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WAVE SPREADING IN RESILIENT VISCOUS-PLASTIC LAYER WITH CAVITY ON RIGID BASE

Abstract. The research paper is devoted to studying wave spreading in resilient viscous-plastic layer with cavity, situated on a rigid base under the dynamic pressure of day surface. This problem is solved by "discontinuity disintegration" method of S. K. Godunov. The "discontinuity disintegration" method of S.K.Godunov is effective for hyperbolic type quasilinear systems equations numerical solution, for wide range problems solution of gas dynamics, aerodynamics, as well as for other problems of solid medium. Research results allow estimate the wave process parameters around rectangular cavity, and also define their tensed and deformed condition that can be used at assessment of underground construction durability and stability, as well as geodynamic task solution. And also at construction in particularly complex conditions (when soil is characterized as structural and unstable soil) industrial, civil, transport, and military constructions, where geomechanical processes differ from normally condensed soil.

Key words: layer, resilient viscousplasticity, cavity, pressure, wave, tensed and deformed condition.

Problem definition and aspen equations creation on resilient viscous-plastic wave spreading in layer with cavity situated on rigid base. Wave source is dynamic pressure, which affects on semi-sphere under the day surface at initial conditions [1, 2]:

$$u = \upsilon = \sigma_x = \sigma_y = \tau = 0; \quad \begin{cases} -\infty \le x \le \infty \\ 0 \le y \end{cases} \quad npu \quad t = 0,$$

under the boundary conditions:

$$\begin{cases} \upsilon = tBe^{-At} \\ u = 0; \end{cases} \text{ if } \begin{cases} n_1 \le x \le n_2; \\ y = 0; \\ t \ge 0, \end{cases}$$

where $A, B = const$; υ, u – respectively, the medium components velocity along the axis x and y; $\sigma_x, \sigma_y, \sigma_z, \tau$ – respectively tension components.

From the day surface on cavity situated on depth h the dynamic pressure affect on segment $n_1 \le x \le n_2$ [3, 4].

There is need to determine the normal and tangential tension quantity at given initial and boundary conditions around a rectangular cavity situated on a rigid base. To solve this task the following output data is used [4, 5]: $\Delta t = dt = 0.0024888$; $\Delta x = dx = 0.005$; $\Delta y = dy = 0.005$; $\gamma = \dfrac{a}{b} = 1.6$; $a = 320\dfrac{m}{s}$;
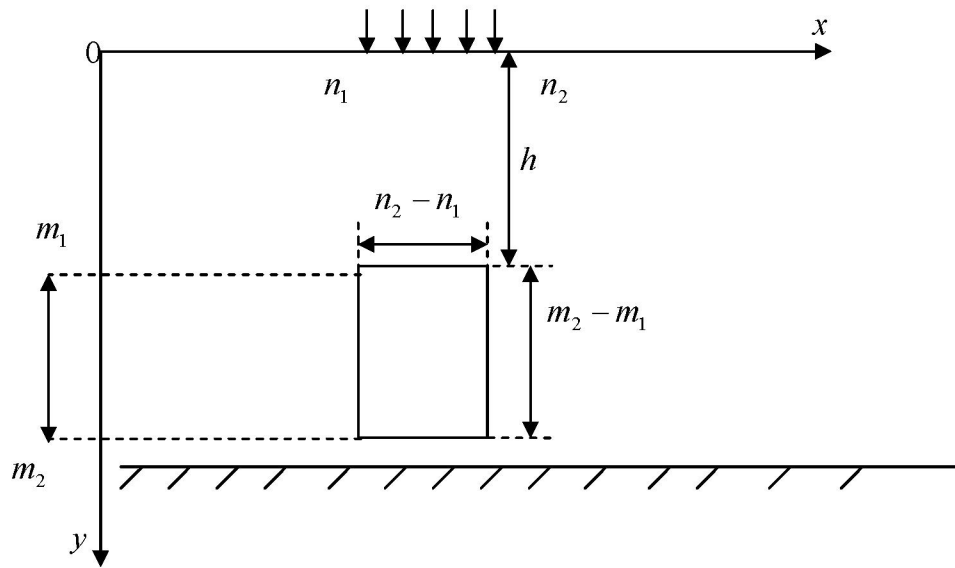
Figure 1 – Cavity in resilient viscous-plastic layer situated on a rigid base

