K. A. Kabylbekov, Kh. K. Abdarakhmanova, G. Sh. Omashova, B. Kedelbaev, J. A. Abekova

M. Auezov South Kazakhstan State University, Shymkent, Kazakhstan.
E-mail: kenkab@mail.ru, khadi_kab@mail.ru, gauhar_omashova@mail.ru

CALCULATION AND VISUALIZATION
OF THE ELECTRIC FIELD OF A SPACE-CHARGED SPHERE

Abstract. The article presents the calculations and visualization of the electric field of a space-charged sphere (i.e., a sphere charged throughout its volume) done using the MATLAB program. There is the calculation of the electric fields and potentials inside and outside of a space-charged sphere as a function of the distance from the center of the sphere. The electric field inside the sphere is proportional to the distance of the field point from the center of the sphere and the potential inside the sphere is inversely proportional to the squared distance from the center of the sphere. The electric field outside the sphere decreases inversely proportionally to the square of the distance from the center of the sphere, and the potential decreases inversely proportionally to the distance from the center of the sphere. There is the calculation of the electric field and potential at the surface and at the center of the sphere. The equipotential lines of the electric field outside and inside the sphere are represented in three-dimensional space and in projection on the X-Y plane.

Key words: visualization, the space-charged sphere, potential, electric field, equipotential lines.

Introduction. Nowadays all educational institutions of Kazakhstan are provided with computer hardware and software, interactive boards and internet. Almost all teachers have completed language and computer courses for professional development. Hence the educational institutions have all conditions for using computer training programs and models for performing computer laboratory works. During several years we have been conducting the work on organization computer laboratory works on physics with use of resources of the Fizikon Company [1, 2] which are developed at Al-Farabi Kazakh National University by V.V.Kashkarov and his group. Some of worksheet templates for computer laboratory works are introduced in educational processes of our university and schools of the Southern Kazakhstan [3-28]. Students of the physics specialties 5B060400 and 5B011000 successfully master the discipline “Computer modeling of physical phenomena” which is the logical continuation of the disciplines “Information technologies in teaching physics” and “Use of electronic textbooks in teaching physics”. The aim of this discipline is to study and learn the MATLAB program language [29] system, acquaintance with its huge opportunities for modeling and visualization of physical processes. This article is devoted to organization of performance of the laboratory work "Calculation and visualization of the electric field of a space-charged sphere" in the MATLAB language.

Here is the brief theory of the electric field of a space-charged sphere. If \( \rho = \frac{q}{V} = \frac{q}{\frac{4}{3} \pi R^3} \) is the volume charge density of the sphere, then the electric field inside the space-charged sphere according the Gauss’s theorem will be:

\[
4 \pi (r)^2 E = \frac{4 \pi (r)^3 \rho}{3 \varepsilon_0} \quad \Rightarrow \quad E(r \leq R) = \frac{l}{4 \pi \varepsilon_0 \frac{q}{R^3}} \quad \text{or} \quad E(r \leq R) = \frac{\rho r}{3 \varepsilon_0} \quad \text{(1)}
\]
and the electric field outside the space-charged sphere will be

\[ E(r > R) = \frac{q}{4\pi \varepsilon_0 r^2} \quad \text{or} \quad E(r > R) = \frac{\rho r}{3\varepsilon_0 \varepsilon_0^2} \]  \tag{2}

The potential difference inside the space-charged sphere is given by the expression:

\[ \int_{\varphi_1}^{\varphi_2} \int E dr = \frac{q}{8\pi \varepsilon_0 R^3} \left( R^2 - r^2 \right) \quad \text{or} \quad \varphi_1 - \varphi_2 = \frac{\rho R^3}{6\varepsilon_0} - \frac{\rho r^3}{6\varepsilon_0} \quad (r < R) \]  \tag{3}

and the potential difference outside the space-charged sphere is

\[ \varphi(r > R) = \frac{q}{4\pi \varepsilon_0 r^2} \quad \text{or} \quad \varphi(r > R) = \frac{\rho R^3}{3\varepsilon_0 \varepsilon_0^2} \]  \tag{4}

In the given formulae, \( \rho \) is the volume charge density of the sphere, \( q \) is the charge of the sphere, \( R \) is the radius of the sphere, \( r \) is the distance from the center of the sphere, \( \varepsilon_0 \) is the electric constant.

