of which unrestricted stresses arise. Using the properties of the generalized Dirac and Heaviside functions, analytical expressions for the stress tensor components are obtained. The resulting equations are derived and the boundary conditions are formulated. The results obtained serve as the basis for formulating a mathematical model of the corresponding thermo-dynamic problem for an elastoplastic metal strip.

Key words: cutting, cutting tool, resistance to rupture, concentrated force, deformation, mathematical model, thermodynamic problem.

Introduction. Creating technologies for processing new parts and designing relevant processing tools requires a lot of time and money. Organization of applied research, especially the experimental part, has become one of the expensive stages in research activities in recent years [1].

With the development of information technology, when solving knowledge-intensive problems, scientists and technologists are increasingly using software [2-4]. To use existing software products, it is necessary to introduce a lot of initial data for which additional research or a lot of time to search for them is required [5-8].

In this case, the task for technologists is simple, to produce a part from a new alloy, it is necessary to assign the appropriate cutting conditions and select the necessary cutting tool. In the course of the research, it was established that production workers spend huge amounts of time and money to master just one new part. The lack of specific reference information and the urgency of resolving the issue forces the technologist to use the trial method.

In almost all studies, whatever the model, the determination of the cutting process indicators is based on experimental data on the standard mechanical properties of the material being processed (resistance to rupture, flow limit, relative elongation and contraction), on the machinability properties, for example, chip shrinkage, determined directly when cutting in the studied conditions [9-11].

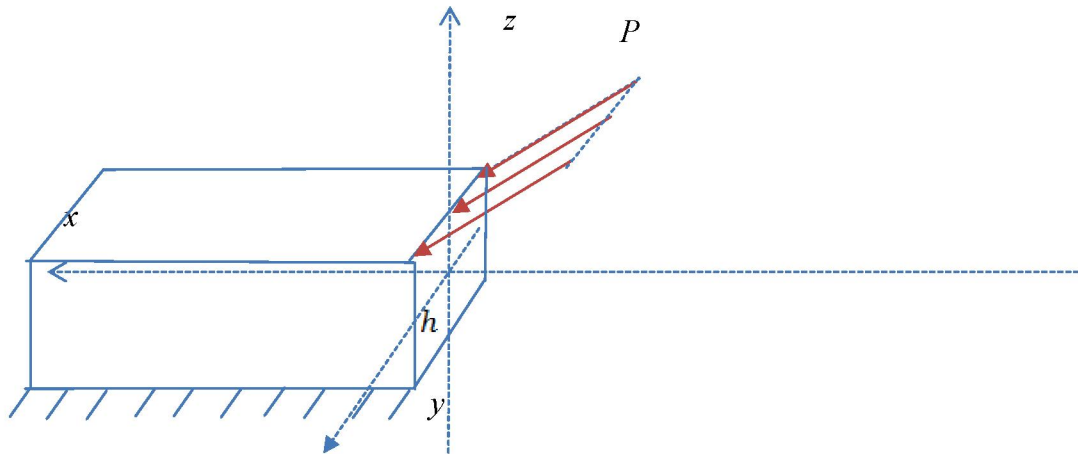
As a result, all models using these approaches have a significant limited application due to the assumption about the shape of the zone of primary deformations and the authors “tune” their models to the specified cutting conditions [12-15]. In this connection, in the published works there are significant differences in the views of different schools in relation to individual aspects of the cutting process. This hinders the creation of a comprehensive predictive theory of cutting, which would allow carry out the study of working processes in various conditions without prior experiments [16].

To solve the problems arising in the field of metal processing a group of scientists for the project AP05132157 “Development of simulation models of cutting processes and forecasting on their basis the optimal parameters of the tool and processing conditions” by the grant financing of the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan proposed a software product development methodology predicting values of the arising force and temperature phenomena, the amount of tool wear, which are accompanied by imitations of the cutting process.

Based on the methodology, the software product is developed based on the cutting process from the position of a systematic approach as a process that has a certain structure with certain functional properties and interrelationships of structural elements, which are: cutting tool, cutting conditions and the material being processed.

