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THE MOTION WITH AIR DRAG OF A BODY LAUNCHED AT AN ANGLE ABOVE THE HORIZONTAL

Abstract. The article offers calculations and visualization of motion of a body launched at an angle above the horizontal for two cases without air drag and with air drag by using the MATLAB software. It presents the motion trajectories of the body launched at an angle above the horizontal for two cases without air drag and with air drag at different values of air drag coefficient. The path of the body launched at a certain angle above the horizontal with no air drag is a parabola. In case of motion with air drag the path is not parabolic, i.e. not symmetric relative the vertical line drawn from the upper point of the trajectory, with less height and range. With the increase of air drag coefficient the maximum height reached by the body decreases. When the air drag coefficient increases from 0.1 up to 0.5 the maximum height reached by the body decreases approximately by 1.8 times. Students are offered to study this phenomenon by changing the air drag coefficient at various initial velocities and to analyze them. Results of this article are used at the practical classes on classical and theoretical mechanics and at the laboratory classes on the discipline "Modeling the physical phenomena".

Key words: motion, at an angle above the horizontal, initial height, initial velocity, path, parabola, medium drag coefficient.

Introduction. Nowadays all educational institutions of Kazakhstan are provided with computer hardware and software, interactive boards and internet. Almost all teachers have completed language and computer courses for professional development. Hence the educational institutions have all conditions for using computer training programs and models for performing computer laboratory works. In recent years the new computer system of carrying out mathematical calculations MATLAB is being widely used in many universities and engineering institutions throughout the world [1-7]. Unfortunately, the numerical calculations carried out by students are often done by means of the calculator. Modern computers are frequently used only for presentation of the work. Actually students should be able not only to solve these or other engineering problems, but also do it by using modern methods, that is, using personal computers.

Students of the physics specialties 5B060400 and 5B011000 successfully master the discipline "Computer modeling of physical phenomena" which is the logical continuation of the disciplines "Information technologies in teaching physics" and "Use of electronic textbooks in teaching physics". The aim of this discipline is to study and learn the program language of the MATLAB system, acquaintance with its huge opportunities for modeling and visualization of physical processes.

In our early works [8-25] we used the MATLAB system for modeling and visualization of physical processes related with mechanics, molecular physics, and electromagnetism and quantum physics. This software has enabled us to solve ordinary differential equations (ODE), visualize equipotential lines of charged conductors' system, describe the motion of charged particles in electric, magnetic and gravitational fields and etc.

The present article is devoted to calculation and visualization of motion of the body launched at an angle above the horizontal for two cases without air drag and with air drag by using the MATLAB software.

Formulation of the problem 1. Let's consider the movement of the body thrown with initial velocity of υ_0 at an angle α to the horizon in the assumption that the air drag is proportional to squared velocity. In a vector form the equation of motion is given as

$$m\ddot{\vec{r}} = -\gamma\vec{\upsilon}|\vec{\upsilon}| - m\vec{g},$$

where $\vec{r}$ is the position vector of the moving body, $\vec{\upsilon}$ is the body's velocity, γ is the drag coefficient, $m\vec{g}$ is the gravity force, m is the mass of the body and $\vec{g}$ is the acceleration due to gravity.

The specific feature of this problem is in ending of the body's motion at an unknown time, when the body falls on the ground.

If to denote $k=\gamma/m$, then we have a system of equations for x- and y- components of the motion, to which it is necessary to add initial conditions: $x(0)=0$, $y(0)=h$ (h is an initial height), $\dot{x}(0)=\upsilon_0\cos\alpha$, $\dot{y}(0)=\upsilon_0\sin\alpha$.

Let us have the following denotations:

$$y(1)=x, \quad y(2)=\dot{x}, \quad y(3)=y, \quad y(4)=\dot{y}.$$

Then the corresponding system of ordinary differential equations (ODE) of the first order for x- and y-components of the motion is given as

$$\begin{aligned}
&\dot{y}(1)=y(2)\\
&\dot{y}(2)=-k\cdot y(2)\cdot\sqrt{(y(2))^2+(y(4))^2}\\
&\dot{y}(3)=y(4)\\
&\dot{y}(4)=-k\cdot y(4)\cdot\sqrt{(y(2))^2+(y(4))^2}-g
\end{aligned}$$

Methods and results. To obtain the solution of this problem we introduce the function "bodyangle.m" for calculation of the right side of the system of ODE.