$$b = 200\frac{m}{s}; \quad \alpha = -0.02; \quad \eta = 350s^{-1}; \quad \rho = 1.8\frac{g}{cm^3}; \quad k_0 = 0.62kG/cm^3; \quad \upsilon = kdtB\exp(-Akdt),$$

$A = 133.93335$; $B = 0.1193379$; $t = kdt$ and the research area size around the cavity $N_x = 400$; $M_y = 75$; $n_1 = 198$; $n_2 = 202$; $m_1 = 10$; $m_2 = 16$, $T = 300dt$.

As known, wave spreading equation in resilient viscous-plastic layer with cavity on rigid base has the type [6-8]:

$$\dot{\varepsilon}_{ij} = \frac{1}{2\mu}\dot{s}_{ij} + \frac{1}{2k_0}\dot{s}\delta_{ij} + \eta\langle\Phi(F)\rangle\left(\alpha\delta_{ij} + \frac{s_{ij}}{2\sqrt{J_2}}\right), \tag{1}$$

where μ - Lame coefficient, η - viscosity coefficient, k_0 - yield strength at pure shear, α - parameter characterizing the soil expansion rate.

In order to predict the tension and deformed medium state, the equation describing plane deformation has been considered [9]:

$$\varepsilon_{11} = \varepsilon_{xx} = \frac{\partial u_x}{\partial x}; \quad \varepsilon_{22} = \varepsilon_{yy} = \frac{\partial u_y}{\partial y}; \quad \varepsilon_{33} = \varepsilon_{zz} = 0;$$

$$\varepsilon_{12} = \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right); \quad \varepsilon_{13} = \varepsilon_{xz} = \varepsilon_{23} = \varepsilon_{yz} = 0,$$

where u_x, u_y - shift along the axis x and y.

The equation describing volume deformation of resilient viscous-plastic medium has the type [10]:

$$\dot{\varepsilon}_{ii} = \frac{1}{3k_0}\dot{\delta}_{ii} + 3\alpha\eta\langle\Phi(F)\rangle. \tag{2}$$

In equations (1) and (2), the quantity F is a plastic flow function describing the resilient viscous-plastic medium dynamic behavior and expressed by following formula [11]:

$$F = \frac{\alpha J_1 + \sqrt{J_2}}{k_0} - 1,$$

where $J_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}$ - first invariant of tensor pressure and $J_2 = \dfrac{1}{2}s_{ij}s_{ij}$ - second invariant of tensor pressure, $s_{ij} = \sigma_{ij} - \dfrac{1}{3}\sigma_{kk}\delta_{ij}$ - deviator of tensor pressure [12, 13].

Function $\langle\Phi(F)\rangle$ in equations (1) and (2) is determined basing on an experimental researches on dynamic characteristics and medium properties, and generally presented as nonlinear function, which is defined by following formula:

$$\langle\Phi(F)\rangle = \begin{cases} 0 & \text{if } F \le 0, \\ F & \text{if } F \succ 0. \end{cases}$$