The formulae (2) and (4) show that outside the space-charged sphere the electric field and the potential are the same as if all of the charge were concentrated at the center of the sphere.

**Methods.** The program for calculation and visualization of the electric field of a space-charged sphere is written in the MATLAB language. This program calculates the electric fields and potentials of the field inside and outside of a space-charged sphere and visualizes its equipotential lines.

% The electric field inside a space-charged sphere
>> q=9e-9; % input the charge of the sphere
>> e0=8.85e-12; % input the electric constant
>> R=0.1; % input the radius of the sphere
>> r=0:0.01*R:0.1*R; % input the radius vector
>> ro=q./(4*pi*R.^3./3); % calculation of the volume charge density of the sphere
>> E1=q.*r./(4*pi*e0*R.^3); % calculation of the electric field inside the sphere
>> E11=q./(4*pi*e0*R.^2) % calculation of the electric field at the sphere surface
    E11 = 8.0926e+003% result
>> f11=ro.*R.^2./(2*e0)-ro.*r.^2./(6*e0); % calculation of the field potential inside the sphere
>> plot(r,E1,k-) % visualization of the electric field inside the sphere
>> plot(r,f1,k-) % visualization of the field potential inside the sphere
>> grid on % drawing the coordinate grid
>> xlabel('r(m)') % input the name of the axis
>> ylabel('E1(V/m)') % input the name of the axis
>> title('E1=F(r)') % input the name of the graph
The result is presented in the figure 1.

>> f1=ro.*R.^2./(2*e0)-ro.*r.^2./(6*e0); % calculation of the field potential inside the sphere
>> plot(r,f1,k-) % visualization of the field potential inside the sphere
>> grid on % drawing the coordinate grid
>> xlabel('r(m)') % input the name of the axis
>> ylabel('f1(V)') % input the name of the axis
>> title('f1=F(r)') % input the name of the graph
The result is presented in the figure 2.

Figure 1 shows that the electric field inside the sphere increases linearly from zero at the center up to its maximum at the surface of the sphere. But the field potential inside the space-charged sphere falls from 1214 V at its center to 809 at its surface (figure 2).

% calculation of the field potential at the center of the sphere
>> f10=ro.*R.^2./(2*e0) % calculation of the field potential at the center of the sphere
f10 = 1.2139e+003% result
>> r=R:0.1*R:5*R; % input the vector of the distance
>> f12=ro.*R.^3./(3*e0.*r); % calculation of the field potential outside the sphere

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Figure 1 – The electric field inside the sphere as a function of the distance from the center of the sphere

Figure 2 – The field potential inside the sphere as a function of the distance from the center of the sphere

```matlab
>> f22 = ro*R^2/(3*e0) % calculation of the field potential at the surface of the sphere
f22 = 809.2624
```

```matlab
>> plot(r,f22,'k-') % visualization of the field potential outside the sphere
>> grid on % drawing the coordinate grid
>> xlabel('r, m') % input the name of the axis
>> ylabel('f2,V') % input the name of the axis
>> title('f2=F(r)') % input the name of the graph

Results. The result is presented in the figure 3.

