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**V. Ya. Tsvetkov**

Research and Design Institute of design information, automation and communication on railway transport,
JSC NIIAS, Moscow, Russia.
E-mail: cvj2@mail.ru

# TRADE-OFF TRANSPORTATION PROBLEM

**Abstract.** The purpose of the work is to offer a new solution to the transportation problem. The article looks into a new type of transportation problems that take into account supply and demand. The classical transportation problem expresses the interests of one party - goods supplier. Supply and demand exist in real market conditions. The article introduces a new concept of dual transportation problem. Double transportation problem respects the interests of supplier and consumer. Such problem is called double, since it gives two solutions. The article analyzes different criteria for solving game-theoretic problems and chooses a suitable method for solving a transportation problem. The article provides a solution to the transportation problem based on pure strategies using the minimax criterion. Solving a double transportation problem allows finding the saddle point. The saddle point determines the equilibrium point between supply and demand and is a market solution to the transportation problem. This approach gives more possibilities for solving transportation problems in market conditions.
**Keyword:** transport problem, optimal solution, game theory, demand. proposal, optimization.

**Introduction.** Mathematical linear programming problem is called a transportation problem (Monge – Kantorovich) (Rachev, 1985, Levin, 2006). The problem situation is that there are many cargo or goods suppliers and many consumers. Transportation problem involves finding an optimal cargo transportation plan from suppliers to points of consumption with minimal transportation costs (Benamou, Brenier, 2000). Transportation problem is called unbalanced or open if the total volume of cargo offers is not equal to the volume of demand, necessary for points of consumption. A classical transportation problem is a problem with two types of optimality criteria. The first type is a cost criterion that requires a minimum of transportation costs. The second type of criteria is a time criterion that requires a minimum of transportation time. In logistics, the second criterion is implemented as a "just in time" paradigm (Aycock, 2003).

The title 'transportation problem' includes a wide range of problems with a general mathematical model (Champion, De Pascale, 2010, Chang et al., 2010, Davidsson, P., et al., 2005). These problems are solved by linear programming methods (Dantzig, 2016). Solving the problem involves optimization methods. The classical transportation problem can be solved by the simplex method (Nelder, Mead, 1965, Tsvetkov, 2001). Taking into account the specifics of the transportation problem provides additional solutions. Currently there are many fuzzy transportation problem options. A common weakness of stating and solving the transportation problem is that it is not a market one.

The classical transportation problem represents the interests of one market participant and characterizes the market supply. Besides supply, there is demand in the market, which is an equally important independent factor. This factor is disregarded when solving transportation problems. Such information situation gives rise to looking into a transportation problem, factoring in both supply and demand. Such a transportation problem can be called a double one, since it provides two solutions: one for the supplier, and one for the consumer.

**Game-Theoretic Approach when Solving the Transportation Problem.** Conditions for solving a matrix problem can be called an information situation (Tsvetkov, 2012). An information situation is a model of conditions for solving a problem or a model of a situation, under which a problem must be solved. The game-theoretic approach (Friedman, 1990, Camerer , 2011) accounts for the interests of two parties. Therefore, it can be used to analyze solutions of an open transportation problem. The game-theoretic approach takes into account market demand and supply. To support decision-making, game theory uses a set of mathematical models and rules for their application under uncertainty. Along with the rules, the game may be characterized by the goals that each player seeks to achieve. To achieve the goals, the player employs strategy, tactics and prompt actions. This approach brings the game theory and management together.

In game theory, strategy is defined as a generalized plan for achieving one or several goals. In solving a transportation problem, it is a basis (Tsvetkov, 2001). The following methods are used for solving a transportation problem: the potential method, the Vogel's method (Samuel, Venkatachalapathy, 2011) and others.

