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CONDITIONS OF EXTREME STRESS STATE

Abstract. In continuum mechanics, the theory of stresses contains the spreading of Newton's laws for point masses to a continuous medium, and the theory of deformations contains a geometric description of the changes that occur when the points of the body move. Stress is a concept used to determine how loads are transmitted through a solid body. In the three-dimensional coordinate system x, y, z , stresses acting on planes with normals parallel to the coordinate axes are known as components of the stress tensor. To calculate the stability of open and underground mines, it is necessary to know the conditions in which the destruction occurs. The mechanical theory of strength is devoted to the analysis of the boundaries of the stress state at which stability loss and destruction occur.

Key words: stress theory, continuous medium, rocks, stability, nonlinear dependence.

Cohesive rocks up to a certain level of stress and strain generally retain their properties. Any small change in shear strain $d\gamma$ corresponds to a change in the shear stress $d\tau$ of the same sign $d\tau/d\gamma > 0$ (figure 1). The deformed state at the point B is characterized by a plastic component γ^p and an elastic component γ^y of general deformation. The unloading of material from point B will be accompanied by the restoration of elastic deformations, and upon repeated loading to the achieved level τ_B , it will occur purely elastic, without the appearance of additional plastic deformations. Thus, the achieved stress level during reloading after preliminary unloading will serve as the boundary of the elastic state region and is called the yield stress. The stress τ_{np} at point C (figure 1) is called the ultimate strength. As long as the level of τ does not exceed τ_{np} the loading process is accompanied by an increase in the yield strength, called hardening, after reaching τ_{np} in the rock, the process of reducing resistance ($d\tau/d\gamma < 0$), called softening, begins.

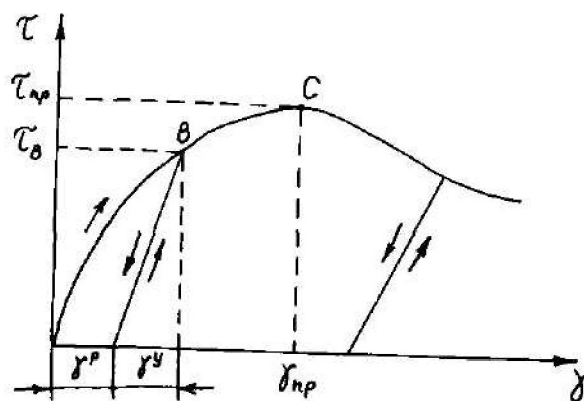


Figure 1 – Nonlinear dependences of stresses and strains in axes γ - τ

In the space of principal stresses, the yield strength will form a certain surface, which is called the yield surface. The equation of this surface is the symmetric function of the principal stresses, in general terms it is written as follows:

$$F(\sigma_1, \sigma_2, \sigma_3) = K, \quad (1)$$

where K is a constant associated with ultimate strength.

Since the basic symmetric functions of the stress components are its invariants, then equation (1) can be represented as:

$$F(J_1(T_\sigma), J_2(T_\sigma), J_3(T_\sigma)) = K, \quad (2)$$

where

$$J_1(T_\sigma) = \sigma_1 + \sigma_2 + \sigma_3 = 3\sigma,$$

$$J_2(T_\sigma) = -(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1),$$

$$J_3(T_\sigma) = \sigma_1\sigma_2\sigma_3$$

– are the linear, quadratic, and cubic invariants of the stress tensor.

Concretization of dependence (2) leads to one or another criteria of strength. For rocks, it is of interest to consider the mechanical theories of Tresca, Coulomb, and Mohr [41].

The Tresca criterion states that the ultimate tangential stress in the body is equal to some constant value C :

$$\tau_{\text{up}} = C. \quad (3)$$

Since $\tau_{\text{up}} = (\sigma_1 - \sigma_3) / 2$ then we have

$$\sigma_1 - \sigma_3 - 2C = 0. \quad (4)$$

Formula (4) describes a plane parallel to the hydrostatic axis. If we consider all the main stresses to be equal, then the Tresca criterion describes a regular hexagonal prism in the space of the main stresses.