For wave spreading movement equation definition in resilient viscous-plastic layer with cavity on rigid base the differential equation system has been considered [14, 15]:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial \sigma_{xx}}{\partial x} + \dfrac{\partial \tau}{\partial y}; \\[2mm] \dfrac{\partial \upsilon}{\partial t} = \dfrac{\partial \tau}{\partial x} + \dfrac{\partial \sigma_{yy}}{\partial y}; \\[2mm] \dfrac{\partial \sigma_{xx}}{\partial t} = \dfrac{\partial u}{\partial x} + \left(1 - \dfrac{2}{\gamma^2}\right)\dfrac{\partial \upsilon}{\partial y} + \Phi_1; \\[2mm] \dfrac{\partial \sigma_{yy}}{\partial t} = \left(1 - \dfrac{2}{\gamma^2}\right)\dfrac{\partial u}{\partial x} + \dfrac{\partial \upsilon}{\partial y} + \Phi_2; \\[2mm] \dfrac{\partial \tau}{\partial t} = \dfrac{1}{\gamma^2}\dfrac{\partial u}{\partial y} + \dfrac{1}{\gamma^2}\dfrac{\partial \upsilon}{\partial x} + \Phi_3. \end{cases} \tag{3}$$

The "discontinuity disintegration" method of S. K. Gudunov is used for differential equations system (3) solving. That is why the considering medium is divided onto elementary cells [16].
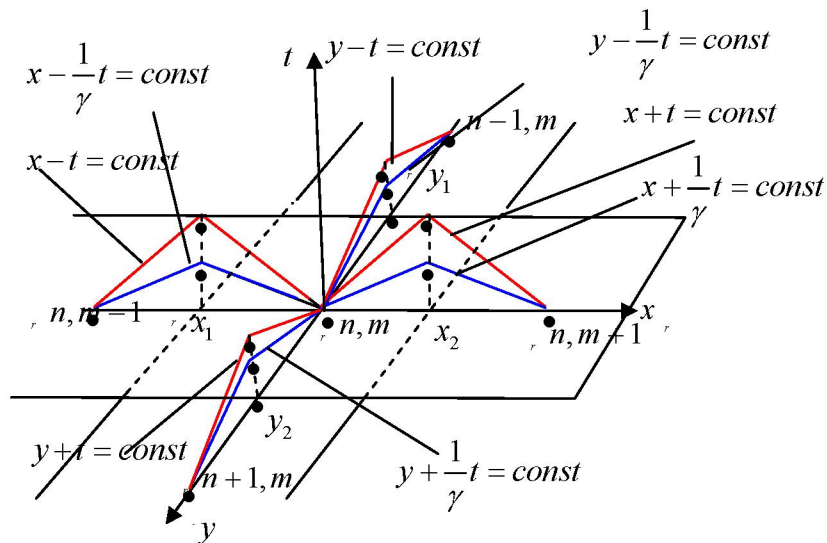

Figure 2 – Characteristic scheme on the template

The studying medium characteristics and properties show that it consists of layers. As it is mentioned above the considering medium is divided onto cells. It is well-known that wave spreading in cells is different. Therefore, the discontinuity disintegration occurs at the adjacent cells boundaries. In this case, the motion between two neighboring cells can be considered as one-dimensional. Consequently, the equations system (3) can be divided onto two equations systems, one of which depends only on x, and the other on y.

After some elementary transformations the equations system (3) can be presented as characteristic equations and their proportions [17, 18]:

$$\begin{cases} d(u + \sigma_{xx}) = \Phi_1 dt & \text{on the line} \quad x + t = \text{const}; \\ d(u - \sigma_{xx}) = -\Phi_1 dt & \text{on the line} \quad x - t = \text{const}; \\ d(\upsilon + \gamma\tau) = \gamma\Phi_3 dt & \text{on the line} \quad x + \frac{1}{\gamma}t = \text{const}; \\ d(\upsilon - \gamma\tau) = -\gamma\Phi_3 dt & \text{on the line} \quad x - \frac{1}{\gamma}t = \text{const}. \end{cases} \tag{4}$$

$$\begin{cases} d(u + \gamma\tau) = \gamma\Phi_3 dt & \text{on the line} \quad y + \frac{1}{\gamma}t = \text{const}; \\ d(u - \gamma\tau) = -\Phi_3 dt & \text{on the line} \quad y - \frac{1}{\gamma}t = \text{const}; \\ d(\upsilon + \sigma_{yy}) = \Phi_2 dt & \text{on the line} \quad y + t = \text{const}; \\ d(\upsilon - \sigma_{yy}) = -\Phi_2 dt & \text{on the line} \quad y - t = \text{const}. \end{cases} \tag{5}$$

