N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
SERIES OF GEOLOGY AND TECHNICAL SCIENCES
ISSN 2224-5278
Volume 3, Number 429 (2018), 258 – 265

UDC 539.3

A. Seitmuratov[1], Z. Seylova[2], K. Kanibaikyzy[1], A. Smakhanova[1], M. Serikbol[1]

[1]The Korkyt Ata Kyzylorda State University, Kyzylorda, Kazakhstan,
[2]The Sh. Ualikhanoya Kokshetau State University, Kokshetau, Kazakhstan.
E-mail: angisin_@mail.ru, stsoias62@mail.ru, VIP kundyz@mail.ru,
Smakanova84@mail.ru, makpal.serikbol@mail.ru

APPROXIMATE EQUATION PLATE OSCILLATION FOR TRANSVERSE DISPLACEMENT OF POINTS OF THE MEDIAN PLANE

Abstract. The materials used in building structures have elastic and viscoelastic properties, are anisotropic, multilayer and other mechanical characteristics. Two-dimensional elements are components of many structures. Construction of general and approximate equations of oscillations of a various kind of plane elements is an actual problem in the development of theoretical bases of calculation of building designs and construction in general. These problems include the objectives of improved models of non-stationary nature of the structures and their elements, the materials of which are difficult mechanical, rheological properties inherent to various building designs under the influence of various external factors.

Keywords: plate, oscillation, mixed points, dynamic motion, approximate equation, three-dimensional problem, stress.

Let limitless in terms of plate thickness $2h_1$ is below the surface of a semi-infinite medium at depth $(h_0 - h_1)$. Put the plane XY in the median plane of the plate at $z = 0$. Axis OZ to the right towards the outer surface of the outer layer. Denote the layer parameters index" 1", the upper layer $[-\infty < (x, y) < \infty;\ h_1 \le z \le (h_0 - h_1)]$ by the index" 2", and the lower half-space $[-\infty < (x, y) < \infty;\ -h_1 \le z \le 0]$ - index" 3".

We assume that the materials of the upper layer of the plate and the grounds are homogeneous, isotropic, show the viscous properties.

We introduce the potentials of $\Phi^{(l)}$ and $\Psi^{(l)}$ transverse and longitudinal waves well-known formulas

$$\vec{u}^{(l)} = grad\Phi^{(l)} + rot\vec{\Psi}^{(l)}, \tag{1}$$

where $\vec{u}^{(l)}$ – the displacement vector points in the layer of the plate and the foundation.

In potentials $\Phi^{(l)}$ and $\Psi^{(l)}$ equations of motion of a layer of the plate and the foundation will take the form of:

$$N_l\left(\Delta\Phi^{(l)}\right) = \rho_l \frac{\partial^2\Phi^{(l)}}{\partial t^2} \qquad M_l\left(\Delta\vec{\Psi}^{(l)}\right) = \rho_l \frac{\partial^2\Psi^{(l)}}{\partial t^2}, \tag{2}$$

where the operator N_l is equal to:

$$N_l = L_l + 2M_l,$$

Δ – three-dimensional Laplace operator; $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ L_l, M_l – viscoelastic operators.

The Helmholtz theorem, in the absence of internal sources of the vector potential $\vec{\Psi}$ of transverse waves must satisfy the condition:

$$div\vec{\Psi}^{(l)} = 0 \quad (3)$$

a closing equation for finding four unknown potentials $\Phi^{(l)}, \Psi_1^{(l)}, \Psi_2^{(l)}, \Psi_3^{(l)}$.

Displacement, u, v, w deformation ε_{ij} and strains in Cartesian coordinates through the potentials Φ and $\vec{\Psi}$ the longitudinal and transverse waves are determined by the well-known formulas.

In [1] it is shown that the boundary value problem vibrations of the plate under the surface are reduced to the integro-differential equations (2) with boundary and initial conditions: on the outer surface $(z = h_0)$

$$\sigma_{zz}^{(2)} = f_z^{(2)}(x,y,t); \; \sigma_{jz}^{(2)} = f_{zj}^{(2)}(x,y,t); \quad (4)$$

on the contact boundary between the upper layer – plate $(z = h_1)$

$$\sigma_{zz}^{(1)} = \sigma_{zz}^{(2)}; \; \sigma_{jz}^{(1)} = 0; \; \sigma_{jz}^{(2)} = 0; \; w^{(1)} = w^{(2)} \quad (5)$$