The difference between the created software product and the existing ones is that the models are based on the data of approximation dependences obtained on the basis of the fundamental laws and postulates of the mechanics of a deformable solid body in the contact area of bearing surfaces in dynamics.

Main part. The action of the cutter on the metal strip with the width – b in the line of Oy , with the length – l in the line of Ox and the thickness – h in the line of Oz occurs as follows (figure).



Metal cutting pattern

The cutting process is modeled as the oblique movable load action in the coordinate plane Oxz (here Oxy is located in the middle plane of the strip), $q_z(x, t) = P_z \delta(x - ct)$ $p_x(x, t) = P_x \delta(x - ct)$, where $\delta(x - ct)$ is the Dirac delta function, c characterizes the cutting motion rate of the metal strip. The metal strip lies on a completely non-deforming base fixedly, while the other facets are free from loads.

In this case, it is considered that there is a flat stress-strain state, the desired variables do not depend on y , therefore

$$U_y = 0, \quad e_{yy} = e_{yx} = e_{yz} = 0.$$

The stress state of the body under consideration in the Oxz rectangular coordinates with the absence of bulk forces is described by the following equilibrium equation [17-19].

$$\begin{cases} \sigma_{xx'} + \sigma_{xz'z} = 0, \\ \sigma_{xz'x} + \sigma_{zz'z} = 0. \end{cases} \quad (1)$$

The defining relations between symmetric stress and strain tensors have the form

$$\begin{cases} \sigma_{xx} = \frac{E}{1-\nu^2} [\varepsilon_{xx} + \nu \varepsilon_{zz}], \\ \sigma_{zz} = \frac{E}{1-\nu^2} [\varepsilon_{zz} + \nu \varepsilon_{xx}], \\ \sigma_{xz} = \frac{E}{2(1+\nu)} \varepsilon_{xz} \end{cases} \quad (2)$$

The desired solution is presented in the following form [20]

$$\begin{cases} U_x = u + \psi z - \Phi_1(z)A - \Phi_2(z)B \\ U_z = W + Vz - \Phi_1(z)\theta \end{cases} \quad (3)$$

where A, B, θ are unknown coordinate functions to be determined and

$$u = \frac{1}{h} \int_{-h/2}^{h/2} U_x dz, \quad w = \frac{1}{h} \int_{-h/2}^{h/2} U_z dz, \quad \psi = \frac{12}{h^3} \int_{-h/2}^{h/2} U_x z dz,$$

$$V = \frac{12}{h^3} \int_{-h/2}^{h/2} U_z z dz, \quad \Phi_1(z) = \frac{h^2}{12} \left[1 - 12 \left(\frac{z}{h} \right)^2 \right], \quad \Phi_2(z) = \frac{1}{4} \left[1 - \frac{20}{3} \left(\frac{z}{h} \right)^2 \right] z$$

Taking into account the kinematic relations between the strain tensor and the components of the displacement vector, the equation of the state for the problem in question will take the following form

$$\begin{cases} \sigma_{xx} = \frac{E}{1-\nu^2} [u' + z\psi' - \Phi_1(z)A' - \Phi_2(z)B' + \nu V + 2\nu\theta z], \\ \sigma_{zz} = \frac{E}{1-\nu^2} [\nu(u' + z\psi' - \Phi_1(z)A' - \Phi_2(z)B') + V + 2\theta z], \\ \sigma_{xz} = \frac{E}{2(1+\nu)} \left[\psi + 2zA - \frac{3h^2}{20} \left(1 - 20 \frac{z^2}{h^2} \right) B + W' + zV' - \Phi_1(z)\theta' \right] \end{cases} \quad (4)$$