Listing of the function Movifriction.m.

```
function F=Movifriction(x,y)
k=0.01;
g=9.81;
F=[y(2); -k.*y(2).*sqrt(y(2).^2+y(4).^2);...
y(4); -k.*y(4).*sqrt(y(2).^2+y(4).^2)-g];
```

The algorithm of the solution ex14_1.m of our boundary-value problem can be as the following:

```
>> % motion of the body launched at an angle to the horizon
>> alph=pi/4; % the angle of launch
>> k=0;
>>v0=1; % initial velocity
>>h=0; % initial height
>>tmax=0.2; % time interval
>>Y0=[0; v0.*cos(alph); h; v0.*sin(alph)]; % vector of initial conditions
>> [T,Y]=ode45(@Movifriction,[0 tmax],Y0);% approximate solution
>>plot(Y(:,1),Y(:,3), 'LineWidth',2); % the graph of the curve x(t), y(t)
>>axis equal;
>>grid on
```

The result is presented in the figure 1.

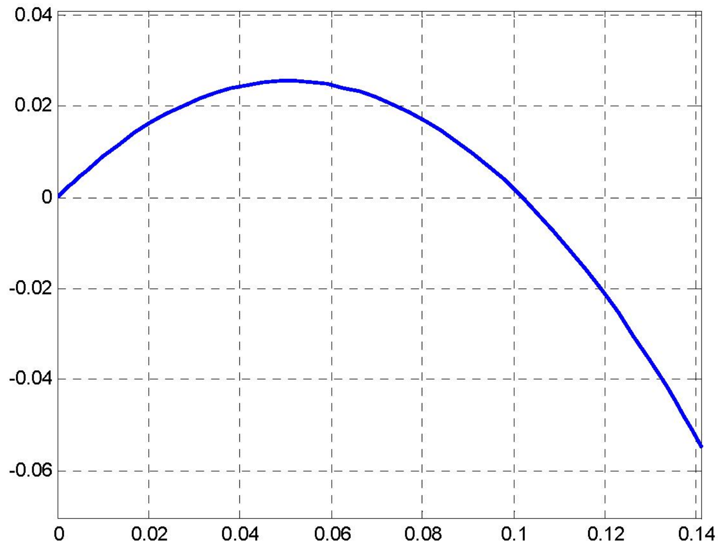

Figure 1 – The path of body's motion without air drag (k=0) at initial conditions
$\alpha=pi/4$ – launch angle, $\upsilon 0=1$ – initial velocity, $h=0$ – initial height

For determination of motion duration it is necessary to select the value of *tmax*. Also it is necessary to determine approximately the maximum height and the range. The ODE solver of MATLAB gives an opportunity to determine the moments of occurring of the events corresponding to some special solutions and reactions to them. For this aim a special function-event processor is used. During solution MATLAB recognizes events and calls the user's processor.

The format of the function of event processor must be as following:

[value, isterminal, direction] = eventsfun(t, y)

Function has to return three vectors -value, isterminal, direction- of identical length in which the i-th component corresponds to equalizing to zero of some expression depending on t and $y(k)$ components of a vector-function of ODE system solution (arguments t, y functions - events). Components of these vectors have the following meanings:

value(i) is the expression made of an argument t and the $y(k)$ components of a vector function y which in a solver will be checked on equality to zero;

isterminal(i) = 1 if an integration of the ODE system has to be stopped at reaching the condition - value (i) = 0, or = 0 if it isn't required to stop calculations;

direction(i) = 0 if it is necessary "to catch" all zeros of the expression - value (i), + 1 when passing through zero the expression - value (i) increases, and – 1 when passing through zero the expression - value(i) decreases.