Characteristic equations and their proportions (4) and (5) are used to derive finite-difference equations systems for calculating the medium parameters at various points and different instants of time [19, 20].

$$\begin{cases} u^{n,m} = u_{n,m} + \Delta t \frac{[\sigma_{xx,n2} - \sigma_{xxn1}]}{\Delta x} + \Delta t \frac{[\tau_{m2} - \tau_{m1}]}{\Delta y}; \\ \upsilon^{n,m} = \upsilon_{n,m} + \Delta t \frac{[\tau_{n2} - \tau_{n1}]}{\Delta x} + \Delta t \frac{[\sigma_{yy,m2} - \sigma_{yy,m1}]}{\Delta y}; \\ \sigma_{xx}^{n,m} = \sigma_{xx,n,m} + \Delta t \frac{[u_{n2} - u_{n1}]}{\Delta x} + \Delta t \left(1 + \frac{2}{\gamma^2}\right) \frac{[\upsilon_{m2} - \upsilon_{m1}]}{\Delta y} + \Delta t \Phi_{1n,m}; \\ \sigma_{yy}^{n,m} = \sigma_{yy,n,m} + \Delta t \left(1 + \frac{2}{\gamma^2}\right) \frac{[u_{n2} - u_{n1}]}{\Delta x} + \Delta t \frac{[\upsilon_{m2} - \upsilon_{m1}]}{\Delta y} + \Delta t \Phi_{2n,m}; \\ \tau^{n,m} = \tau_{n,m} + \Delta t \frac{[u_{m2} - u_{m1}]}{\gamma^2 \Delta y} + \Delta t \frac{[\upsilon_{n2} - \upsilon_{n1}]}{\gamma^2 \Delta x} + \Delta t \Phi_{3n,m}, \end{cases} \tag{6}$$

where $\quad \Phi_1 = -\frac{1}{\gamma^2}\eta\langle\Phi(F)\rangle\left[(3\gamma^2 - 4)\alpha + \frac{\frac{1}{3}(2\sigma_{xx} - \sigma_{yy} - \sigma_{zz})}{\sqrt{J_2}}\right], \quad \Phi_2 = -\frac{1}{\gamma^2}\eta\langle\Phi(F)\rangle \times$

$\times\left[(3\gamma^2 - 4)\alpha + \frac{\frac{1}{3}(2\sigma_{yy} - \sigma_{xx} - \sigma_{zz})}{\sqrt{J_2}}\right], \quad \Phi_3 = -\frac{1}{\gamma^2}\eta\langle\Phi(F)\rangle\frac{\tau}{\sqrt{J_2}}.$

Thus, the obtained finite-difference characteristic proportions (4), (5) and the equations system (3) constitute a complete equations system for posed task solution.