In order to simplify the scheme for solving the problem in question, let's integrate the equilibrium equation (1) through the thickness, as a result of which there will be the following system of ordinary differential equations

$$\begin{cases} N_{xx,x} + P_x \delta(x - vt) - \sigma_{zx}(x; -0, 5h) = 0 \\ M_{xx,x} - Q_x + 0, 5h [P_x \delta(x - vt) + \sigma_{zx}(x; -0, 5h)] = 0 \\ Q_{x,x} + P_z \delta(x - vt) - \sigma_{zz}(x; -0, 5h) = 0 \end{cases} \quad (5)$$

where $N_{xx} = \int_{-0.5h}^{0.5h} \sigma_{xx} dz$ – normal force,

$Q_x = \int_{-0.5h}^{0.5h} \sigma_{zx} dz$ – intensity of shear,

$M_{xx} = \int_{-0.5h}^{0.5h} \sigma_{xx} z dz$ – internal bending moment, which taking into account (2)-(4) can be

expressed through the integral quantities

$$\begin{cases} N_{xx} = \frac{Eh}{1-\nu^2}(u' + \nu V'), \\ M_{xx} = \frac{Eh^3}{12(1-\nu^2)}(\psi' + 2\theta), \\ Q_x = \frac{Eh}{2(1+\nu)}\left[\psi + \frac{h^2}{20}B + W'\right]. \end{cases} \quad (6)$$

At that

$$\begin{cases} \sigma_{zz}\left(x, -\frac{h}{2}\right) = \frac{E}{1-\nu^2}\left[\nu\left(u' - \frac{h}{2}\psi' + \frac{h^2}{6}A' - \frac{h^3}{20}B'\right) + V - \theta h\right], \\ \sigma_{xz}\left(x, -\frac{h}{2}\right) = \frac{E}{2(1+\nu)}\left[\psi - hA + \frac{3h^2}{5}B + W' - \frac{h}{2}V' + \frac{h^2}{6}\theta'\right] \end{cases} \quad (7)$$

The obtained expressions (6) and (7) inserting in (5) will be

$$\begin{cases} \frac{Eh}{1-\nu^2}(u' + \nu V')' + P_x\delta(x-ct) - \frac{E}{2(1+\nu)}\left[\psi - hA + \frac{3h^2}{5}B + W' - \frac{h}{2}V' + \frac{h^2}{6}\theta'\right] = 0, \\ \frac{Eh^3}{12(1-\nu^2)}(\psi' + 2\theta)' - \frac{Eh}{2(1+\nu)}\left[\psi + \frac{h^2}{10}B + W'\right] + \\ \frac{h}{2}\left\{P_x\delta(x-ct) + \frac{E}{2(1+\nu)}\left[\psi - hA + \frac{3h^2}{5}B + W' - \frac{h}{2}V' + \frac{h^2}{6}\theta'\right]\right\} = 0 \\ \frac{Eh}{2(1+\nu)}\left[\psi + \frac{h^2}{10}B + W'\right]' + P_z\delta(x-ct) - \frac{E}{1-\nu^2}\left[\nu\left(u' - \frac{h}{2}\psi' + \frac{h^2}{6}A' - \frac{h^3}{20}B'\right) + V - \theta h\right] = 0 \end{cases} \quad (8)$$

To determine the unknown functions, let's use the boundary conditions for the metal strip under consideration:

$$N = 0, \quad M = 0, \quad Q = 0, \quad x = 0, L \quad (9)$$

$$\begin{cases} U_x = U_z = 0, \\ U_x = U_z = 0, \end{cases} \quad \text{at} \quad z = \pm \frac{h}{2}, \quad (10)$$

$$\begin{cases} \sigma_{xz} = -P_x\delta(x-ct) \\ \sigma_{zz} = P_z\delta(x-ct) \end{cases} \quad \text{at} \quad z = \pm \frac{h}{2} \quad (11)$$