Then the descriptor of this function should be transferred to a solver, having specified it in structure of parameters - options - of the function- odeset (see above) as the value of the parameter - Events.

options=odeset('Events', eventsfun)

After that a solver should be called with 5 exit parameters

[T,Y,TE,YE,IE] = solver(odefun, tspan, y0, options)

Problem 2. Let's return to the problem 1 in which we we considered the motion of the body thrown at an angle to the horizon. Let's create the bodyanglek.m function of a right side of the system of equations having additional parameter k – the medium drag coefficient.

Listing function bodyanglek.m.

```
function F=bodyanglek(x,y,k)
g=9.81;
F=[y(2); -k.*y(2).*sqrt(y(2).^2+y(4).^2);...
y(4); -k.*y(4).*sqrt(y(2).^2+y(4).^2)-g];
```

If the function -processor of an event - Events is used, then it also has to have this additional argument.

```
function [value,isterminal,direction] = ex8eventsk(t,y,k)
% Determine the time when the height
% is equal to 0 during the body's falling and to end calculations, also
% determine the time of body's reaching the maximum height (without stopping calculations),
% when the y-component of the velocity is equal to 0
value = [y(3), y(4)]; % determine zeros for y(3) and y(4)
isterminal = [1,0]; % stop calculations at y(3)=0
direction = [-1,0]; % function y(3) decreases, for y(4) it makes no difference
```

When calling the solver we also should give it an additional argument. Then the algorithm of the solution ex16_1.m of our boundary- value problem will be as following

```
% motion of the body launched at an angle to the horizon ex16_1.m
>>alph=pi/4; % the launch angle
>>v0=10; % initial velocity
>>Y0=[0; v0.*cos(alph); 0; v0.*sin(alph)]; % vector of initial conditions
>>ak=[0.1 0.2 0.3 0.5]; % medium drag coefficients
>>opts = odeset('Events',@ex8eventsk,'Refine',16); % parameters
>>newplot; hold on;
>>for i=1:4
>> [t,Y,te,ye,ie] = ode45(@bodyanglek,[0 Inf],Y0,opts,ak(i)); % solution
>>plot(Y(:,1),Y(:,3), 'LineWidth',2); % path graph
>>end
>>grid on; hold off;
```

The result is presented in the figure 2.

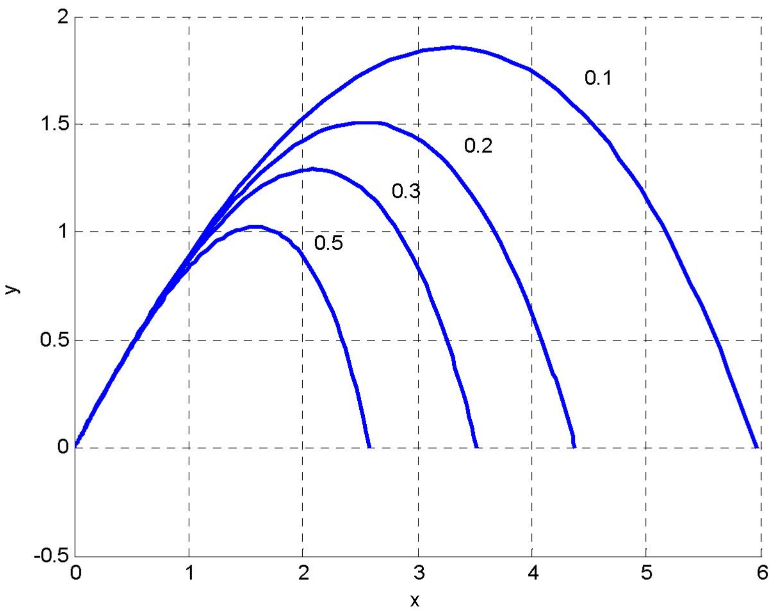

Figure 2 – A trajectory of motion of a body launched at an angle to the horizon at various values of the medium drag coefficient

Students are offered to study this phenomenon by changing the initial conditions (the launch angle, initial velocities and heights) and at various values of the medium drag coefficient and to analyze them.