The finite-difference equations systems numerical solution results are presented in figures 3- 7, which correspond to changes in tension for different instants of time along the coordinates.
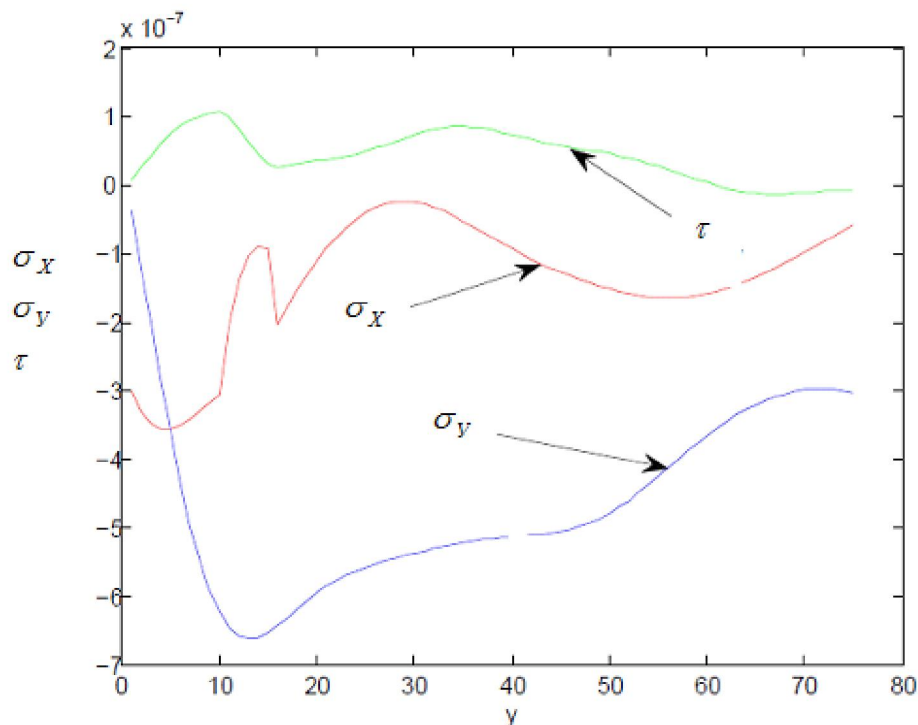

Figure 3 – Changes of indicated tension along the axis y, $x = 190dx$,
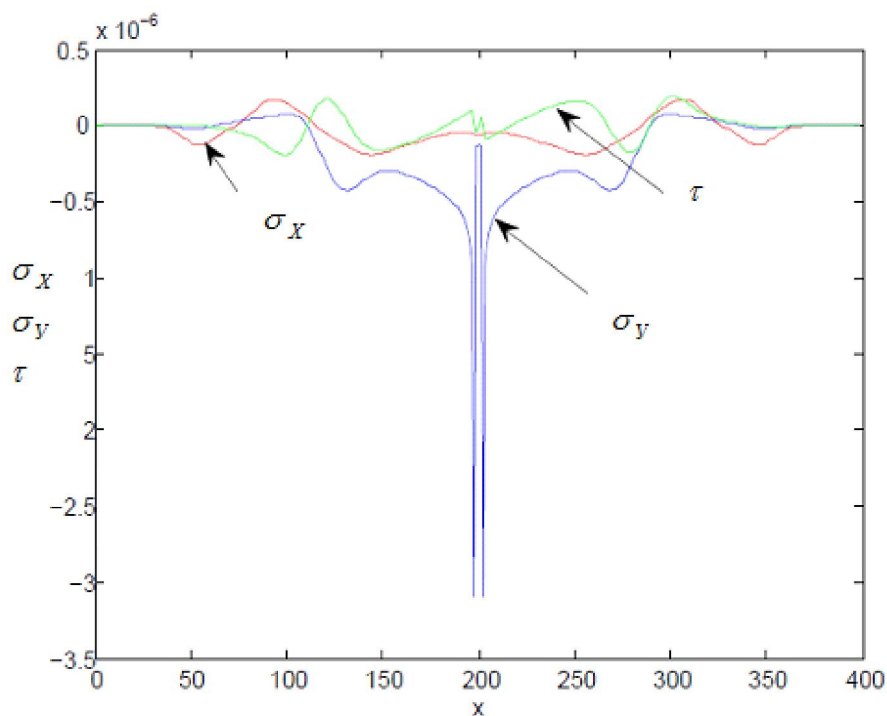at the depth below the cavity $y = 1dy - 75dy$, at the time $t = 295dt$