Using the boundary conditions on the face metal strip planes, there will be

$$\begin{cases} \nu\left(u' + \frac{h}{2}\psi' + \frac{h^2}{6}A' + \frac{h^3}{20}B'\right) + V + \theta h = -\bar{P}_z\delta(x-ct), \quad \bar{P}_z = P_z \frac{1-\nu^2}{E} \\ \left[\psi + hA + \frac{3h^2}{5}B + W' + \frac{h}{2}V' + \frac{h^2}{6}\theta'\right] = \bar{P}_x\delta(x-ct), \quad \bar{P}_x = P_x \frac{2(1+\nu)}{E} \end{cases} \quad (12)$$

$$\begin{cases} h^2B = 20\left(\frac{u}{h} - \frac{1}{2}\psi + \frac{h}{6}A\right) \\ h\theta = -6\left(\frac{W}{h} - \frac{1}{2}V\right) \end{cases} \quad (13)$$

Then, instead of (8) and (12) there will be

$$\left\{ \begin{aligned} & \frac{Eh}{1-\nu^2} (u' + \nu V)' + P_x \delta(x-ct) - \frac{E}{2(1+\nu)} \left[\psi - hA + \frac{3h^2}{5} B \right] = 0, \\ & \frac{Eh^3}{12(1-\nu^2)} (\psi' + 2\theta)' - \frac{Eh}{2(1+\nu)} \left[\psi + \frac{h^2}{10} B + W' \right] + \\ & \frac{h}{2} \left\{ P_x \delta(x-ct) + \frac{E}{2(1+\nu)} \left[\psi - hA + \frac{3h^2}{5} B \right] \right\} = 0 \\ & \frac{Eh}{2(1+\nu)} \left[\psi + \frac{h^2}{10} B + W' \right]' - P_z \delta(x-ct) - \frac{E}{1-\nu^2} [V - \theta h] = 0 \\ & \nu \left(2u' + \frac{h^2}{3} A' \right) + V + \theta h = -\bar{P}_z \delta(x-ct), \\ & \left[\psi + hA + 12 \left(\frac{u}{h} - \frac{1}{2} \psi \right) + W' + \frac{h}{2} V' + \frac{h^2}{6} \theta' \right] = \bar{P}_x \delta(x-ct), \end{aligned} \right. \quad (14)$$

In the first equation (13), neglecting the members of a high order of smallness, which greatly simplifies the mathematical calculations, there will be:

$$\left\{ \begin{aligned} & V = \frac{1}{4} \left[\frac{1-\nu}{2} \bar{P}_z \delta(x-ct) - \frac{1}{2} \nu u' + \frac{3}{2} \frac{W}{h} \right], \\ & hA = \bar{P}_x \delta(x-ct) - 12 \frac{u}{h} + 5\psi - hV', \end{aligned} \right. \quad (16)$$

Taking into account the boundary condition (13), the equilibrium equation is reduced to the form

$$\left\{ \begin{aligned} & \frac{Eh}{1-\nu^2} (u' + \nu V)' + P_x \delta(x-ct) - \frac{E}{2(1+\nu)} \left[hA - 5\psi + 12 \frac{u}{h} \right] = 0, \\ & \frac{Eh^3}{12(1-\nu^2)} (\psi' + 2\theta)' - \frac{Eh}{2(1+\nu)} \left[W' + 2 \frac{u}{h} + \frac{h}{3} A \right] + \\ & \frac{h}{2} \left\{ P_x \delta(x-ct) + \frac{E}{2(1+\nu)} \left[hA - 5\psi + 12 \frac{u}{h} \right] \right\} = 0, \\ & \frac{Eh}{2(1+\nu)} \left[W' + 2 \frac{u}{h} + \frac{h}{3} A \right]' - P_z \delta(x-ct) - \frac{E}{1-\nu^2} \left[4V - 6 \frac{W}{h} \right] = 0. \end{aligned} \right. \quad (17)$$