```matlab
>> E2=q./(4*pi*e0*r.^2); % calculation of the electric field outside the space-charged sphere
>> plot(r,E2,'k-') % visualization of the electric field outside the space-charged sphere
>> grid on % drawing the coordinate grid
>> xlabel('r,m') % input the name of the axis
>> ylabel('E2,V/m') % input the name of the axis
>> title('E2=F(r)') % input the name of the graph
The result is presented in the figure 4.
```
Figure 3 – The field potential outside the sphere as a function of the distance from the center of the sphere

Figure 4 – The electric field outside the space-charged sphere as a function of the distance from the center of the sphere

Comparison of the curves given in the figures 3 and 4 demonstrates that the electric field outside the space-charged sphere decreases with distance quicker than the field potential.

% Visualization of equipotential lines (inside the sphere)
>> q=9e-9; % input the charge of the sphere
>> e0=8.85e-12; % input the electric constant
>> R=0.10; % input the sphere radius
>> x=-R:0.01*R:R; % input the vector of the coordinate x
>> y=-R:0.01*R:R; % input the vector of the coordinate y
>> [X,Y]=meshgrid(x,y); % drawing of a grid at knots of which x and y coordinates are recorded; % (arrays X and Y).
>> r1 = ((X -0.01* R).^ 2 + (Y+0.01* R).^ 2).^0.5; % calculation of the distance
>> r2 = ((X +0.01* R).^ 2 + (Y-0.01* R).^ 2).^0.5; % calculation of the distance
>> r=sqrt(r1.^2+r2.^2); % calculation of the distance
>> ro=q./(4*pi*R.^3./3); % calculation of the volume charge density
>> f1=ro*R.^2./(2*e0)-ro*r.^2./(6*e0); % calculation of the potential
>> Z=f1; redesignation
>> contour3(X,Y,Z,100); % visualization of equipotential lines
>> xlabel('X(m)') % input the name of x axis
>> ylabel('Y(m)') % input the name of y axis
>> zlabel('f12=V') % input the name of z axis
>> title('f12=F(x,y)') % input the name of the figure

The result is presented in the figure 5.

![Figure 5 – Equipotential lines inside the sphere in three-dimensional space](image1)

>> view([0 0 100]) % command for drawing the projection on the X-Y plane.
The result is presented in the figure 6.

![Figure 6 – Equipotential lines inside the sphere in projection on the X-Y plane](image2)

% The electric field outside a sphere
% Program of visualization of equipotential lines
>> q=9e-9; % input the charge of the sphere
>> e0=8.85e-12; % input the electric constant
>> R=0.10; % input the sphere radius
>> x=R:.01*R.5*R; % input the vector of the coordinate x
>> y = R:0.01*R:5*R; % input the vector of the coordinate y
>> fi2=ro*R.^3./(3*e0.*r); % calculation of the potential outside the sphere
>> Z=fi2; % redesignation
>> [X,Y]=meshgrid(x,y); % drawing of a grid at knots of which x and y coordinates are recorded; % (arrays X and Y).
>> r2 = ((X + 0.01*R).^2 + (Y-0.01*R).^2).^0.5; % calculation of the distance
>> r1 = ((X - 0.01*R).^2 + (Y+0.01*R).^2).^0.5; % calculation of the distance
>> r = sqrt(r1.^2+r2.^2); % calculation of the distance
>> ro=q./((4*pi*R.^3./3)); % calculation of the volume charge density
>> fi2=ro*R.^3./(3*e0.*r); % calculation of the potential outside the sphere
>> Z=fi2; % redesignation
>> contour3(X,Y,Z,100); % visualization of equipotential lines
>> xlabel(X,m') % input the name of x axis
>> ylabel(Y,m') % input the name of y axis
>> zlabel('fi2=F(x,y)') % input the name of the z axis
>> title('fi2=F(x,y)') % input the name of the figure

The result is presented in the figure 7.

![Figure 7](image)

Figure 7 – Equipotential lines outside the sphere in three-dimensional space

>> view([0 0 100]) % command for drawing the projection on the X-Y plane.
The result is presented in the figure 8.

![Figure 8](image)

Figure 8 – Equipotential lines outside the sphere in projection on the X-Y plane
Conclusion. The MATLAB program for calculation and visualization of the electric field of a space-charged sphere allowed to
1) calculate the electric field and electric potential inside a space-charged sphere as a function of the distance from the center of the sphere. The electric field inside the sphere is proportional to the distance of the field point from the center of the sphere and the potential inside the sphere is inversely proportional to the squared distance from the center of the sphere. (The electric field inside a space-charged sphere increases proportionally to the distance from zero to $E_1=8092.6$ V/m at its surface (figure 1). The field potential decreases as the reciprocal of the squared distance from $f10=1213.9$ V at the center up to $f122=809.2624$ V at its surface (figure 2));