on the boundary of the plate - foundation $(z = -h_1)$

$$\sigma_{zz}^{(1)} = \sigma_{zz}^{(3)} + f_{3z}^{(3)}(x,y,t);$$

$$\sigma_{jz}^{(1)} = 0; \; \sigma_{ij}^{(3)} + f_{jz}^{(3)}(x,y,t) = 0;$$

$$w^{(1)} = w^{(3)} + f_0^{(3)}(x,y,t) \qquad (j = x,y) \quad (6)$$

Furthermore, there should be conditions decay at infinity, i.e. at $z \to -\infty$

$$\Phi^{(3)} = 0; \; \Psi_1^{(3)} = \Psi_2^{(3)} = \Psi_3^{(3)} = 0. \quad (7)$$

The initial conditions are zero, i.e.

$$\Phi^{(l)} = \frac{\partial \Phi^{(l)}}{\partial t} = \frac{\partial \vec{\Psi}_j^{(l)}}{\partial t} = \vec{\Psi} = 0 \quad (8)$$

$$\left(l = \overline{1,3}\right), \qquad t = 0 \quad (j = 1,2,3).$$

The task of the vibrations of the plate in a differentiable environment is reduced to the study of equation (2) satisfying the boundary (4), (5), (6) and initial conditions (8).

In the study of oscillations of plate's accurate three-dimensional problem is replaced by simpler, two-dimensional points of median plane of the plate, which imposes limitations on the external conditions. These restrictions are that external forces cannot be high frequency.

The above problem is solved with the use of Fourier *X* and *Y* Laplace at *t*.

In the work [1] found the General solution of the formulated three-dimensional tasks for zero initial conditions and the General expressions for the displacement and strain.

$$u^{(l)} = \sum_{n=0}^{\infty} \left\{ \left[\left(\lambda_2^{(n)} + C_1 Q_{1n} \frac{\partial^2}{\partial x^2} \right) U^{(l)} + C_1 Q_{1n} \frac{\partial}{\partial x} \left(\frac{\partial V^{(l)}}{\partial y} + W^{(l)} \right) \right] \times \right.$$

$$\left. \times \frac{z^{2n}}{(2n)!} \right\} + \sum_{n=0}^{\infty} \left\{ \left[\left(\lambda_2^{(n)} - D_1 Q_{1n} \frac{\partial^2}{\partial x^2} \right) U_1^{(l)} - D_1 Q_{1n} \frac{\partial}{\partial x} \times \left(\frac{\partial V_1^{(l)}}{\partial y} + \lambda_2^{(1)} W_1^{(l)} \right) \right] \frac{z^{2n+1}}{(2n+1)!} \right\};$$

$$v^{(l)}=\sum_{n=0}^{\infty}\left\{\left[\left(\lambda_2^{(n)}+C_1Q_{1n}\frac{\partial^2}{\partial y^2}\right)V^{(l)}+C_1Q_{1n}\frac{\partial}{\partial y}\left(\frac{\partial U^{(l)}}{\partial x}+W^{(l)}\right)\right]\times\right.$$

$$\left.\times\frac{z^{2n}}{(2n)!}\right\}+\sum_{n=0}^{\infty}\left\{\left[\left(\lambda_2^{(n)}-D_1Q_{1n}\frac{\partial^2}{\partial y^2}\right)V_1^{(l)}-D_1Q_{1n}\frac{\partial}{\partial y}\times\right.\right.$$

$$\left.\left.\times\left(\frac{\partial U_1^{(l)}}{\partial x}+\lambda_2^{(1)}W_1^{(l)}\right)\right]\frac{z^{2n+1}}{(2n+1)!}\right\};\qquad(9)$$

$$w^{(l)}=\sum_{n=0}^{\infty}\left\{\left[\left(\lambda_2^{(n)}+C_1Q_{1n}\lambda_1^{(1)}\right)W^{(l)}+C_1Q_{1n}\lambda_1^{(1)}\left(\frac{\partial U^{(l)}}{\partial x}+\frac{\partial V^{(l)}}{\partial y}\right)\right]\times\right.$$

$$\left.\times\frac{z^{2n+1}}{(2n+1)!}\right\}+\sum_{n=0}^{\infty}\left\{\left[\left(\lambda_2^{(n)}-D_1Q_{1n}\lambda_2^{(1)}\right)W_1^{(l)}-D_1Q_{1n}\times\left(\frac{\partial U_1^{(l)}}{\partial x}+\frac{\partial V_1^{(l)}}{\partial y}\right)\right]\frac{z^{2n}}{(2n)!}\right\};$$