Decision-making in game theory means an analytical approach to choosing the next best (Trizano-Hermosilla, Alvarado, 2016) or an action sequence (Tikhonov, Tsvetkov, 2001). In solving a transportation problem, it is called an optimal plan. Transportation problem is described based on the matrix model. Game theory calls such problems matrix games. Matrix game (Vijay, et al., 2005) – is a matrix model representing a two-player zero-sum game. In a matrix game, the strategies of one player *A* are displayed as rows, and the strategies of another player *B* are displayed as columns. For the purpose of a transportation problem, let us consider the option when player *A* represents demand, and player *B* represents supply. Matrix entries $a_{ij}$are called gains or payoffs, which is why the matrix of relations between the players is called the payoff matrix (figure 1). Hereinafter we will denote the payoff matrix as *AA*. Additionally, the matrix is also called the decision matrix.

| | | | |
|---|---|---|---|
| | $B_1$ | | $B_n$ |
| $A_1$ | $a_{11}$ | | $a_{n1}$ |
| | | $a_{ij}$ | |
| $A_m$ | $a_{m1}$ | | $a_{mn}$ |

Figure 1 – Payoff Matrix

Figure 1 shows a decision matrix with an auxiliary column on the left and an auxiliary row at the top. These column and row show the interests of consumers *A* and the interests of suppliers *B*.

In game theory, strategy *A* is seen as an action, another strategy *B*as a reaction. Sometimes each cell or column of a matrix is characterized by a possible state called an external state. In this case, in addition to the gain, each cell of the matrix will have another characteristic - the probability of an external state. Let's denote the probability of external state $F_j$ as $q_i$ – (figure 2). Figure 2 shows a payoff matrix with states.

| | | | |
|---|---|---|---|
| | $B_1 F_1$ | $B_i F_j$ | $B_n F_n$ |
| $A_1$ | $a_{11} q_1$ | | $a_{n1} q_n$ |
| | | $a_{ij} q_i$ | |
| $A_m$ | $a_{m1} q_1$ | | $a_{mn} q_n$ |

Figure 2 – Payoff Matrix with Payoff and State Values

It is proved for matrix games that any of them has a solution. A solution can be found by reducing the game model to a linear programming problem. Transportation problem is also solved by linear programming methods; therefore, the information situation taken has algorithmic similarity, but a different problem statement.

**Finding Tradeoff Solutions.** In the payoff matrix (figure 1) solutions are found based on certain rules or criteria. In game theory, there are several proven criteria used in decision-making. Each criterion is based on a particular strategy and applies under specific conditions. For the sake of simplicity, further analysis is carried out for strategy $A$, while strategy $B$ is considered to be opposing. Strategy $A$ involves cash payment, while strategy $B$ can only agree or disagree with the payment. There are a number of criteria for finding solutions for the payoff matrix (figure 1). Let's highlight some of the criteria.

*The minimax principle* (MM) (Lehmann, Romano, 2005). According to this criterion, strategy $A$, where the minimum gain is maximum, is chosen as the optimal strategy. Consumers most often use this criterion.

*The Bayes–Laplace Criterion* (Aldrich, 2008). The criterion is based on calculating the average gain for each row of the payoff matrix and choosing the maximum therefrom. Consumers do not use this criterion when the demand can be fully met.

*The Savage criterion* (Tikhonov, Tsvetkov, 2001). The concept of risk is introduced in the statistical decision theory. Savage's criterion is also called Savage's minimax risk criterion. Consumers do not use this criterion when the demand can be fully met.

*The Hurwitz criterion* (Gil, et al., 2004). Hurwitz suggested a criterion at a point lying between the point of view of extreme optimism and extreme pessimism.

*The Hodges–Lehmann criterion* (Tikhonov, Tsvetkov, 2001). The criterion is based simultaneously on the MM criterion and the Bayes-Laplace criterion. The ν parameter expresses the degree of confidence in the probability distribution used. If confidence is high, the Bayes-Laplace criterion will dominate, otherwise the – MM-criterion will.

