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STABILIZATION OF THE MOVEMENT OF A SMALL SPACECRAFT IN A GEOSTATIONARY ORBIT

Abstract. A system for controlling the movement of a small spacecraft, which will later be part of a formation located in the geostationary orbit for the remote sensing of the Earth is being considered. The article assesses the effect of acting moments on the small spacecraft in the geostationary orbit and justifies the choice of these moments in the motion simulation. A motion control system for small spacecraft in the geostationary orbit using a P-controller (a controller with a proportional control law) is proposed. The results of the simulation to verify the correctness of the proposed control law are given.

Keywords: small spacecraft, geostationary orbit, remote sensing of the Earth, p-controller.

Introduction. The current state and level of development of microelectronics makes it possible to miniaturize the electronic equipment used in the space industry. In connection with this trend, the use of spacecraft for solving various scientific and technological problems is currently relevant [1-3]. The small spacecraft plays a very important technological role in space science. Compared with the traditional single large spacecraft, the small spacecraft demonstrates increased reliability [4], lower costs and a shorter development time period [5]. The tasks of remote sensing of the Earth, usually realized with the help of expensive bulky vehicles, are currently possible to solve with several miniature spacecrafts [6–10]. As practice shows, Earth remote sensing is and remains one of the most important areas of space activity in which the most innovative technologies are being introduced. The main objective of this activity is to obtain information about the object or phenomenon on the surface of the Earth. Earth remote sensing in most cases is carried out with the help of spacecraft located in low Earth orbit [11].

The monitoring of the Earth’s surface in real time necessitates the use of the geostationary orbit for the implementation of remote sensing [12, 13]. The geostationary orbit plays an irreplaceable role in various missions, including communications, navigation, research, as well as the recent Earth remote sensing.

In order to remote sensing the Earth’s surface from the geostationary orbit with a single large spacecraft a large aperture is used, the dimensions of which affect both the creation cost and the cost of spacecraft injection to a given orbit. The same results, which gives one large spacecraft, can be achieved with the help of several small spacecrafts in the formation with a specific configuration [14].

When studying the motion of the small spacecraft in the geostationary orbit, it becomes necessary to study the influence of external disturbances [15, 16], which in turn requires the development of a system for controlling the motion and stabilization of the small spacecraft [17]. This paper proposes a mathematical model of the stabilization system and orientation of the small spacecraft in the geostationary orbit and simulation results.
Estimation of moments acting on small spacecrafts in the geostationary orbit. To describe the motion of the small spacecraft relative to the center of mass, the Euler’s dynamic equations were used [18]:

\[
\begin{align*}
A\dot{p} + (C - B)qr &= \sum M_x, \\
B\dot{q} + (A - C)rp &= \sum M_y, \\
C\dot{r} + (B - A)qp &= \sum M_z.
\end{align*}
\]

(1)

where \( A, B, C \) - general inertia moments of the spacecraft; \( p, q, r \) - projections of the angular velocity vector on the axes of the body coordinate system; \( \vec{M} = (M_x, M_y, M_z)^T \) - moment of external forces relative to a fixed point.

Figure 1 – Environmental torques acting on the small spacecraft, depending on the height of the orbit.

Figure 1 shows the dependence graphs of the magnitude of the moments of forces acting on the small spacecraft on the height of the orbit. The gravitational moment, magnetic moment, moment of solar radiation pressure (SPR) force and moments arising from the other other spacecrafts in the formation were considered. The calculation was performed for satellites with mass 50 kg in accordance with the following formulas:

\[
\vec{M} = 3\frac{\mu_e}{R^3}\vec{e}_r \times \left[ A\left(\vec{e}_r \cdot \vec{i}\right)\vec{i} + B\left(\vec{e}_r \cdot \vec{j}\right)\vec{j} + C\left(\vec{e}_r \cdot \vec{k}\right)\vec{k}\right]
\]

(2)

\[
\bar{M}_{\text{spr}} = \sum_i dM_i,
\]

(3)

\[
\bar{M}_m = \bar{H} \times \vec{i}
\]

(4)

\[
\bar{H} = \frac{\mu_e}{R^3} \left\{ k_e - 3\left(k_e \vec{e}_r\right)\vec{e}_r \right\},
\]

(5)
where $\vec{e}_r$ - unit vector, $\vec{i}^r, \vec{j}^r, \vec{k}^r$ - unit vectors of the symmetry axes directions of the small spacecraft.

