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<https://doi.org/10.32014/2019.2518-170X.182>**A. Seitmuratov¹, A. Dauitbayeva¹, K. M. Berkimbaev², K. N. Turlugulova¹, E. Tulegenova¹**¹Korkyt Ata Kyzylorda State University, Kazakhstan,²Khoja Akhmet Yassawi International Kazakh-Turkish University, Kentau, Kazakhstan.

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**CONSTRUCTED TWO-PARAMETER STRUCTURALLY
STABLE MAPS**

Abstract. In this paper we consider a linear stationary closed control system, which describes the equation of state with undefined parameters. This system will help to solve the problem of constructing a linear stationary observer system in the class of two-parameter structurally stable maps for objects with one input and one output, as well as the conditions of asymptotic robust stability of steady-state control systems corresponding to the critical points of Morse from the theory of catastrophes.

Key words: stationary, closed system, critical point, object matrix, stability.

Consider a linear stationary closed control system, describing the following equation of state with undefined parameters

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t) \quad y(t) = Cx(t) + V(t), \quad x(t_0) = x_0, \quad t \geq t_0. \quad (1)$$

Here $x(t) \in R^n$ the state vector of the object, $u(t) \in R^m$, $y(t) \in R^l$ input and output vectors, A, B, C - respectively, the matrix of the object of control and observation. The object is subject to disturbances $f(t)$ and "noise (error) measurements" $\vartheta(t)$. It is believed that when the system is available to measure the processes $u(t), y(t)$, $a x(t), f(t), \vartheta(t)$ - not available. The problem of obtaining an assessment of the state of the object is considered $x(t)$. A process $x(t)$ obtained with the help of some algorithm must in a certain (for example, in an asymptotic) sense approach the process $x(t)$ ($x(t) \rightarrow \hat{x}(t)$ on $t \rightarrow \infty$) regardless of the initial state of the object x_0 .

Let the matrix of the control A object of dimension $n \times n$ and the matrix b and c - respectively control and output have the form.

$$A = \begin{vmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{vmatrix}; \quad b = \begin{vmatrix} 0 \\ 0 \\ \vdots \\ b_n \end{vmatrix}; \quad c = \begin{vmatrix} c_1 & 0 & \dots & 0 \end{vmatrix}$$

For stationary systems the observer is described by the equation

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)), \\ \hat{y}(t) &= C\hat{x}(t), \quad \hat{x}(t_0) = \hat{x}_0, \quad t \geq t_0. \end{aligned} \quad (2)$$

Here $\hat{x}(t) \in R^n$ - the state vector of the observer, which serves as an assessment of the state of the object; $\hat{y}(t) \in R^l$ - the output vector; L - the feedback operator on the residual between the outputs of the object and the observer.

The synthesis of the observer lies in the choice of the operator L . We will consider an observer whose dimension of the state vector is the same as that of the object (the so-called full-order observer, or Kalman observer).

To construct an observer consider the estimation errors

$$\varepsilon(t) = (x(t) - \hat{x}(t))$$

Subtracting from (1) equation (2), we obtain the equation for the error

$$\dot{\varepsilon}(t) = A\varepsilon(t) - LC\varepsilon(t) + f(t) - Lv(t), \quad (3)$$

$$\varepsilon(t_0) = \varepsilon_0 = x_0 - \hat{x}_0, \quad t \geq t_0.$$

As can be seen from this equation, the sources of error $\varepsilon(t)$ are the initial mismatch $\varepsilon_0 = x_0 - \hat{x}_0$, perturbation and interference measurements $\vartheta(t)$. The dynamics of the transient error $\varepsilon(t)$ is determined by the operator $\zeta(t) = A - L(t)C$.

It is necessary to investigate the behavior of the process $\varepsilon(t)$.

The synthesis of the observer lies in the choice of the operator L . Select the operator L in the form:

$$L(t) = \varepsilon_1^3 - k_1\varepsilon_1^2 - k_2; \quad (4)$$

$$\begin{cases} \dot{\varepsilon}_1(t) = \varepsilon_2(t) \\ \dot{\varepsilon}_2(t) = \varepsilon_3(t) \\ \vdots \\ \dot{\varepsilon}_n(t) = -\frac{1}{4}c_1\varepsilon_1^4 - \frac{1}{2}c_1k_1\varepsilon_1^2 + c_1(k_2 - a_n)\varepsilon_1 - a_{n-1}\varepsilon_2 - \dots - a_2\varepsilon_{n-1} - a_1\varepsilon_n \end{cases} \quad (5)$$