$$C_1=1-\frac{N_1}{M_1};\quad Q_{1n}=\sum_{m=0}^{n-1}\lambda_1^{(n-m-1)}\cdot\lambda_2^{(m)};\quad D_1=1-\frac{M_1}{N_1}$$

where operators $\lambda_1^{(1)}$ and $\lambda_2^{(1)}$ are equal

$$\lambda_1^{(1)}=\left[\rho_1N_1^{-1}\left(\frac{\partial^2}{\partial t^2}\right)-\Delta\right];\quad \lambda_2^{(1)}=\left[\rho_1M_1^{-1}\left(\frac{\partial^2}{\partial t^2}\right)-\Delta\right];\quad \Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\qquad(10)$$

$$\sigma_{xx}^{(l)}=M_1\left[\sum_{n=0}^{\infty}\left\{\left[(1-C_1)\lambda_2^{(n)}+C_1Q_{1n}\left(\lambda_2^{(1)}-2\lambda_2^{(1)}+\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\right]\right.\right.$$

$$\times\frac{\partial U^{(l)}}{\partial x}+\left[C_1Q_{1n}\left(\lambda_2^{(1)}-2\lambda_2^{(1)}+\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)-(1+C_1)\lambda_2^{(n)}\right]\times$$

$$\left.\times\left(\frac{\partial V^{(l)}}{\partial y}+W^{(l)}\right)\right\}\frac{z^{2n}}{(2n)!}+\sum_{n=0}^{\infty}\left\{\left[2D_1Q_{1n}\left(\lambda_2^{(1)}+\frac{\partial^2}{\partial y^2}\right)+(1+\alpha D_1)\lambda_1^{(n)}\right]\times\right.$$

$$\left.\left.\times\frac{\partial U^{(l)}}{\partial x}+\left[(1+\alpha D_1)\lambda_1^{(n)}-2D_1Q_{1n}\frac{\partial^2}{\partial x^2}\right]\left(\frac{\partial V_1^{(l)}}{\partial y}+\lambda_2^{(1)}W_1^{(l)}\right)\right\}\frac{z^{2n+1}}{(2n+1)!}\right];$$

$$\sigma_{yy}^{(l)}=M_1\left[\sum_{n=0}^{\infty}\left\{\left[(1+C_1)\lambda_2^{(n)}+C_1Q_{1n}\left(\lambda_2^{(1)}-2\lambda_2^{(1)}-\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]\right.\right.$$

$$\times\frac{\partial V^{(l)}}{\partial y}+\left[C_1Q_{1n}\left(\lambda_2^{(1)}-2\lambda_1^{(1)}-\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)-(1+C_1)\lambda_2^{(n)}\right]\times$$

$$\left.\times\left(\frac{\partial V^{(l)}}{\partial x}+W^{(l)}\right)\right\}\frac{z^{2n}}{(2n)!}+\sum_{n=0}^{\infty}\left\{\left[1+2D_1\left(\lambda_1^{(n)}\right)+2D_1Q_{1n}\left(\lambda_2^{(n)}+\frac{\partial^2}{\partial x^2}\right)\right]\times\right.$$

$$\left.\left.\times\frac{\partial V_1^{(l)}}{\partial y}+\left[(1+D_1)\lambda_1^{(n)}-2D_1Q_{1n}\frac{\partial^2}{\partial y^2}\right]\left(\frac{\partial U_1^{(l)}}{\partial x}+\lambda_2^{(1)}W_1^{(l)}\right)\right\}\frac{z^{2n+1}}{(2n+1)!}\right];$$

$$\sigma_{zz}^{(l)} = M_1\left[\sum_{n=0}^{\infty}\left\{\left[C_1Q_{1n}\left(\lambda_2^{(1)}-\Delta\right)-\left(1+C_1\right)\lambda_2^{(n)}\right]\left(\frac{\partial U^{(l)}}{\partial x}+\frac{\partial V^{(l)}}{\partial y}\right)+\right.\right.$$

$$+\left[\left(1+C_1\right)\lambda_2^{(n)}+C_1Q_{1n}\left(\lambda_2^{(1)}-\Delta\right)\right]W^{(1)}\left.\right\}\frac{z^{2n}}{(2n)!}+\sum_{n=0}^{\infty}\left\{-\left[2D_1Q_{1n}\lambda_2^{(1)}+\lambda_1^{(n)}\right]\times\right.$$