$\vec{H}$ - Earth magnetic vector, $\mu_E$ - magnitude of the magnetic moment of the earth dipole, $\vec{k}$ - the direction of the Earth’s magnetic dipole axis, $R$ - orbit height. $\vec{I} = \vec{k} I_0$, where $I_0$ - constant magnetic moment arising due to the presence of current systems on the small spacecraft. The moment $d\vec{M}$, created by the SRP for a particular area is calculated as the vector product of the radius vector of the center of mass of the area and the force vector acting on this area, i.e.:

$$\sum_i d\vec{M}_i = \vec{r}_x \times d\vec{F}_i.$$  \hspace{1cm} (6)

Here $\vec{r}_x$ - the radius vector from the center of mass of the small spacecraft to the center of mass of the elementary area. And $d\vec{F}_i$, determined by the equation:

$$d\vec{F} = F_p^b dS \left( \vec{n}_s \cdot \vec{f}_s \right)$$  \hspace{1cm} (7)

where $F_p^b = A^{-1} F_p^d$, and $\vec{F}_p = 2 \pi I_0 \left( \frac{a_0}{\Delta} \right)^2 dS \vec{k}$, where $\vec{k}$ - the unit vector of the “small spacecraft - Sun” direction. $dS$ - the area of the elementary platform (in our case, for a qualitative assessment, the total illuminated plane was taken as 0.25S, where $S$ is the surface area of the small spacecraft); $a_0$ - the average distance from Earth to the Sun; $P_0 = E_0 / c = 4.561 \cdot 10^{-6}$ N/m$^2$ - the pressure on a single area of the reflecting surface; $\Delta$ - the distance from the Sun to the small spacecraft; $A$ - the matrix describing the angular position of the small spacecraft:

$$A = \begin{bmatrix}
\cos \varphi \cos \psi - \sin \varphi \sin \psi \cos \theta & \cos \varphi \sin \psi + \sin \varphi \cos \psi \cos \theta & \sin \varphi \sin \theta \\
-\sin \varphi \cos \psi - \cos \varphi \sin \psi \cos \theta & -\sin \varphi \sin \psi + \cos \varphi \cos \psi \cos \theta & \cos \varphi \sin \theta \\
\sin \psi \sin \theta & -\cos \psi \sin \theta & \cos \theta
\end{bmatrix}$$  \hspace{1cm} (8)

As can be seen from figure 1, gravitational moments prevail in low earth orbits. Moments of SRP forces in low earth orbits are less than gravitational moments for an order. However, at altitudes of 10 000 km and higher, they are comparable with gravitational moments. Magnetic moments are more important in low earth orbits. The effect of the moments gravity field arising from the $2^{nd}$, $3^{rd}$ and $4^{th}$ satellites in the formation is much less than all other moments. According to that, the gravitational moment and the moment of SRP forces were chosen as main acting moments on the small spacecrafts. Since in the geostationary orbits the moments of the SRP forces and the gravitational moments prevail over the other moments, in equations (1), the right-hand side will be the following sum:

$$\vec{M} = \vec{M}_G + \vec{M}_{SRP}.$$  \hspace{1cm} (9)

where $\vec{M}_G$ and $\vec{M}_{SRP}$ - gravitational moment and moment of SRP forces.

**An orientation control system for small spacecraft based on the P-controller.** The task of building an orientation control system to maintain the orbital orientation of the small spacecraft is considered.

$$\vec{\omega}^b = J^{-1} \left[ -\vec{\omega}^b \times \left( \vec{J} \vec{\omega}^b + \vec{h}_R^b \right) + \vec{M}_c + \vec{M} \right].$$  \hspace{1cm} (10)

where $\vec{\omega}^b$ - small spacecraft’s angular velocity in the body coordinate system; $\vec{h}_R^b$ - kinetic moment of reaction wheels; $J$ - inertia tensor of the small spacecraft; $\vec{M}_c$ - control moment.
The equation (10) in scalar form is:

$$
\begin{align*}
\dot{\omega}_x &= \frac{1}{J_x} \left[ (J_y - J_z)\omega_y^b \omega_z^b - H_y^b \omega_z^b + H_z^b \omega_y^b + M_{C_x} + M_{x} \right], \\
\dot{\omega}_y &= \frac{1}{J_y} \left[ (J_z - J_x)\omega_x^b \omega_z^b - H_x^b \omega_z^b + H_z^b \omega_x^b + M_{C_y} + M_{y} \right], \\
\dot{\omega}_z &= \frac{1}{J_z} \left[ (J_x - J_y)\omega_y^b \omega_x^b - H_y^b \omega_x^b + H_x^b \omega_y^b + M_{C_z} + M_{z} \right].
\end{align*}
$$