The Coulomb criterion is based on the assumption that the rock's resistance to shear in the fracture plane is equal to the adhesion C plus a value proportional to the normal stress in this plane:

$$|\tau| = C + f\sigma, \quad (5)$$

where $|\tau|$ - the absolute value of the ultimate shear stress; f is the coefficient of proportionality.

The coefficient f is called the coefficient of internal friction, since the expression $f\sigma$ is similar to the dry friction force.

We write criterion (5) in terms of the principal stresses σ_1 и σ_3 .

For this, the normal σ and tangent τ stress on the considered site are expressed in terms of the principal stresses:

$$\begin{aligned} \sigma &= \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\beta \\ \tau &= \frac{1}{2}(\sigma_1 - \sigma_3) \sin 2\beta \end{aligned} \quad (6)$$

Where the angle β between the normal N to the site and the direction of stress σ_1 (figure 2).

Putting (6) into (5) gives

$$C = |\tau| - f\sigma = \frac{1}{2}(\sigma_1 - \sigma_3)(\sin 2\beta - f \cos 2\beta) - \frac{1}{2}(\sigma_1 + \sigma_3)f. \quad (7)$$

This expression has a minimum value as a function β when

$$\operatorname{tg} 2\beta = -\frac{1}{f}. \quad (8)$$

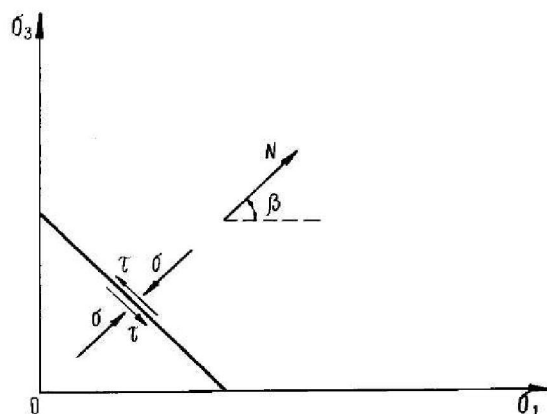


Figure 2 – Graphical representation of stresses on an inclined plane

Since $\tan 2\beta < 0$, obviously that the angle β lies within $45^\circ - 90^\circ$ and

$$\begin{aligned}\sin 2\beta &= (f^2 + 1)^{-1/2}, \\ \cos 2\beta &= -f(f + 1)^{-1/2}\end{aligned}\quad (9)$$

Putting (9) into (7), we obtain the Coulomb criterion expressed in terms of principal stresses:

$$\sigma_1 \left[(f^2 + 1)^{1/2} - f \right] - \sigma_3 \left[(f^2 + 1)^{1/2} + f = 2C \right]. \quad (10)$$

Concerning (10) it implies that if the left side of the equation is less than $2C$, then destruction will not occur; if more - then it will happen.

In the coordinates σ_1, σ_3 the equation (10) describes the line BSC (figure 3).

The uniaxial compression strength S is obtained if we put in equation (10)

$\sigma_3 = 0$:

$$\sigma_1 = S = 2C[(f^2 + 1)^{1/2} + f]. \quad (11)$$

The criterion assumes $\sigma_1 > \sigma_3 > 0$, i.e., it corresponds to the conditions of volume compression.

In the tensile site, we supplement the Coulomb criterion with the durability condition at tension:

$$\sigma_3 = T, \quad (12)$$

where T is a durability for tension ($T < 0$).

The segment ATV corresponds to the formula (12) in figure 3.