Using the expression (16) there will be:

$$\left\{ \begin{aligned} & u' + (1+\nu)V = v_0, \\ & \frac{h^2}{6(1-\nu)} \left(\psi' - 12 \frac{W}{h^2} \right)' - \left[W' - 2 \frac{u}{h} + \frac{5}{3} \psi \right] - h \frac{1+4\nu}{1-\nu} V' = -\frac{2h}{3} \bar{P}_x \delta(x-ct), \\ & W' - 2 \frac{u}{h} + \frac{1}{3} \bar{P}_x \delta(x-ct) + \frac{5}{3} \psi - \frac{h}{3} V' + \frac{4}{1-\nu} \nu \frac{u}{h} = W_1, \end{aligned} \right. \quad (18)$$

where W_1, v_0 – the unknown integration constants to be determined from the boundary conditions. Taking into account the first equation (18) there will be

$$V = \frac{1}{8 - \nu(1 + \nu)} \left[(1 - \nu) \bar{P}_z \delta(x - ct) - \nu v_0 + 3 \frac{W}{h} \right],$$

Differentiating the first equation (17) and substituting the last (18) into the resulting system after some mathematical calculations, there will be

$$\left\{ \begin{aligned} & u'' + 6 \frac{1 - 3\nu}{1 - \nu} \frac{1 + \nu}{7 - \nu(1 + \nu)} \frac{u}{h^2} - 5 \frac{1 + \nu}{7 - \nu(1 + \nu)} \frac{X}{h^2} = \\ & - \frac{1 - \nu^2}{8 - \nu(1 + \nu)} (1 - \nu) \left[1 - \frac{1}{8 - \nu(1 + \nu)} \frac{(1 + \nu)}{7 - \nu(1 + \nu)} \right] \bar{P}_z \delta'(x - ct) \\ & - 3 \frac{1 - \nu^2}{8 - \nu(1 + \nu)} \frac{(1 + \nu)}{7 - \nu(1 + \nu)} \left[\frac{W_1}{h} - \frac{1}{3} \frac{\bar{P}_x}{h} \delta(x - ct) \right], \\ & X'' - 10(1 - \nu) \left[1 + \frac{3 - \nu}{1 - \nu} \frac{8 - \nu(1 + \nu)}{7 - \nu(1 + \nu)} + \frac{1}{1 - \nu} \frac{5 + 20\nu}{7 - \nu(1 + \nu)} \right] \frac{X}{h^2} + \\ & 12(1 - \nu) \left[1 - \frac{1 - 3\nu}{1 - \nu} \left(\frac{3 - \nu}{1 - \nu} - 6 \frac{1 + 4\nu}{1 - \nu} \frac{1}{8 - \nu(1 + \nu)} \right) \frac{8 - \nu(1 + \nu)}{7 - \nu(1 + \nu)} \right] \frac{u}{h^2} = \\ & - 2(1 - \nu) \left[2 + \frac{3 - \nu}{1 - \nu} \frac{8 - \nu(1 + \nu)}{7 - \nu(1 + \nu)} + 3 \frac{1 + 4\nu}{1 - \nu} \frac{1}{8 - \nu(1 + \nu)} \right] \frac{\bar{P}_x}{h} \delta(x - ct) - \\ & 6(1 - \nu) \left[\frac{3 - \nu}{3(7 - \nu - \nu^2)} + \frac{1 + 4\nu}{8 - \nu(1 + \nu)} - \frac{1 - \nu}{(7 - \nu - \nu^2)} \right] \bar{P}_z \delta'(x - ct) + \\ & 6(1 - \nu) \left[\frac{3 - \nu}{1 - \nu} \frac{8 - \nu(1 + \nu)}{7 - \nu(1 + \nu)} - 3 \frac{1 + 4\nu}{1 - \nu} \frac{1}{8 - \nu(1 + \nu)} \right] \frac{W_1}{h}, \quad X = h\nu, \end{aligned} \right. \quad (19)$$

which can be written in vector-matrix and unified form

$$\vec{Y}'' + \frac{1}{h^2} \tilde{B} \vec{Y} = \frac{W_1}{h} \vec{F}_0 + \frac{P_x}{h} \vec{F} \delta(x - ct) + P_z \vec{G} \delta'(x - ct), \quad (20)$$