Figure 4 – Changes of indicated tension axis x, $x = 1dx - 400dx$,
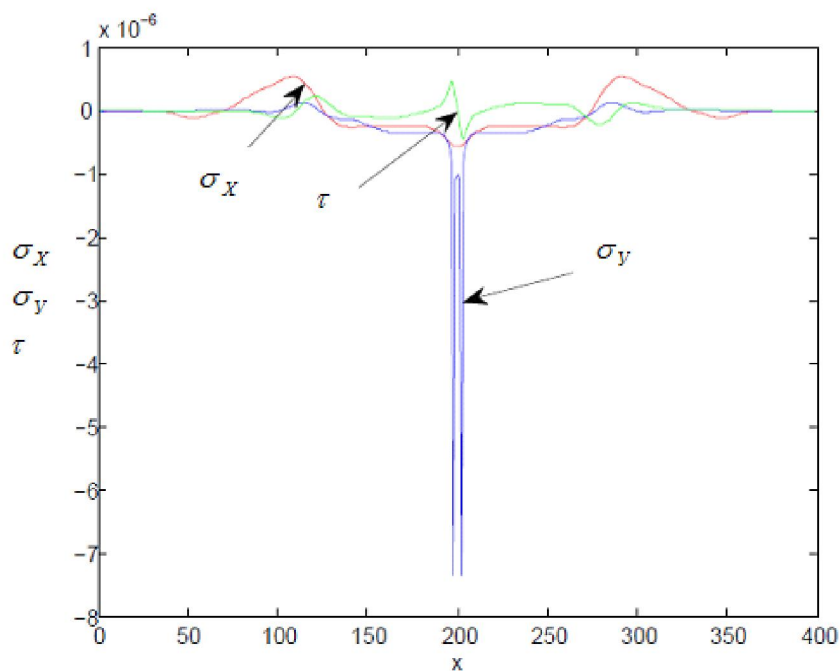at a depth under the cavity $y = 25dy$ at the time $t = 295dt$

Figure 5 – Changes of indicated tension along the axis x, $x = 1dx – 400dx$,
at a depth above the cavity $y = 5dy$ at the time $t = 295dt$
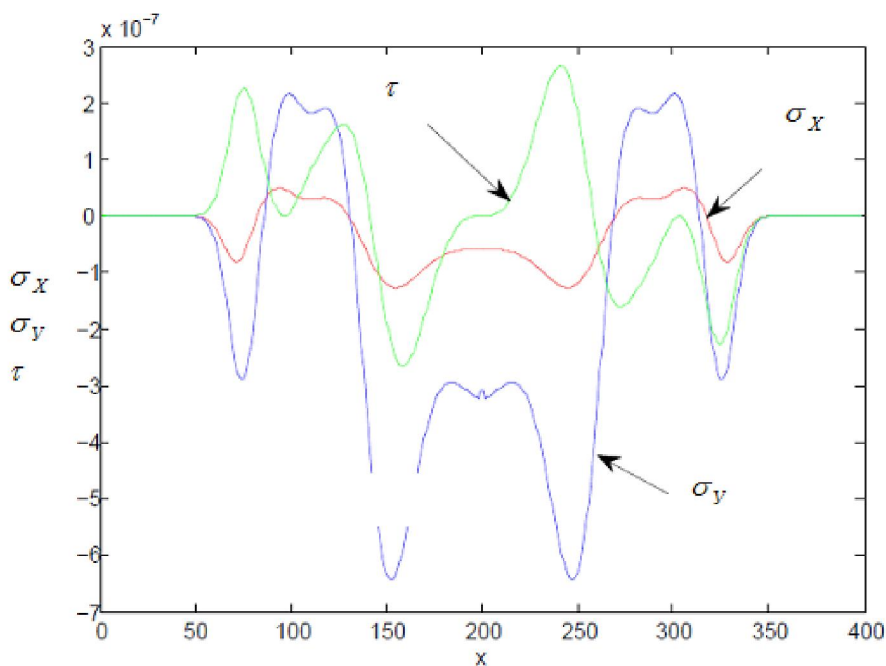

Figure 6 – Changes of indicated tension along the axis x, $x = 1dx – 400dx$,
along the brink with rigid base at a depth $y = 75dy$ at the time $t = 295dt$

Conclusion. The finite-difference equation system and numerical results were received for resilient viscous-plastic wave spreading in layer situated on rigid base under the dynamic non-stationary pressure from a free surface. Analyzing the received numerical results shows up that at $198 \leq n \leq 200$ interval the gap occurs. It means vertical walls are under the maximum physical impact.