**Application of the Minimax Principle for Solving a Double Transportation Problem.** Pursuant to the criterion, the rule for finding a solution to the transportation problem can be interpreted as follows:

*One more column with the smallest $a_{ir}$ results of each row is added to the decision matrix. It is then necessary to choose those options, in the rows of which there is the highest $a_{ir}$ value of this column.*

With this criterion, the strategy when choosing a solution is based on the fact that player $A$ tries to lose as little as possible, that is, to eliminate the risk. This means that a decision maker cannot face a result worse than the one he relies on. This defines the name of this criterion as maximin. It is reasonable to use the maximin (MM) criterion if the following information situation occurs:

1. The possibility of occurrence of external states $F_j$ is unknown;
2. The solution is implemented only once (no refund is possible);
3. It is necessary to eliminate the risk or minimize it.

The use of this criterion is based on consumer *A's* assumption that the supplier will answer every move with a move where the gain is minimal. This is called a pessimistic point of view. Applying the criterion involves analyzing the *AA* payoff matrix.

The first step involves finding the minimum. For each value $i$ ($i$= 1, m is the number of rows) the minimum value of the gain is determined depending on the supplier $B$'s strategies used.

$$\alpha = \min_i a_{ij} \quad (i=1,m). \tag{1}$$

The $i$ index in the description (1) of the $\min_i$i function defines the search range. In this case, the $i$ index varies in rows and the search is made by rows. The search determines the minimum gain for player $A$, provided that he adopts his $i$ th pure strategy.

The next step involves searching among these minimum gains. At the second step, such a strategy $i = i_o$ is found, at which this minimum gain will be maximum, i.e.

$$\max_i \min_j a_{ij} = a_{iojo} = \underline{\alpha} \qquad (2) \text{ is found.}$$

The number $\underline{\alpha}$ defined by the formula (2) is called the net lower value of the game or maximin. It shows what minimum gain consumer $A$ can secure himself by applying his pure strategies with all sorts of actions taken by supplier $B$. The value $\underline{\alpha}$ in expression (2) is also called the maximin gain. This strategy guarantees a gain of at least $\underline{\alpha}$.

You can choose supplier *B's* side and reason for it. Supplier $B$ should strive to minimize player $A$'s gain by applying its strategies. Therefore, for player $B$, the maximum is found according to expression (3)

$$\alpha = \max_j a_{ij} \tag{3}$$

The max gain of player *A* is determined in expression (3), provided that player *B* applies his *j*th pure strategy, then player *B* searches for his $j = j_1$ strategy where player *A* will receive min gain, i.e. finds

$$\min_j \max_i a_{ij} = a_{i1j1} = \bar{\alpha} . \qquad (4)$$

The $\bar{\alpha}$ number, determined from the expression (4), is called the net upper value of the game or the minimax. It shows the maximum gain supplier *B* can secure with its strategies. In other words, consumer *A,* by applying his pure strategies, can secure a gain of at least $\underline{\alpha}$, and player *B,* by applying his pure strategies, can prevent player *A* from gaining more than $\bar{\alpha}$.

The principle dictating the players the choice of appropriate strategies (maximin and minimax) is called the minimax principle.

**Solving a Double Transportation Problem in Pure Strategies.** Let's analyze the fundamentals of solving a double transportation problem in pure strategies. A zero-sum matrix game of two players *A*, *B*can be considered as follows. Consumer *A* has *m* strategies $i = 1,2,...,m$, supplier *B* has *n* strategies $j = 1,2,...,n$. Each pair of strategies $(i,j)$ is assigned a number $a_{ij}$, which expresses player *A*'s gain at player *B*'s expense if the first player uses his *i*th strategy, and *B*– his *j*th strategy. Figure 1 shows the payoff matrix.

If we take a look at the payoff matrix *AA* in figure 1, then each matrix game with the *AA* matrix comes down to consumer *A* choosing the A *i*th row, and supplier *Bj*th column. In such a game, consumer *A* gets a $a_{ij}$ gain. In this situation, if $a_{ij} < 0$, this means that consumer *A* pays a sum of $|a_{ij}|$ to supplier *B*. If $a_{ij} > 0$, this means that supplier *B* sells products to consumer *A*for the amount of$|a_{ij}|$. This ends the game. Each player strategy $i=1,m; j=1, n$ is called a pure strategy. The main thing in game theory is to find the optimal players' strategies. A player's strategy is considered optimal if application thereof provides him with the biggest guaranteed gain with all sorts of strategies of another player.