The stationary state of the system is determined by the solution of the equation

$$\begin{cases} \varepsilon_{1s}^1 = 0, \varepsilon_{2s} = 0, \dots, \varepsilon_{n-1,s} = 0, \varepsilon_{ns} = 0 \\ -\frac{1}{4}c_1\varepsilon_{1s}^4 - \frac{1}{2}c_1k_1\varepsilon_{1s}^2 + c_1(k_2 - a_n)\varepsilon_{1s} - a_{n-1}\varepsilon_{2s} - \dots - a_2\varepsilon_{n-1,s} - a_1\varepsilon_{ns} = 0 \end{cases} \quad (6)$$

$$f(\varepsilon_{1s}, C_1, k_1, a_n, k_2) = -c_1\varepsilon_{1s}^4 + c_1k_1\varepsilon_{1s}^2 + c_1(k_2 - a_n)\varepsilon_{1s} = 0 \quad (7)$$

The critical, twice-degenerate and thrice-degenerate critical points of the assembly (7) are determined by equating the first, second and third derivatives (7) to zero, respectively. Condition (7) is satisfied at critical points

$$-4c_1\varepsilon_{1s}^3 + 2c_1k_1\varepsilon_{1s} + c_1(k_2 - a_n) = 0 \quad (8)$$

and

$$-12c_1\varepsilon_{1s}^2 + 2c_1k_1 = 0 \quad (9)$$

The points of the control parameter space that parameterize functions with twice degenerate critical points are determined from equations (9) and (8)

$$(k_1 = 6\varepsilon_{1s}^2 \Rightarrow a_n + k_2 = -8\varepsilon_{1s}^3) \quad (10)$$

If the position of a twice-degenerate critical point is denoted by ε_{1s} , then formula (10) gives the values of the control parameters k_1 and $a_n + k_2$, which describes a function with a twice-degenerate critical point ε_{1s}

Equation (10) defines a parametric representation of the relationship between k_1 and $a_n + k_2$. A more direct expression for the relation between k_1 and $a_n + k_2$ can be obtained by excluding ε_{1s} from (10):

$$\begin{aligned} \left(\frac{k_1}{6}\right)^{\frac{1}{2}} = \varepsilon_{is} &= \left(-\frac{a_n + k_2}{8}\right)^{\frac{1}{3}} \text{ or } \left(\frac{k_1}{3}\right)^{\frac{1}{2}} = \varepsilon_{is} = \left(-\frac{a_n + k_2}{2}\right)^{\frac{1}{3}}, \left(\frac{k_1}{3}\right)^3 = -\left(\frac{a_n + k_2}{2}\right)^2 \\ \left(\frac{k_1}{3}\right)^3 + \left(\frac{a_n + k_2}{2}\right)^2 &= 0 \end{aligned} \quad (11)$$

Hence, given (11), equation (7) has the solution:

$$\varepsilon_{1s}^2 = \sqrt[3]{a_n + \frac{k_2}{2}}, \quad \varepsilon_{is} = 0, \quad i = 2, \dots, n \quad (12)$$

$$\varepsilon_{1s}^{3,4} = -\sqrt[3]{a_n + \frac{k_2}{2}}, \quad \varepsilon_{is} = 0, \quad i = 2, \dots, n \quad (13)$$

The full time derivative of the Lyapunov $V(\varepsilon)$ vector functions taking into account the equations of state (3) is defined as the scalar product of the gradient of the Lyapunov function on the velocity vector i.e.

$$\begin{aligned} \frac{dV(\varepsilon)}{dt} &= -\sum_{i=1}^n \left(\sum_{j=1}^n \frac{\partial V_i(\varepsilon)}{\partial \varepsilon_j} \right) \frac{d\varepsilon_i}{dt} = \\ &= -\sum_{i=1}^n \left[-\frac{1}{4} c_1 \varepsilon_1^4 + \frac{1}{2} c_1 k_1 \varepsilon_1^2 - c_1 (k_2 - a_n) \varepsilon_1 - a_{n-1} \varepsilon_2 - a_{n-3} \varepsilon_3 - \dots - a_2 \varepsilon_{n-1} + \dots a_1 \varepsilon_n \right]^2 \end{aligned} \quad (14)$$

It follows from equation (14) that the full time derivative of the Lyapunov function will always be a sign-negative function, i.e. a sufficient stability condition will always be satisfied for any stationary state.