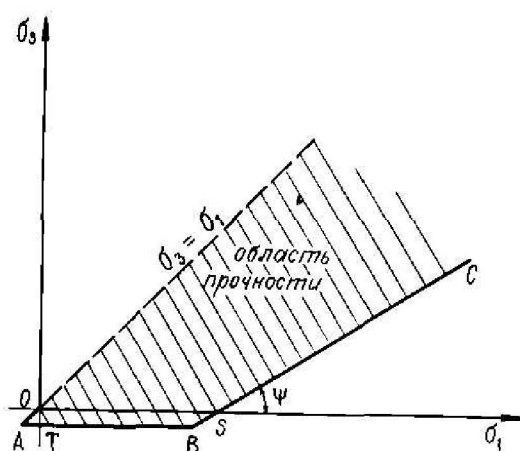


Figure 3 – Coulomb criterion on the axes of principal stresses

Equation (10) can be written by introducing the notation $f = \operatorname{tg} \varphi$ and after simple trigonometric transformations we obtain

$$\sigma_1 = S + \sigma_3 \operatorname{ctg}^2 \left(\frac{\pi}{4} - \frac{\varphi}{2} \right), \quad (13)$$

where φ is the angle of internal friction.

The slope angle Ψ of the straight line BSC to the axis σ_1 is determined by the ratio:

$$\operatorname{ctg} \Psi = \operatorname{ctg}^2 \left(\frac{\pi}{4} - \frac{\varphi}{2} \right) = \frac{1 + \sin \varphi}{1 - \sin \varphi}. \quad (14)$$

The Coulomb criterion in the space of three principal stresses is a hexagonal pyramid, in which the axis $\sigma_1 = \sigma_3 = \sigma_3$ is the axis of symmetry.

Mohr's criterion states that the resistance to shear along the site is a function of the normal stress on it:

$$\tau / \sigma = F(\sigma). \quad (15)$$

If the function F is linear, then the Mohr's and Coulomb's criteria coincide. The form of the function $F(\sigma)$ is determined by the test results under conditions of triaxial compression. In the space $\sigma_1, \sigma_2, \sigma_3$ the Mohr criterion will describe a surface resembling the Coulomb pyramid, but with curved boundaries.

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АҚЫРЛЫ КЕРНЕУЛІ ЖАҒДАЙДЫҢ ШАРТТАРЫ

Аннотация. Тұтас орта механикасында кернеу теориясы нүктелі массалар үшін Ньютон заңдарының тұтас ортаға таралуынан тұрады, ал деформация теориясы дененің нүктелерін ауыстыруда болатын өзгерістердің геометриялық сипаттамасынан тұрады. Кернеу бұл тұтас дене арқылы жүктемелердің қалай берілуін анықтау үшін қолданылатын ұғым. x, y, z координаттарының үш өлшемді жүйесінде нормалармен, параллель координаттық осьтерге параллель жазықтықта әрекет ететін кернеу тензорының компоненттері ретінде белгілі. Ашық және жер асты тау-кен қазбаларының тұрақтылығын есептеу үшін тау-кен қазбаларының күйреу жағдайларын анықтау қажет. Мақала орнықтылықтың жоғалуы және күйреуі орын алатын кернеулі күй шекарасын талдау кезіндегі беріктіктің механикалық теорияларына арналған.

Түйін сөздер: кернеу теориясы, тұтас орта, тау жыныстары, тұрақтылық, сызықсыз тәуелділік.

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УСЛОВИЯ ПРЕДЕЛЬНОГО НАПРЯЖЕННОГО СОСТОЯНИЯ

Аннотация. В механике сплошной среды теория напряжений содержит распространение законов Ньютона для точечных масс на сплошную среду, а теория деформаций- геометрическое описание изменений, происходящих при перемещениях точек тела. Напряжение это понятие, используемое для определения того, как передаются нагрузки через сплошное тело. В трехмерной системе координат x, y, z напряжения, действующие на плоскостях с нормальными, параллельными координатным осям, известны как компоненты тензора напряжений. Для расчета устойчивости открытых и подземных горных выработок необходимо знать условия, в которых происходит разрушение. Анализ границы напряженного состояния, на которых происходят потеря устойчивости и разрушение, посвящены механические теории прочности.

Ключевые слова: теория напряжений, сплошная среда, горные породы, устойчивость, нелинейная зависимость.

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