$$\times\left(\frac{\partial U_1^{(l)}}{\partial x}+\frac{\partial V_1^{(l)}}{\partial y}\right)+\lambda_2^{(1)}\left[\lambda_1^{(n)}+2D_1Q_{1n}\Delta\right]W_1^{(1)}\left.\right\}\frac{z^{2n+1}}{(2n+1)!}\left.\right];$$

$$\sigma_{xy}^{(l)} = M_1\left[\sum_{n=0}^{\infty}\left\{\left[2C_1Q_{1n}\frac{\partial^2}{\partial x^2}+\lambda_2^{(n)}\right]\frac{\partial V^{(1)}}{\partial y}+\left[\lambda_2^{(n)}+2C_1Q_{1n}\frac{\partial^2 W^{(1)}}{\partial x\partial y}\right\}\times\right.\right.$$

$$\times\frac{z^{2n}}{(2n)!}+\sum_{n=0}^{\infty}\left\{\left[\lambda_1^{(n)}+D_1Q_{1n}\left(\lambda_2^{(1)}-\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]\frac{\partial U_1^{(l)}}{\partial y}+\right.$$

$$+\left[D_1Q_{1n}\left(\lambda_2^{(1)}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)+\lambda_1^{(n)}\right]\frac{\partial V_1^{(l)}}{\partial x}-2D_1Q_{1n}\lambda_2^{(1)}\frac{\partial^2 W_1^{(1)}}{\partial x\partial y}\left.\right\}\times\frac{z^{2n+1}}{(2n+1)!}\left.\right];$$

$$\sigma_{xz}^{(1)} = M_1\left[\sum_{n=0}^{\infty}\left\{\left[C_1\left[2\lambda_1^{(1)}Q_{1n}+\lambda_2^{(n)}\right]\frac{\partial^2 V^{(1)}}{\partial x\partial y}+\left[2C_1Q_{1n}\lambda_1^{(1)}\frac{\partial^2}{\partial x^2}+\right.\right.\right.\right.$$

$$+\lambda_2^{(n)}\left[\left(1-C_1\right)\lambda_1^{(1)}-C_1\frac{\partial^2}{\partial y^2}\right)\left.\right]V^{(1)}+\left[\left(1+C_2\right)\lambda_2^{(n)}+2C_1Q_{1n}\lambda_1^{(1)}\right]\frac{\partial W^{(1)}}{\partial x}\left.\right\}\times$$

$$\times\frac{z^{2n+1}}{(2n+1)!}\left.\right]+\sum_{n=0}^{\infty}\left\{\left[\left(\lambda_2^{(1)}-\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)D_1Q_{1n}+\lambda_1^{(n)}\right]V_1^{(1)}-\right.$$

$$-2D_1Q_{1n}\frac{\partial^2 V_1}{\partial x\partial y}-\left[D_1Q_{1n}(\lambda_2^{(1)}-\Delta)-\lambda_1^{(n)}\right]\frac{\partial W_1^{(1)}}{\partial x}\left.\right\}\frac{z^{2n}}{(2n)!}$$

$$\sigma_{yz}^{(1)} = M_1\left[\sum_{n=0}^{\infty}\left\{\left[C_1\left[2\lambda_1^{(1)}Q_{1n}+\lambda_2^{(n)}\right]\frac{\partial^2 U^{(1)}}{\partial x\partial y}+\left[2C_1Q_{1n}\lambda_1^{(1)}\frac{\partial^2}{\partial y^2}+\right.\right.\right.\right.$$

$$+\lambda_2^{(n)}\left[\left(1-C_1\right)\lambda_1^{(1)}-C_1\frac{\partial^2}{\partial x^2}\right)\left.\right]V^{(1)}+\left[2C_1Q_{1n}\lambda_1^{(1)}+\left(1+C_2\right)\lambda_2^{(n)}\right]\frac{\partial W^{(1)}}{\partial x}\left.\right\}\times$$

$$\times\frac{z^{2n+1}}{(2n+1)!}\left.\right]+\sum_{n=0}^{\infty}\left\{\left[\left(\lambda_2^{(1)}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)D_1Q_{1n}+\lambda_1^{(n)}\right]V_1^{(1)}-\right.$$

$$-2D_1Q_{1n}\frac{\partial^2 V_1}{\partial x\partial y}-\left[D_1Q_{1n}(\lambda_2^{(1)}-\Delta)-\lambda_1^{(n)}\right]\frac{\partial W_1^{(1)}}{\partial y}\left.\right\}\frac{z^{2n}}{(2n)!}, \qquad (11)$$

where unknown $U^{(1)}, V^{(1)}, W_1^{(1)}$ are the tangent and normal displacements of the points in the plane $z = 0$ and points of median plane of the plate, $U_1^{(1)}, V_1^{(1)}, W^{(1)}$ - value of derivatives Z transverse displacement or values type of deformation ($W^{(1)}$-deformation at $Z = 0$).