We investigate the stability of the stationary state (13). Equation of state (4) in deviations relative to the stationary state (13). To do this, calculate:

$$\begin{aligned} \left(\frac{\partial F_1}{\partial \varepsilon_2} \right)_{\varepsilon_s} &= \varepsilon_2, \left(\frac{\partial F_2}{\partial \varepsilon_3} \right)_{\varepsilon_s} = \varepsilon_3, \left(\frac{\partial F_{n-1}}{\partial \varepsilon_n} \right)_{\varepsilon_s} = \varepsilon_n \\ \left(\frac{\partial F_n}{\partial \varepsilon_2} \right)_{\varepsilon_s} &= a_{n-1}, \left(\frac{\partial F_n}{\partial \varepsilon_3} \right)_{\varepsilon_s} = a_{n-2}, \left(\frac{\partial F_n}{\partial \varepsilon_1} \right)_{\varepsilon_s} = a_{n-3}, \dots, \left(\frac{\partial F_n}{\partial \varepsilon_1} \right)_{\varepsilon_s^4} = 2(a_n + c_1 k_2) + \\ &+ 3(a_n + c_1 k_2) + (a_n + c_1 k_2) + (a_n + c_1 k_2) = 6(a_n + c_1 k_2), \\ \left(\frac{\partial^2 F_1}{\partial \varepsilon_1^2} \right)_{\varepsilon_s^{3,4}} &= \left[-3c_1 \varepsilon_1^2 + 2c_1 k_1 \right]_{\varepsilon_s^{3,4}} = -5c_1 \sqrt[3]{\left(\frac{a_n + c_1 k_2}{2c_1} \right)^2} \\ \left(\frac{\partial^3 F_1}{\partial \varepsilon_1^3} \right)_{\varepsilon_s} &= -6c_1 \varepsilon_1 \Big|_{\varepsilon_s^3} = -6c_1 \sqrt[3]{\frac{a_n + c_1 k_2}{2c_1}} \\ \frac{\partial^3 F_1}{\partial \varepsilon_i \partial \varepsilon_j \partial \varepsilon_k} &= 0, i \neq j \neq k, i = 2, \dots, n, j = 1, \dots, n, k = 1, \dots, n \\ \left(\frac{\partial^4 F_n}{\partial \varepsilon_1^4} \right) &= -6c_1, i = 1, 2, \dots, n \end{aligned}$$

Equations of state (4) in deviations relative to the stationary state (13) is written:

$$(15) \quad \begin{cases} \dot{\varepsilon}_1(t) = \varepsilon_2(t) \\ \dot{\varepsilon}_2(t) = \varepsilon_3(t) \\ \dots \\ \dot{\varepsilon}_n(t) = -c_1\varepsilon_1^4(t) + 4c_1\sqrt{\frac{a_n + c_1k_2}{2c_1}}\varepsilon_n^3(t) - 9c_1\sqrt{\left(\frac{a_n + c_1k_2}{2c_1}\right)^2}\varepsilon_1^2(t) + 6(a_n + c_nk_2)\varepsilon_n(t) - a_{n-1}\varepsilon_2 \\ - a_{n-3}\varepsilon_3 - \dots - a_1\varepsilon_n \end{cases}$$

The full time derivative of the Lyapunov vector functions $\nabla V_i(\varepsilon)$, taking into account the equations of state (15), will be equal to:

$$(16) \quad \frac{dV(\varepsilon)}{dt} = -\frac{1}{2}\varepsilon_2^2 - \frac{1}{2}\varepsilon_3^2 - \dots - \frac{1}{2}\varepsilon_n^2 - \left[c_1\varepsilon_1^4 + 4c_1\sqrt{\frac{a_n + c_1k_2}{2c_1}}\varepsilon_1^3 - 9c_1\sqrt{\left(\frac{a_n + c_1k_2}{2c_1}\right)^2}\varepsilon_1^2 + 6(a_n + c_nk_2)\varepsilon_1 + \right. \\ \left. + a_{n-1}\varepsilon_2 - a_{n-3}\varepsilon_3 - \dots - a_1\varepsilon_n \right]^2$$

A sufficient stability condition will always be satisfied. We find the gradient components of the Lyapunov vector functions $\nabla V_i(\varepsilon)$ by the components of the velocity vector, i.e. by the equation of state.

$$\begin{aligned} \frac{\partial V_1}{\partial \varepsilon_1} &= 0, \frac{\partial V_1}{\partial \varepsilon_2} = \varepsilon_2, \dots, \frac{\partial V_n}{\partial \varepsilon_n} = 0 \\ \frac{\partial V_2}{\partial \varepsilon_1} &= 0, \frac{\partial V_2}{\partial \varepsilon_2} = 0, \frac{\partial V_2}{\partial \varepsilon_3} = \varepsilon_3, \dots, \frac{\partial V_2}{\partial \varepsilon_n} = 0 \\ \dots \\ \frac{\partial V_n}{\partial \varepsilon_2} &= a_{n-2}\varepsilon_2, \frac{\partial V_n}{\partial \varepsilon_3} = a_{n-3}\varepsilon_3, \dots, \\ \frac{\partial V_n}{\partial \varepsilon_1} &= c_n\varepsilon_n^4 - 4c_1\sqrt{\frac{a_n + c_1k_2}{2c_1}}\varepsilon_1^3 + 9c_1\sqrt{\left(\frac{a_n + c_1k_2}{2c_1}\right)^2}\varepsilon_1^2 - 6(a_n + c_nk_2)\varepsilon_1, \dots, \frac{\partial V_n}{\partial \varepsilon_n} = -a_1\varepsilon_n \end{aligned}$$