(11)

Since the controlling influence of the executive bodies far exceeds the external moments, for solving the problem of building a control system, they can be neglected. It is assumed that, $M_{\omega} + M_{srb} = 0$, then the differential equation characterizing the angular velocity changing of the small spacecraft in the projections on the axes of the body coordinate system will take the form:

$$
\ddot{\omega}^b = J^{-1} \left[ -\omega^b \times \left( J\dot{\omega}^b + H^b \right) + M_{\omega} \right]
$$

(12)

where the control moment $M_{\omega}$ is a function of coordinates characterizing the angular position and the angular velocity of the small spacecraft:

$$
\overline{M}_{\omega} = \overline{M}_{\omega} \left( \omega_{bo}^b, \overline{Q}_{bo}, K_{a,}, K_{q} \right)
$$

(13)

where $\omega_{bo}^b$ - angular velocity of small spacecraft in body frame with respect to the orbital coordinate system, $Q_{bo}$ - quaternion defining the current angular position of the small spacecraft in the orbital coordinate system; $K_{a}, K_{q}$ - unknown control parameters.

The specific task of building a control system with a known form of the control law will be directed to determining unknown $K_{a}, K_{q}$ parameters of the control law (P-controller), based on the conditions of stability and quality of control processes.

As a result of the study, the values of the coefficients of the P-controller $k_1, k_2, k_3$ were determined to stabilize the motion of the small spacecraft relative to the center of mass. In the calculations, the following data was taken: small spacecraft orbit – geostationary (36,000 km), moments of inertia of the spacecraft $J = [0.04, 0.04, 0.01]$, initial angular position and angular velocity are assumed to be equal $\overline{\omega}^b = [1, 2, 3]$ rad/s, $\psi = \pi/3$ rad, $\phi = \pi/3$ rad, $\theta = \pi/3$ rad.

The coefficient of the P-controller was set in the range of $0.03 \leq k_1 \leq 3$. For the component of the angular velocity along the axis $Ox$ with an increase in the coefficient of the regulator, the damping of the oscillations of the angular velocity arising due to the disturbing moments is observed (figure 2). In the case when $k_1 = 0.03$, the total damping of the angular velocity component is observed at $t = 250$ s. By increasing the coefficient almost 3 times, i.e. when $k_1 = 0.1$, it can be noted that the total damping of the angular velocity component is observed at $t = 100$ s, which is 2.5 times faster than the first case. Further, with an increase of the coefficient to 0.3, damping is observed at $t = 40$ s. When the coefficient values $k_1 = 1$ and $k_1 = 3$, the angular velocity is damped at 5 and 2 seconds, respectively. From the above results, we can conclude that with an increase of the coefficient value $k_1$, stabilization of the component of angular velocity along the $Ox$ axis occurs faster.

For the angular velocity component along the axis $Oy$, with an increase of the regulator coefficient, the damping of the angular velocity oscillations, which arises due to disturbing moments, is observed (figure 3). The range of values of the regulator coefficients, as before, is $0.03 \leq k_2 \leq 3$. In this case, there are some limitations, since there is no direct relationship between stabilization and the value of the regulator coefficient. When $k_2 = 0.03$, total damping of the angular velocity component is observed at $t = 250$ s. In the case when $k_2 = 0.1$, total damping of the angular velocity component is observed at
t = 100 s. With an increase in the coefficient to 0.3, the damping is observed at \( t = 30 \) s. Further, as the coefficient increases to 1 and 3, the damping of the angular velocity is not observed, for example, at \( k_2 = 1, q = 0.2 \) rad/s, and at \( k_2 = 3, q = 0.6 \) rad/s, i.e. free stationary rotation of the small spacecraft relative to the center of mass is observed.

![Graph](image)

**Figure 2** – Stabilization of the angular velocity component along the \( Ox \) axis

![Graph](image)

**Figure 3** – Stabilization of the angular velocity component along the \( Oy \) axis
For the angular velocity component along the axis $Oy$, with an increase of the regulator coefficient, the damping of the angular velocity oscillations, which arises due to disturbing moments, is observed (figure 4). The range of values of the regulator coefficients, as before, is $0.03 \leq k_2 \leq 3$. In this case, when $k_3 = 0.03$, the total damping of the angular velocity component is observed at $t = 200$ s. In the case when $k_3 = 0.1$, the total damping of the angular velocity component is observed at $t = 150$ s. With an increase of the coefficient to 0.3, the damping is observed at $t = 50$ s. Further, with an increase of the ratio to 1 and 3, the damping of the angular velocity occurs fast enough. For example, when $k_2 = 1$, oscillation damping occurs at 15 s, and for $k_2 = 3$ at 5 s. Euler angles in the range of regulator coefficients values $0.03 \leq k_1, k_2, k_3 \leq 3$ behave ambiguously (figures 5, 6).