Operators $\lambda_1^{(2)}, \lambda_2^{(1)}$ are two-dimensional integro-differential describing the propagation of longitudinal and transverse waves in a plane $z = 0$.

For finding the unknown $U^{(1)}, V^{(1)}, W_1^{(1)} U_1^{(1)}, V_1^{(1)}, W^{(1)}$ we have boundary conditions (4)–(6).

Using the formulas (9) and (11) of strains and displacements substituting these expressions into the boundary conditions (4) - (6) equations are obtained for determining the unknown functions that are general solutions of the considered problem and describing oscillations of a three-dimensional environment.

For the study of oscillations of rectangular plates in terms of the need to formulate boundary problems.

Under the boundary tasks fluctuations limited plate in the plan, which is under the surface, means the conclusion of the equation of oscillations of plates; the formulation of boundary conditions on the edges of the plate and the initial conditions for the functions.

As a general equations of oscillations of plates obtained by the author [1] contain derivatives of any order of the coordinates x, y and time t, stacked on the structure and therefore are not suitable for solving applied tasks and of engineering calculations.

For this purpose it is necessary to formulate approximate boundary value problems of oscillation.

In [2-5] is obtained approximate equation of transverse vibrations of the plate for transverse shift $W_1^{(1)}$ points of median plane of the plate in the shape of

$$A_1\left(\frac{\partial^2 W_1^{(1)}}{\partial t^2}\right)+A_2\left(\frac{\partial^4 W_1^{(1)}}{\partial t^4}\right)+A_3\left(\Delta\frac{\partial^2 W_1^{(1)}}{\partial t^2}\right)+A_4\left(\Delta^3 W_1^{(1)}\right)+P\left(W_1^{(1)}\right)=\Phi(x,y,t), \tag{12}$$

where $A_j, P, \Phi(x,y,t)$ are equal to

$$A_1=\rho_1 M^{-1}h_1+\rho_2 N_2^{-1}(h_0-h_1);$$

$$A_2=\rho_1^2 M_1^{-1}\left(N_1^{-1}+3M_1^{-1}\right)\frac{h_1^3}{6}+\rho_2 N_2^{-1}\left[\rho_2 N_2^{-1}\frac{(h_0-h_1)^3}{6}-\rho_1 N_1^{-1}\frac{h_1^2(h_0-h_1)}{2}\right];$$

$$A_3=\left[\left(3-4M_2N_2^{-1}\right)\frac{(h_0-h_1)^3}{6}-\left(2M_1N_1^{-1}\right)\frac{h_1^2(h_0-h_1)}{2}\right]\rho_2 N_2^{-1}-2\rho_1\left(3M_1^{-1}-2N_1^{-1}\right)\frac{h_1^3}{3}$$

$$A_4=4\left(1-M_1N_1^{-1}\right)\frac{h_1^3}{3}-4\left(1-M_2N_2^{-1}\right)\left(M_2N_2^{-1}\right)\frac{(h_0-h_1)^3}{6}$$

$$P=\frac{S}{2}\rho_1 M_1\left\{\frac{\partial}{\partial t}+\frac{h_1^2}{2}\left[\rho_1\left(M_1^{-1}+3N_1^{-1}\right)\left(\frac{\partial^3}{\partial t^3}\right)-4\left(\frac{\partial}{\partial t}\right)\Delta\right]+2\left(M_1N_1^{-1}\right)\left(\rho_2N_2^{-1}\right)\left(\frac{\partial^3}{\partial t^3}\right)h_1(h_0-h_1)\right\};$$

$$\Phi(x,y,t)=\left[1-\left(3-2M_1N_1^{-1}\right)\frac{h_1^2}{2}\Delta+\left(\rho_1M_1^{-1}\right)\left(\frac{\partial^2}{\partial t^2}\right)\frac{h_1^2}{2}\left\{F_3+M_2^{-1}f_z^{(3)}\left[\left(M_1N_1^{-1}\right)\left(\rho_2N_2^{-1}\right)\left(\frac{\partial^3}{\partial t^3}\right)(h_0-h_1)h_1\right]\right. \tag{13}$$

Reaction of the basis of P, is determined by the formula (13) includes both the velocity of the points transverse displacement of the plane $z=0$ and the odd derivatives time.