The potential function has the form:

$$(17) \quad V(\varepsilon) = \frac{1}{5}c_1\varepsilon_1^5 - c_1\sqrt{\frac{a_n + c_1k_2}{2c_1}}\varepsilon_1^4 + 3c_1\sqrt{\left(\frac{a_n + c_1k_2}{2c_1}\right)^2}\varepsilon_1^2 - 3(a_n + c_nk_2)\varepsilon_1^2 - \frac{1}{2}\varepsilon_2^2 - \frac{1}{2}\varepsilon_3^2 - \dots - \frac{1}{2}\varepsilon_n^2 - \frac{1}{2}a\varepsilon_n^2$$

According to Morse's Lemma, the potential function (17) can be carried out by replacing the variables to a quadratic form with a Hess matrix of diagonal form.

$$V_{i,j} = \left\| \begin{pmatrix} \frac{\partial^2 V(\varepsilon)}{\partial \varepsilon_i \partial \varepsilon_j} \end{pmatrix}_{\varepsilon_{is}^2} \right\| = \begin{vmatrix} \lambda_1 & 0 & 0 & \dots & 0 \\ 0 & \lambda_2 & 0 & \dots & 0 \\ 0 & 0 & \lambda_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \lambda_n \end{vmatrix},$$

where

$$\begin{cases} \lambda_1 = -3(a_n - c_1 k_2) \\ \lambda_2 = -(a_{n-1} + 1) \\ \lambda_3 = -(a_{n-2} + 1) \\ \dots \\ \lambda_n = -(a_1 + 1) \end{cases}$$

The conditions of positive definiteness of the Lyapunov function or the stability conditions of the Hess matrix will be expressed by a system of inequalities:

$$\begin{cases} -3(a_n - c_1 k_2) > 0 \\ -(a_{n-1} + 1) > 0 \\ -(a_{n-2} + 1) > 0 \\ \dots \\ -(a_1 + 1) > 0 \end{cases} \quad (18)$$

Thus, the observing device constructed in the class of two-parameter structurally stable mappings will be stable within an unlimited range of changes in the undefined parameters of the control object ε_i ($i = 1, 2, \dots, n$). The stationary state of the observing device (6) exists and is stable when the undefined parameters of the object in the region (2) change, and the stationary states (12) and (13) appear when the state (6) loses stability. These stationary states do not exist simultaneously and among stationary states (12) and (13) is stable. It should be noted that the stationary state (13) in the region (18) does not exist.

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ҚОС ПАРАМЕТРЛІК ҚҰРЫЛЫМДЫ-ТҮРАҚТЫ КЕСКІНДЕРДІ ТҮРФЫЗУ

Аннотация. Берілген жұмыста анықталмаған параметрлері бар күй тендеуін сипаттайтын сыйықтық стационарлы тұйықталған басқару жүйесі қарастырылады. Бұл жүйе бір кіру және бір шығу жолы бар обьектілер үшін екі параметрлік құрылымдық-тұрақты бейнелеудер класында бақылаушының желілік стационарлық жүйесін құру кезінде тапсырманы шешуге көмектеседі, сондай-ақ апарттар теориясынан Морстің сынни нүктелеріне сәйкес келетін Басқару жүйелерінің қалыптасқан жай-күйінін асимптотикалық робастикалық тұрақтылығының шарттары алынды.

Түйін сөздер: стационарлық, тұйық жүйе, сынни нүктесі, обьект матрицасы, тұрақтылық.

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ПОСТРОЕНИЕ ДВУХПАРАМЕТРИЧЕСКИХ СТРУКТУРНО-УСТОЙЧИВЫХ ОТОБРАЖЕНИЙ

Аннотация. Рассматривается линейная стационарная замкнутая система управления, которая описывает уравнение состояния с неопределенными параметрами. Эта система поможет решать задачу при построении линейной стационарной системы наблюдателя в классе двухпараметрических структурно-устойчивых отображений для объектов с одним входом и одним выходом, а также получать условия асимптотической робастной устойчивости установившихся состояний систем управления, соответствующие критическим точкам Морса из теории катастроф.

Ключевые слова: стационарная, замкнутая система, критическая точка, матрица объекта, устойчивость.

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