where

$$\begin{aligned} \tilde{B} &= \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \quad b_{11} = \frac{61 - 3\nu}{7} \frac{1 + \nu}{1 - \nu(1 + \nu)/7}; \quad b_{12} = -\frac{5}{7} \frac{1 + \nu}{1 - \nu(1 + \nu)/7}; \\ b_{21} &= 12(1 - \nu) \left[1 - \frac{1 - 3\nu}{1 - \nu} \left(\frac{3 - \nu}{1 - \nu} - \frac{3}{4} \frac{1 + 4\nu}{1 - \nu} \frac{1}{1 - \nu(1 + \nu)/8} \right) \frac{8 - \nu(1 + \nu)}{7 - \nu(1 + \nu)} \right]; \\ b_{22} &= -10(1 - \nu) \left[1 + \frac{3 - \nu}{1 - \nu} \frac{8 - \nu(1 + \nu)}{7 - \nu(1 + \nu)} + \frac{5}{7} \frac{1}{1 - \nu} \frac{1 + 4\nu}{1 - \nu(1 + \nu)/7} \right]. \end{aligned}$$

$$\vec{Y} = \begin{pmatrix} u \\ X \end{pmatrix}, \quad \vec{F}_0 = \begin{pmatrix} -\frac{3}{56} \frac{1 - \nu^2}{1 - \nu(1 + \nu)/8} \frac{(1 + \nu)}{1 - \nu(1 + \nu)/7} \\ 18(1 - \nu) \left[\frac{1 - \nu/3}{1 - \nu} \frac{8 - \nu(1 + \nu)}{7 - \nu(1 + \nu)} - \frac{1 + 4\nu}{1 - \nu} \frac{1}{1 - \nu(1 + \nu)/8} \right] \end{pmatrix},$$

$$\vec{G} = \begin{pmatrix} -\frac{1(1-\nu)(1-\nu^2)}{8(1-\nu(1+\nu)/8)} \left[1 - \frac{1}{56(1-\nu(1+\nu)/8)} \frac{1}{1-\nu(1+\nu)/7} \right] \\ -6(1-\nu) \left[\frac{1-\nu/3}{(7-\nu-\nu^2)} + \frac{1+4\nu}{8-\nu(1+\nu)} - \frac{1-\nu}{(7-\nu-\nu^2)} \right] \end{pmatrix},$$

$$\vec{F} = \begin{pmatrix} \frac{1}{56} \frac{1-\nu^2}{1-\nu(1+\nu)/8} \frac{(1+\nu)}{1-\nu(1+\nu)/7} \\ -4(1-\nu) \left[1 + \frac{3}{2} \frac{1-\nu/3}{1-\nu} \frac{8-\nu(1+\nu)}{7-\nu(1+\nu)} + \frac{3}{2} \frac{1+4\nu}{1-\nu} \frac{1}{8-\nu(1+\nu)} \right] \end{pmatrix}$$

The desired solution is presented in the form

$$\vec{Y} = \vec{X} + \vec{Z} + h W_1 B^{-1} \vec{F}_0, \quad (21)$$

where B^{-1} is the inverse matrix. At the same time, \vec{X} is a particular solution (20) corresponding only to the right part without taking into account the boundary conditions:

$$\vec{X}'' + B\vec{X} = \frac{P_x}{h} \delta(x-ct) \vec{F} + P_z \delta'(x-ct) \vec{G}, \quad (22)$$

Since, for the generalized Dirac and Heaviside functions, the following relations take place:

$$\delta(x-ct) = \frac{1}{2} [H(x-ct) - H(ct-x)]' = \frac{1}{2} [\delta(x-ct) + \delta(ct-x)] = \delta(x-ct).$$

If take into account that $(x-ct)^n \delta(x-ct) = 0$, $n \in \mathbb{N}$, $x-ct \in \mathbb{V}$ the solution (22) can be presented in the following form

$$\vec{X} = \frac{1}{2} [H(x-ct) - H(ct-x)] \sum_{n=0}^{\infty} \vec{X}_n \frac{(x-ct)^n}{n!}, \quad 0! = 1,$$

at that

$$\vec{X}' = \vec{X}_0 \delta(x-ct) + \frac{1}{2} [H(x-ct) - H(ct-x)] \sum_{n=0}^{\infty} \vec{X}_{n+1} \frac{(x-ct)^n}{n!},$$

$$\vec{X}'' = \vec{X}_0 \delta'(x-ct) + \vec{X}_1 \delta(x-ct) + \frac{1}{2} [H(x-ct) - H(ct-x)] \sum_{n=0}^{\infty} \vec{X}_{n+2} \frac{(x-ct)^n}{n!}.$$