Thus, the law of resistance $P\left(W_1^{(1)}\right)$ (13) explicitly contains the parameters of the plate, foundation and topcoat.

Despite the fact that the equation (12) is approximate, it is quite difficult. In the operators (13) contains all the parameters and operators characterizing the mechanical and rheological properties of the materials of plates - and the base layers and their thicknesses.

We derive the boundary conditions on the edges of a rectangular plate. For simplicity, consider a flat edge $x=const$, the boundary conditions for easy record of the conditions for $x=const$, and for an arbitrary curved edge of the known formulas through the boundary conditions at $x=const$, $y=const$.

Boundary conditions will display on the theory of thick platform or plates.

Based on the boundary conditions on a surface of a plate $z = h$ or $z = -h$ we obtain the dependence of the quantities $u_1^{(1)}, V_1^{(1)}$, from transverse shift $W_1^{(1)}$.

$$u_1^{(1)} = -\frac{\partial W_1^{(1)}}{\partial x}; \qquad V_1^{(1)} = -\frac{\partial W_1^{(1)}}{\partial y}. \tag{14}$$

Rigid fixation of the edge $x = const$. As it is known from the theory of thick plates are two types of such fastening

$$u_1^{(1)} = v_1^{(1)} = w_1^{(1)} = 0 \tag{15}$$

or

$$u_1^{(1)} = w_1^{(1)} = \sigma_{xy}^{(1)} = 0 \tag{16}$$

hinge-supported edge $x = const$.

For this fix also two ways of fastening.

$$v_1^{(1)} = w_1^{(1)} = \sigma_{xx}^{(1)} = 0 \tag{17}$$

or

$$w_1^{(1)} = \sigma_{xx}^{(1)} = \sigma_{xy}^{(1)} = 0 \tag{18}$$

Edge, free from strain.

For the free edge stringent conditions are

$$\sigma_{xx}^{(1)} = \sigma_{xz}^{(1)} = \sigma_{xy}^{(1)} = 0. \tag{19}$$

Rigid and hinged fixations are quite simple, and using the approximate formulas (13) and the dependence (15), we obtain for the transverse displacement of the boundary conditions:

For rigid fixation

$$W_1^{(1)} = \frac{\partial W_1^{(1)}}{\partial x} = 0. \tag{20}$$

For hinged fixation

$$W_1^{(1)} = \frac{\partial^2 W_1^{(1)}}{\partial x^2} = 0. \tag{21}$$

For the edge, which is free from strain, in the formula of- σ_{xx} take the first additive component | z, and for $\sigma_{xz}^{(1)}$ the first two, as the first identical equation is zero due to conditions (15)

It is obtained

$$(2+3D_1)\frac{\partial^2 W_1^{(1)}}{\partial x^2} + (1+D_1)\left[2\frac{\partial^2 W_1^{(1)}}{\partial y^2} - \rho M^{-1}\left(\frac{\partial^2 W_1^{(1)}}{\partial t^2}\right)\right] = 0$$

$$\frac{\partial}{\partial x}\left[2\Delta W_1^{(1)} - \rho M^{-1}\left(\frac{\partial^2 W_1^{(1)}}{\partial t^2}\right)\right] = 0 \tag{22}$$

excluding $\frac{\partial^2 W_1^{(1)}}{\partial t^2}$ in the second condition (22) from the first, we obtain

$$(2+3D_1)\frac{\partial^2 W_1^{(1)}}{\partial x^2} + (1+D_1)\frac{\partial^2 W_1^{(1)}}{\partial y^2} - \rho(1+D_1)M^{-1}\left(\frac{\partial^2 W_1^{(1)}}{\partial t^2}\right) = 0$$

$$\frac{\partial^3 W_1^{(1)}}{\partial x^3} = 0. \tag{23}$$

The third of the conditions (19) gives $\frac{\partial^3 W_1^{(1)}}{\partial x} = F(t)$, i.e. in the first approximation $\frac{\partial W_1^{(1)}}{\partial x}$ does not depend on y, and decide upon a solution to a particular problem.