2) calculate the electric field and potential outside a space-charged sphere as a function of the distance from the center of the sphere. The electric field outside the sphere decreases inversely proportionally to the square of the distance from the center of the sphere (figure 3), and the potential decreases inversely proportionally to the distance from the center of the sphere (figure 4).
3) draw equipotential lines of the electric field outside and inside the sphere in three-dimensional space and in projection on the X-Y plane.

The students can be suggested to work out the similar program for calculation and visualization of the electric fields and electric potentials inside and outside the analogous charged sphere of a certain radius and charge evenly distributed throughout the volume of the sphere.

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К. А. Кабылбеков, Х. К. Абдрахманова, Г. И. Омашова, Б. Келдебаев, Ж. А. Абекова
М. Эузев атындағы Өңгүстік Казакстан мемлекеттік университеті, Шымкент, Казахстан

КОЛЕМДІ ЗАРЯДТАЛГАН ШАР ОРИСІНІ ЕСЕПТЕУ ЖӘНЕ БЕЙНЕЛЕУ

Аннотация. Колемді зарядталған шар (ягины, шар, қолем бойынша біркелкі зарядталған) өрісін есептеу мен бейнелеу Matlab бағдарламасының комитетімен орнайды. Колемді зарядталған шардың ішіндегі және сыртқындағы электр өрісінен кернеуілігі мен потенциалы бөрілген нүктенің шар центрине елшеннен ара қашқытықка тауелділігі анықталады. Шар ішіндегі кернеуілік шардың қолемін дейін ара қашқытыққа пропорционал еседі, ал потенциал сол ара қашқытықтың квадратына кері пропорционалды кеміді. Шар сыртқындағы электр өрісіненің кернеуілігі шардың қолеміндегі саналған ара қашқытықтың квадратына кері пропорционал кеміді, ал потенциалы сол ара қашқытыққа кері пропорционал кеміді. Шар центринде же бетіндеғі потенциал мен кернеуілік есептелген. Шар ішіндегі және сыртқындағы эквиваленттіліктер сыйықтары үш әл- шемді кеңістікте және X-Ү қашқытымен өңірленді.

Түнін сөзір: бейнелеу, колемді зарядталған шар, потенциал, кернеуілік, эквиваленттілік сыйықтар.
К. А. Кабылбеков, Х. К. Абдрахманова, Г. Ш. Омашова, Б. Кедельбаев, Ж. А. Абекова
Южный Казахстанский государственный университет им. М. Ауезова, Шымкент, Казахстан

ВЫЧИСЛЕНИЕ И ВИЗУАЛИЗАЦИЯ ЭЛЕКТРИЧЕСКОГО ПОЛЯ ОБЪЕМНО-ЗАРЯЖЕННОГО ШАРА

Аннотация. Вычисления и визуализация электрического поля объемно-заряженного шара (т.е., шар, равномерно заряженный по объему), выполнены с помощью программы MATLAB. Проведено вычисление напряженности и потенциала поля как внутри, так и снаружи объемно-заряженного шара как функции расстояния от центра сферы. Напряженность поля внутри шара пропорциональна расстоянию точки поля, где определяется напряженность, до центра сферы. Потенциал поля внутри шара обратно пропорционален квадрату расстояния точки поля, где определяется потенциал, до центра сферы. Напряженность вне шара уменьшается обратно пропорционально квадрату расстояния от центра шара, а потенциал уменьшается обратно пропорционально расстоянию от центра шара. Также вычислены напряженность и потенциал поля на поверхности и в центре сферы. Эквипотенциальные линии электрического поля внутри и снаружи шара представлены в трехмерном пространстве в проекции на плоскость X-Y.

Ключевые слова: визуализация, объемно-заряженный шар, потенциал, электрическое поле, эквипотенциальные линии.

Information about the authors:
Кабылбеков К.А. – cand. phys.-math. sciences, associate professor of the department “Physics” of M. Auezov South Kazakhstan State University Shymkent, Kazakhstan, the correspondent member of KazNAS; https://orcid.org/0000-0001-8347-4153
Абдрахманова К.К. – cand. chem. sciences, associate professor of the department “Physics” of M. Auezov South Kazakhstan State University, Shymkent, Kazakhstan; khadi_kab@mail.ru; https://orcid.org/0000-0002-6110-970X
Омашова Г.Ш. – cand. phys.-math. sciences, associate professor of the department “Physics” of M. Auezov South Kazakhstan State University, Shymkent, Kazakhstan; gauhar_omashova@mail.ru; https://orcid.org/0000-0003-2020-8258
Кедельбаев В. – doc. chem. sciences, professor of the department “Chemistry” of M. Auezov South Kazakhstan State University, Shymkent, Kazakhstan; https://orcid.org/0000-0001-7158-1488
Абекова Ж.А. – cand. phys.-math. sciences, associate professor of the department “Physics” of M. Auezov South Kazakhstan State University, Shymkent, Kazakhstan