Conclusion. The article considers calculations and visualization of motion of a body launched at an angle above the horizontal without air drag and with air drag by using the MATLAB software. It presents the motion trajectories of the body launched at an angle above the horizontal at different values of air drag coefficient. The path of the body launched at a certain angle above the horizontal with no air drag is a parabola. In case of motion with air drag the path is not parabolic, i.e. not symmetric relative the vertical line drawn from the upper point of the trajectory, with less height and range. With the increase of air drag

coefficient the maximum height reached by the body decreases. When the air drag coefficient increases from 0.1 up to 0.5 the maximum height reached by the body decreases approximately by 1.8 times. Students are offered to study this phenomenon by changing the air drag coefficient at various initial velocities and to analyze them. Results of this article are used at the practical classes on classical and theoretical mechanics and at the laboratory classes on the discipline "Modeling the physical phenomena".

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MATLAB ЖҮЙЕСІНДЕ «КӨКЖИЕККЕ БҰРЫШ ЖАСАП ЛАҚТЫРЫЛҒАН ДЕНЕНІҢ КЕДЕРГІМЕН ҚОЗҒАЛЫСЫН ЗЕРТТЕУ»

Аннотация. Көкжиекке бұрыш жасап лақтырылған дененің ортаның кедергісі жоқ және кедергісі болған жағдайларындағы қозғалысын есептеу мен бейнелеу ұсынылады. Көкжиекке бұрышпен лақтырылған дененің ортаның кедергісі жоқ және кедергісінің коэффициенттері әртүрлі болған жағдайларындағы қозғалысының траекторияларың графиктері келтірілген.

Ортаның кедергісін есепке алмаған жағдайда дене парабола бойында қозғалады, ал орта кедергісі есепке алынған жағдайда оның траекториясы параболадан ауытқиды және ең жоғарғы биктіктегі нүктеден жүргізілген верикалға салыстырғанда симметриялы болмайды.

Ортаның кедергі коэффиценті 0.1 ден 0.5-ке дейін өскен сайын дененің көтерілу биктігі 1.8 есе төмендейді.

Студенттерге өз бетінше қозғалыстың бастапқы жылдамдығы мен бастапқы биіктігі және лақтыру бұрышы мен кедергі коэффицетінің әр түрлі мәндерінде тәжірибелер жасау және нәтижелерін талдау ұсынылады.

Тәжірибелер нәтижелері классикалық, теориялық механика және «Физикалық процестерді есептеу мен бейнелеу» пәндерін меігеруде қолданылады

Түйін сөздер: қозғалыс, көкжиекке бұрыш жасап лақтырылған, бастапқы жылдамдық, бастапқы биіктік, лақтыру бұрышы, траектория, парабола, ортаның кедергі коэффициенті.

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«ИССЛЕДОВАНИЕ ДВИЖЕНИЯ ТЕЛА С ТРЕНИЕМ, БРОШЕННОГО ПОД УГЛОМ К ГОРИЗОНТУ» В СИСТЕМЕ MATLAB

Аннотация. Предлагается программа расчета и визуализации движения тела, брошенного под углом к горизонту без учета и с учетом сопротивления среды. Представлены траектории движения тела, брошенного под углом к горизонту без учета и с учетом сопротивления среды при различных значения коэффициента сопротивления среды. Траекторией движения тела, брошенного под углом к горизонту без учета сопротивления среды является парабола и с учетом сопротивления среды траектория отличается и не симметрична относительно вертикали проведенной от верхней точки координаты. С увеличением коэффициента сопротивления среды понижается высота подьема тела; увеличение коэффициента сопротивления от 0.1 до 0.5 понижает высоту подъема тела примерно в 1.8 раза. Студентам предлагается самостоятельно поэкспериментировать, изменяя коэффициент сопротивления среды при различных начальных скоростях и их проанализировать. Результаты экспериментов используются при изучении и освоении классической, теоретической механики и дисциплины «Расчет и визуализация физических процессов».

Ключевые слова: движение, под углом к горизонту, начальная высота, начальная скорость, угол броска, траектория, парабола, коэффициент сопротивления среды.

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