The first additive component in (23) differs from the classical and the second matches. The first condition (23) takes into account the deformability of the edge over time and d'Alembert principle similar to the dynamics of a material point.

General initial conditions for the plate as a three-dimensional body are of the form:

$$u^{(1)} = v^{(1)} = w^{(1)} = 0; \qquad \frac{\partial u^{(1)}}{\partial t} = \frac{\partial v^{(1)}}{\partial t} = \frac{\partial w^{(1)}}{\partial t} = 0; \quad (t = 0). \tag{24}$$

Using dependency (14) for movements have

$$\begin{aligned} u^{(1)} &= -\frac{\partial W_1^{(1)}}{\partial x} z + D_1 \frac{\partial}{\partial x} \Delta W_1^{(1)} \frac{z^3}{6}; \\ v^{(1)} &= -\frac{\partial W_1^{(1)}}{\partial y} z + D_1 \frac{\partial}{\partial y} \Delta W_1^{(1)} \frac{z^3}{6}; \\ w^{(1)} &= W_1^{(1)} + \left[(2D_1 - 1)\Delta W_1^{(1)} + (1 - D_1)\rho M^{-1} \frac{\partial^2 W_1^{(1)}}{\partial t^2} \right] \frac{z^2}{2} \end{aligned} \tag{25}$$

In the beginning let us consider the initial conditions from (24) for the movement. Then from the expressions (25) we get

$$\begin{aligned} &\frac{\partial W_1^{(1)}}{\partial x} = 0; \qquad \frac{\partial}{\partial x} \Delta W_1^{(1)} = 0; \\ &\frac{\partial W_1^{(1)}}{\partial y} = 0; \qquad \frac{\partial}{\partial y} \Delta W_1^{(1)} = 0 \ ; \end{aligned} \tag{26}$$

$$W_1^{(1)} = 0; \qquad \frac{\partial^2 W_1^{(1)}}{\partial t^2} = 0. \tag{27}$$

Differentiating the expression (25) and by using the second three initial conditions (24), we similarly obtain

$$\frac{\partial W_1^{(1)}}{\partial t} = \frac{\partial^3 W_1^{(1)}}{\partial t^3} = 0 \ . \tag{28}$$

Initial conditions (26) and (27) provide the necessary number of initial conditions for transverse shift $W_1^{(1)}$ meets with the hyperbolic equation of fourth order in the coordinates and time.

In conclusion, it should be noted that the evaluation and conclusion initial and boundary conditions for the plate under the surface completely coincides with similar initial boundary conditions for a free plate, obtained in the works [1].

Thus, in the formulation of boundary value problems boundary conditions do not depend on the presence of the upper layer and lower foundation.

REFERENCES

[1] Filippov I.G., Filippov S.I. Dynamic stability theory of rods. Proceedings of the Russian-Polish seminar. Theoretical Foundations of construction. Warsaw, 1995. P. 63-69.

[2] Filippov I.G., An approximate method for solving dynamic viscoelastic media // PMM. 1979. 43(1): 133-137.

[3] Filippov I.G., Filippov S.I., Kostin V.I. Dynamics of two-dimensional composites // Proceedings of the International Conference on Mechanics and Materials, USA, Los Angeles, 1995. P. 75-79.

[4] Seytmuratov A.Zh. Determination of natural frequencies of the plate // Bulletin Treasury Series mathematics, mechanics, computer science. 2010. 4(67): 120.

[5] Seytmuratov A.Zh., The impact of the moving load on the surface of a layered elastic half-plane // Herald KazNPU. 2010. 3: 112-11.

[6] Brunelle E.J. Buckling of transversely isotopic Mindlen plates // AIAA. 1977. 9(6): 1018-1022.

[7] Bergman G.G. Elastic waves propagation in fluid saturated porous media G. Asoust. Soc. America. 1981. 2: 416-424.

[8] Bowen P.M., Incompressible porous media models by use of the theory mixtures // Int. J. Engng. Sci. 1980. 18: 1129-1148.

[9] De Boer R. Theory of porous media. Berlin: Springer, 2000.

[10] Ellsiepen P. Zeit- und ortsadaptive Verfahren angewandt auf Mehrphasenprobleme poroser Medien. Bericht Nr. II-3, Universitat Stuttgart, Institute for Mechanik, Lehrstuhl II. 1999.