Therefore, $\vec{X}_{n+2} + \tilde{B} \vec{X}_n = 0$, $n = 0, 1, 2, \dots$. $\vec{X}_0 = \vec{G}$, $\vec{X}_1 = \vec{F}$, then the solution (22) will take the form

$$\vec{X} = \frac{1}{2} [H(x-ct) - H(ct-x)] \sum_{n=0}^{\infty} (-1)^n \tilde{B}^n \left[P_z \vec{G} + \frac{(x-ct)}{(2n+1)} \frac{P_x}{h} \vec{F} \right] \frac{(x-ct)^{2n}}{(2n)!}, \quad \tilde{B}^n = h^{-2n} \tilde{B} \tilde{B} \dots \tilde{B}$$

$$\begin{cases} \vec{Z}'' + \frac{1}{h^2} \tilde{B} \vec{Z} = 0, \\ N_{xx}(\vec{Z}) = -N_{xx}(\vec{X} + h W_1 \tilde{B}^{-1} \vec{F}_0), \\ M_{xx}(\vec{Z}) = -M_{xx}(\vec{X} + h W_1 \tilde{B}^{-1} \vec{F}_0), \\ Q_x(\vec{Z}) = -Q_x(\vec{X} + h W_1 \tilde{B}^{-1} \vec{F}_0), \end{cases} \quad x = 0, l \quad (23)$$

The solution of the problem (23) with regard to the values of the elements of matrix B can be presented as

$$\vec{Z} = \vec{C} \operatorname{sh} k_1 x + \vec{D} \operatorname{ch} k_1 x + \vec{M} \sin k_2 x + \vec{N} \cos k_2 x$$

$$k_1 = \sqrt{\sqrt{(b_{11} + b_{22})^2 - 4b_{12}b_{21}} - b_{11} - b_{22}}, \quad k_2 = \sqrt{\sqrt{(b_{11} + b_{22})^2 - 4b_{12}b_{21}} + b_{11} + b_{22}},$$

Thus, in the first approximation, there is a linear problem for the cutting technology of metal strips. At the same time, the considered thermodynamic problem of cutting metals with a significant heat release in the framework of elastoplastic, in the general case becomes essentially nonlinear [21]. In this case, according to the method of elastic solutions of A.A. Ilyushin, solving the linear problem becomes the first approximation for the general thermodynamic problem.

Conclusion. During the research it was found that existing software products cannot simultaneously predict the occurrence of cutting forces, temperature phenomena of the process, as well as tool wear. In addition, to use these programs, a young scientist or technologist from the production must enter a huge amount of data. To enter these data, it is necessary to have very high skills in higher mathematics, in the mechanics of solid deformable bodies, in dynamic problems of mathematical modeling, and also generating initial data in software tools requires a long time to find data from sources.

In this regard, the phenomena of the cutting process, i.e. the deformed state, are described by mathematical equations, the output elements of which will be the component forces, the cutting temperature and tool wear. The obtained data will be tested by comparing with the values of the calculation of the obtained data using the program developed on the Microsoft Excel application. The correlation coefficients will be adjusted if necessary. Based on the data obtained, the dependencies of the cutting parameters on the processing conditions are constructed. Approximation of the obtained dependencies will help to create probably static models. On the basis of which the software product will be created.