Payment matrix game $\underline{\alpha} = \bar{\alpha}$ is said to have a saddle point in pure strategies and the *net value* of the game.

$$\upsilon = \underline{\alpha} = \bar{\alpha}.$$

Saddle point – is a pair of pure strategies $(i_o, j_o)$ of players *A* and *B* respectively, when equality $\underline{\alpha} = \bar{\alpha}$ is achieved. In market interpretation, this means finding an equilibrium point between supply and demand. When solving a transportation problem, this concept has the following meaning: if a consumer follows a strategy that corresponds to the saddle point, a supplier cannot do better than follow a strategy that corresponds to the saddle point. Mathematically, it may be written differently:

$$a_{ij0} \le a_{i0j0} \le a_{i0j} , \qquad (5)$$

where *i*, *j* – are any pure strategies of players *A* and *B* respectively; $(i_o, j_o)$ – are strategies forming the saddle point. According to (5), saddle element $a_{i0j0}$ is minimal in the $i_o$ th row and maximal in the $j_o$th column in the *AA* matrix. A saddle point is found by finding the minimum element in each row of the payoff matrix and checking it for being the maximum in its column. If so, it is a saddle element, and a pair of strategies corresponding thereto, forms a saddle point, that is, an equilibrium point between supply and demand.

A pair of pure strategies $(i_o, j_o)$ of consumer *A* and supplier *B*, forming the saddle point, and saddle element $a_{i0j0}$, are called the equilibrium solution to the transportation problem. At the same time, $i_o$ and $j_o$are called optimal pure strategies of consumer *A* and supplier *B*.

**Conclusion.** Accounting for the interests of not only the supplier, but also the consumer results in the need to solve the transportation problem of a new type: 'Demand and Supply Transportation Problem' or 'Trade-Off Transportation Problem'. Such a transportation problem can be called a double transportation problem, since it provides two solutions: one for the supplier, and one for the consumer. Game-theoretic methods are used to solve this problem. Using the minimax principle with pure strategies to solve the transportation problem allows for finding a solution to the transportation problem in the form of a saddle point. The saddle point corresponds to the point of market equilibrium between supply and demand and is an equilibrium solution to the transportation problem, while the classical solution to the transportation problem represents the interests of only one party and does not provide an equilibrium solution.

**В. Я. Цветков**

Научно-исследовательский и проектно-конструкторский институт информатизации, автоматизации и связи на железнодорожном транспорте, АО "НИИАС", Москва, Россия

## КОМПРОМИССНЫЕ РЕШЕНИЯ ЗАДАЧ ТРАНСПОРТИРОВКИ

**Аннотация.** Цель работы – предложить новое решение транспортной проблемы. В статье рассматривается новый вид транспортных проблем, учитывающий спрос и предложение. Классическая проблема перевозки выражает интересы одной стороны – поставщика товаров. Спрос и предложение существуют в реальных рыночных условиях. В статье представлена новая концепция двойной транспортной. Двойная транспортировка учитывает интересы поставщика и потребителя. Она называется двойной, поскольку она дает два решения. В статье анализируются различные критерии решения задач теории игр и определяется подходящий метод решения транспортной проблемы. В статье представлено решение транспортной задачи на основе чистых стратегий с использованием минимаксного критерия. Решение двойной транспортировки позволяет найти седловую точку. Седловая точка определяет точку равновесия между спросом и предложением и является рыночным решением проблемы транспортировки. Такой подход дает больше возможностей для решения транспортных проблем в рыночных условиях.

**Ключевые слова:** задача транспортировки, оптимальное решение, теория игр, спрос, предложение, оптимизация.

**Information about author:**

Tsvetkov V. Ya., Professor, Doctor of Technical Sciences. Academician of the International Academy of Sciences of Eurasia (IEAS), Strategic analysis and development center. The deputy head. Research and Design Institute of design information, automation and communication on railway transport, JSC NIIAS, Moscow, Russia; cvj2@mail.ru; https://orcid.org/0000-0003-1359-9799

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