[11] Diebels S., Ehlers W. Dynamic analysis of a fully saturated porous medium accounting for geometrical and material non-linearities // Int. J. Num. Methods Engng. 1996. 39: 81-97.

[12] Diebels S., Ehlers W., Markert B. Neglect of the fluid-extra stresses in volumetrically coupled solid-fluid problems // ZAMM. 2001. 81, S521-S522.

[13] Umbetov U., Seytmuratov A.Zh. Decomposition approach to the modeling of seismic resistance of building structures // M., PAE. Modern high technologies. 2012. 6: 43.

А. Ж. Сейтмұратов[1], З. Т. Сейлова[2], Қ. Қанибайқызы[1], А. Қ. Смаханова[1], М. С. Серікбол[1]

[1]Қорқыт Ата атындағы Қызылорда мемлекеттік университеті, Қызылорда, Қазақстан,
[2]Ш. Уалиханов атындағы Көкшетау мемлекеттік университеті, Көкшетау, Қазақстан

КӨЛБЕУ ЫҒЫСУЛАР ЖАҒДАЙЫНДАҒЫ ЖАЗЫҚТЫҚТЫҢ ОРТАСЫНДА ОРНАЛАСҚАН НҮКТЕЛЕР ҮШІН ПЛАСТИНА ТЕРБЕЛІСІНІҢ ЖУЫҚ ТЕҢДЕУІ

Аннотация. Мақалада сыртқы әсерлер кезінде иілгіш және тұтқыр иілгіш пластиналардың динамикалық қозғалысын анықтау дірілдерін математикалық теория тұрғысынан түсіндіруге талпынған. Осы көзқарасқа негізделген физикалық бейсызықтық материалдар теңдеулеріне жақындатылған, бастапқы ауыстыру мен кернеулерді ескерген және ескерусіз тұтқыр иілгіш пластиналардың бойлық және көлденең дірілдерінің нақты теңдеулері шығарылған. Нақты теңдеулер негізінде сол немесе олардан кейінгі дәлдік дәрежесі бар кейбір жуықтау теңдеулері талданған және олар үшін шекаралық есептер құрастырылған.

Түйін сөздер: пластинка, тербеліс, аралас нүктелер, динамикалық қозғалыс, жуық теңдеу, үш өлшемді есеп, кернеу.

А. Ж. Сейтмуратов[1], З. Т. Сейлова[2], Қ. Қанибайқызы[1], А. К. Смаханова[1], М. С. Серікбол[1]

[1]Кызылординский государственный университет им. Коркыт Ата, Кызылорда, Казахстан,
[2]Кокшетауский государственный университет им. Ш. Уалиханова, Кокшетау, Казахстан

ПРИБЛИЖЕННОЕ КОЛЕБАНИЕ ПЛАСТИНЫ УРАВНЕНИЯ ДЛЯ ПОПЕРЕЧНОГО СМЕЩЕНИЯ ТОЧЕК СРЕДИННОЙ ПЛОСКОСТИ

Аннотация. В настоящей работе предпринята попытка изложения математической теории колебаний упругой или вязкоупругой пластинки для изучения динамического их поведения при нестационарных внешних воздействиях. На основе такого подхода выведены точные уравнения продольных и поперечных колебаний вязкоупругих пластин с учетом и без учета начальных смещений и напряжений, приближенные уравнения физической нелинейности материала. На основе точных уравнений проанализированы некоторые вытекающие из них приближенные уравнения с той или иной

Ключевые слова: пластина, колебание, смешанные точки, динамическая движение, приближенные уравнение, трехмерная задача, напрянение.

Information about authors:

Seitmuratov Angisin – Doktor of Physical and Matematical Sciences, Professoz, The Korkyt Ata Kyzylorda state university, Kyzylorda, Kazakhstan

Seyloya Zoya Tuleubaebna – kandidate of pedagogical sciences, associate professor Kokshetau state university named Sh. Ualikanova, Kazakhstan

Kanibaikyzy Kundyzay – Master degree of pedagogical sciences, The Korkyt Ata Kyzylorda state university, Kyzylorda, Kazakhstan

Smakhanova Aizhan Korganbekovna – Master degree of mathematical sciences, The Korkyt Ata Kyzylorda state university, Kyzylorda, Kazakhstan

Serikbol Makpal Serikbolkyzy – Master degree of mathematical sciences, The Korkyt Ata Kyzylorda state university, Kyzylorda, Kazakhstan