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МЕТАЛЛ КЕСУ ҮШІН МАТЕМАТИКАЛЫҚ ҮЛГІЛЕРДІ ҚҰРУДЫҢ КЕЙБІР АСПЕКТІЛЕРІ

Аннотация. Жоғары сапалы сенімді машиналарды жасау олардың барлық бөлшектерін өндіруге арналған жаңа материалдарды қолдануға алып келеді. Бұл жағдайда, өндірушілер жаңа тұрақты кесу құралдарын пайдаланып, жаңа өңдеу технологияларды енгізуде үлгермейді, себебі ол ұзақ уақытты және қаржы ресурстарын қажет етеді. Осы міндеттерді жедел шешу үшін, зерттеу барысында ғалымдар және технологиялар заманауи ақпараттық технологияларды және қолданбалы зерттеу пакеттерін қолдануда. Әрекеттегі бағдарламалық жасақтама өнімдерін пайдалану үшін, қосымша зерттеулер немесе оларды іздеуге ұзақ уақыт талап етілетін, көптеген бастапқы мәліметтерді ендіру қажет. Әзірленген бағдарламалық өнімдер эмпирикалық деректерге негізделген, соның нәтижесінде олардың қолдану спектрі тарылып кетеді.

Жасалатын бағдарламаның айырмашылығы, әзірленетін модельдер жуықтау тәуелділіктерінің нәтижелеріне кескіш режимдердің параметрлеріне базаланып, ықтималды-статистикалық модельдердің алынған шешімімен түзетілген деректерге негізделетін болады.

Мақалада серпімді металл жолағын классикалық емес жағдайда кесу үшін, металл кесуге арналған математикалық үлгілерді құрудың кейбір аспектілері қарастырылады. Құралдың әрекет етуі қолданылатын нүктеде шектеусіз кернеулер пайда болатын қозғалыстағы бұрыштық концентрацияланған күш ретінде модельденген. Жалпыланған Дирик және Хэвисайд функцияларының қасиеттерін пайдалана отырып, кернеулердің тензорлық компоненттеріне арналған аналитикалық мәліметтер алынады. Рұқсат теңдеулері құрылды және шектік талаптар қалыптастырылды.

Алынған нәтижелер эластопластикалық металл жолаққа арналған тиісті термодинамикалық мәселенің математикалық моделін құру үшін негіз болып табылады.

Түйін сөздер: кесу, кесу құралы, төзімділік шегі, шоғырланған күш, пішіннің өзгеруі, математикалық модель, термодинамикалық міндет.

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НЕКОТОРЫЕ АСПЕКТЫ ПОСТРОЕНИЯ МАТЕМАТИЧЕСКОЙ МОДЕЛИ ДЛЯ РЕЗАНИЯ МЕТАЛЛОВ

Аннотация. Создание надежных высокопроизводительных машин обуславливает применение все новых материалов для изготовления их деталей. При этом производители не успевают с внедрением новых технологий обработки с использованием стойких режущих инструментов, которые требуют немало времени и финансовых средств. Для оперативного решения этих задач при исследованиях ученые и технологи все больше применяют современные информационные технологии и пакеты прикладных программ. Для использования существующих программных продуктов необходимо внести множество исходных данных, для которых требуется дополнительные исследования или много времени их поиска. Разработанные программные продукты базируются на эмпирических данных, вследствие чего сужается спектр их применения.

Отличие создаваемой программы, состоит в том, что разрабатываемые модели будут базироваться на результатах аппроксимационных зависимостей от параметров режимов резания, с скорректированными данными полученными решениями вероятностно-статистических моделей.

В статье рассматриваются некоторые аспекты построения математической модели для резки упругой металлической полосы в неклассической постановке. Действие резца смоделировано как подвижная наклонная сосредоточенная сила, в точке приложения которой возникают неограниченные напряжения. Используя свойства обобщенных функций Дирака и Хэвисайда, получены аналитические выражения для компонент тензора напряжений. Построены разрешающие уравнения и сформулированы граничные условия.

Полученные результаты служат основой к построению математической модели соответствующей термодинамической задачи для упругопластической металлической полосы.

Ключевые слова: резания, режущий инструмент, предел прочности, сосредоточенная сила, деформация, математическая модель, термодинамическая задача.

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