For example, the precession angle stabilizes at $0.03 \leq k_1, k_2, k_3 \leq 1$, respectively, when $k_1, k_2, k_3 \geq 1$ the precession angle increases indefinitely in a linear form, i.e. free rotation takes place (figure 5). The nature of the change of angle of rotation is different from the change of the angle of precession. For example, in this case, the stabilization of the angle occurs when the regulator coefficients lie in the range of $0.1 < k_1,$
$k_2, k_3 \leq 3$, i.e. for small values of $k_1, k_2, k_3$ stabilization, as in the case of the precession angle, does not occur. And a nutation angle has the largest fluctuations. Among the whole range of values of the regulator, a good result is 0.1 (Figure 6). In other cases, the small spacecraft does not stabilize in the nutation angle or the nutation angle increases indefinitely in a linear form, which indicates free rotations. For example, when $k_1, k_2, k_3 = 0.03$, oscillations are damped very slowly. When $k_1, k_2, k_3 = 1$ and $k_1, k_2, k_3 = 3$, the oscillations are not amplitude damped, but there occurs an increase of the period and a reduction of the frequency of rotations. In the case when $k_1, k_2, k_3 = 0.3$, the oscillation amplitude and frequency decrease, and the period increases, then the oscillations become a periodic and acquire an infinitely increasing linear character.

On the basis of the calculations, it can be concluded that to use the regulator the most optimal coefficient range is $0.03 < k_1, k_2, k_3 < 0.3$, since in this case stabilization takes place at all angles and angular velocities.

**Conclusions.** This paper discusses the small spacecraft motion control system, which will later be part of a group located in a geostationary orbit for the purpose of remote sensing of the Earth.

Based on the analysis of the effect of perturbations on an small spacecraft with a mass of 50 kg, it was determined that in geostationary orbit the moments of the SRP forces and the moments of gravitational forces prevail over the magnetic moment. Thus, it turned out that for the small spacecraft motion control system in the geostationary orbit, it is necessary to take into account the moments of the SRP forces and gravitational moments as the main acting factors.

This article also presents the results of the simulation of the small spacecraft motion control system in the geostationary orbit using the P-controller. As a result of the simulation, the values of the coefficients for P-controller $k_1, k_2, k_3$ were determined for stabilizing the motion of the small spacecraft relative to the center of mass.

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ГЕОСТАЦИОНАР ОРБИТАДАГЫ КИШІ ГАРЫШ АППАРАТЫНЫҢ КОЗГАЛЫСЫНЫН ТУРАҚТАНДЫРУ

Аннотация. Кейінірек Жерді қашықтықтан қолдауға арналған геостационар орбитадағы топтаманың бір белігі болатын киши гарыш аппаратының козғалысын басқару жұйесі кәрістірілді. Макадама геостационар орбитадағы киши гарыш аппараттарына сәрттап өмірге өткізіп, ерекесе бірақ, өз козғалысты моделдеу қезіндеб өтті. П-регулятор (пропорционалды басқару және колланылатын регулятор) колдану арқылы геостационарлар орбитадағы киши гарыш аппараттарының козғалысының басқару жұйесі құрылған.

Түйін сөздер: киши гарыш аппараты, геостационар орбита, Жерді қашықтықтан қолдау, П-регулятор.

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СТАБИЛИЗАЦИЯ ДВИЖЕНИЯ МАЛЫГО КОСМИЧЕСКОГО АППАРАТА НА ГЕОСТАЦИОНАРНОЙ ОРБИТЕ

Аннотация. Рассматривается система управления движением малого космического аппарата, который в дальнейшем будет являться частью группировки, расположенной на геостационарной орбите с целью дистанционного зондирования Земли.

В статье проведена оценка влияния действующих моментов на малый космический аппарат на геостационарной орбите и обоснован выбор этих моментов при моделировании движения.

Предложена система управления движением малого космического аппарата на геостационарной орбите с применением П-регулятора (регулятор с пропорциональным законом управления). Приведены результаты проведения моделирования для проверки правильности предложенного закона управления.

Ключевые слова: малый космический аппарат, геостационарная орбита, дистанционное зондирование Земли, П-регулятор.

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