Constructers should take into consideration the calculation results mentioned above during the underground facilities designing.

Stable schemes for finite-difference equations systems solving are presented, in order to algorithm realization the algorithms are developed and applied programs packages are compiled.

The tension distribution patterns under different initial physical and geometric parameters have been established.

The research results allow determining wave processes characteristics around the cavity and their tensed and deformed condition that necessary for underground constructions durability estimation.

The received results and created applied programs package can be applied for wave field's estimation, tensed condition character determination in dealing with similar tasks with boundary conditions, at various underground facilities and constructions designing.

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ҚАТТЫ НЕГІЗДЕ ЖАТҚАН ҚУЫСЫ БАР СЕРПІМДІ-ТҰТҚЫРЛЫ ПЛАСТИКАЛЫҚ ҚАБАТТА ТОЛҚЫННЫҢ ТАРАЛУЫ

Аннотация. Бұл жұмыста қатты негізде жатқан қуысы бар серпімді-тұтқырлы-пластикалық қабатқа бет жағынан әсер еткен динамикалық жүктеменің әсерінен пайда болған толқынның таралуын зерттеуге арналған. Берілген мәселені шешуге С. К. Годуновтың «ыдырау үзілу» әдісі қолданылған. Қазіргі уақытта Годуновтың «ыдырау үзілу» әдісі гиперболалық түрдегі квазисызықты теңдеулер жүйесін сандық түрде шешуге, газ динамикасы, аэродинамика мәселелерін кеңінен қарастыруда, соымен қаттар жалпы орта механикасының көптеген есетерін шешуде кеңінен қолданылады. Зерттеу барысында тікбұрышты қуыстың айналасындағы толқын үдірісінің шамаларын болжауда, оның кернеулі-деформациялық күйін бағалауда анықталған нәтижелер жер асты құрылыстарының беріктілігін және тұрақтылығын анықтауға және геодинамика есептерін шешу үшін қолдануға болады. Сонымен қатар қалыпты жағдайдан өзгешеленетін, геомеханикалық үдірісі күрделі аса қиын жағдайда жүргізілетін (құрылымы тұрақсыз жер) өндірістік, азаматтық, транспорттық және әскери құрылыстар салу барысында қолдануға болады.

Түйін сөздер: қабат, серпімді-тұтқырлыпластикалық, қуыс, жүктеме, толқын, кернеулі-деформациялық күй.

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РАСПРОСТРАНЕНИЯ ВОЛН В УПРУГО-ВЯЗКОПЛАСТИЧЕСКОМ СЛОЕ С ПОЛОСТЬЮ НА ЖЕСТКОМ ОСНОВАНИИ

Аннотация. Данная работа посвящена исследованию распространения волн в упруго-вязкопластическом слое с полостью, лежащей на жестком основании при воздействии динамической нагрузки со стороны дневной поверхности. В основе решения данной задачи использован метод «распада разрыва» С. К. Годунова. Метод «распада разрыва» С. К. Годунова является на сегодня эффективным для численного решения квазилинейных систем уравнений гиперболического типа, для решения широкого круга задач газовой динамики, аэродинамики, а также для других задачи механики сплошной среды. Результаты исследований позволят дать оценку параметрам волновых процессов вокруг прямоугольной полости, а также определить их напряженно-деформированного состояния, которые могут быть использованы при оценке прочности и устойчивости подземных сооружений, при решении задачи геодинамики. А так же при строительстве в особо сложных условиях (когда грунт, характеризуется как структурно-неустойчивый грунт) промышленных, гражданских, транспортных и военных сооружениях, в которых протекание геомеханических процессов существенно отличается от процессов, протекающих в нормально уплотненных грунтах.

Ключевые слова: слой, упруго-вязкопластичность, полость, нагрузка, волна, напряженно-деформируемое состояние.

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