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A. Seitmuratov[1], M. Ramazanov[2], N. Medeubaev[2], B. Kaliev[1]

[1]The Korkyt Ata Kyzylorda State University, Kyzylorda, Kazakhstan,
[2]The E.A. Buketov Karaganda State University, Karaganda, Kazakhstan.
E-mail: angisin_@mail.ru, ramamur@mail.ru, medeubaev65@mail.ru, kaliev_b@mail.ru

MATHEMATICAL THEORY OF VIBRATION OF ELASTIC OR VISCOELASTIC PLATES, UNDER NON-STATIONARY EXTERNAL INFLUENCES

Abstract. In this work, attempt was made to present the mathematical theory of oscillations of an elastic or viscoelastic plate to study its dynamic behavior under nonstationary external influences. On the basis of this approach, exact equations of longitudinal and transverse oscillations of viscoelastic plates are derived with and without allowance for initial displacements and stresses, approximate equations for the physical nonlinearity of the material. For all problems, expressions are obtained for all displacements and stresses along the thickness of the plate and basic boundary-value formulated problems that lead to longitudinal or transverse vibrations of the plate. On the basis of exact equations some approximate equations that follow with some degree of accuracy are analyzed and approximate boundary value problems formulated for them.

Key words: elastic, viscoelastic, nonstationary, oscillations, nonlinear, deformable, longitudinal, transverse.

Investigation of wave processes in restricted deformable bodies are reduced to complicated mathematical problem, which is generally at the present stage can not be solved either analytical or numerical methods.

Even for deformable media which is described by the simplest models, such as elastic and viscoelastic media, many nonstationary problems have not been investigated and there are no methods to solve these problems in accurate formulation. Therefore, many applied problems in various fields of technology are solved on simplified models that reduce the spatial problems of dynamics to two-dimensional or one-dimensional ones. Such simplified models are plates, rods and shells.

All naturally occurring environments on the nature of the dynamic behavior can be divided into perfectly elastic and differential elastic.

. The first group includes environments of mechanical characteristics that are close to each other. When studying dynamics and wave processes, such media can be considered as homogeneous medium with averaged mechanical characteristics.

Для дифференциально-упругих сред механические характеристики, составляющие двухкомпонентную среду, значительно различаются друг от друга.

For differential-elastic media, the mechanical characteristics which constituting the two-component medium differ significantly from each other.

We shall consider two-component media consisting of two elastic components or from an elastic porous skeleton and liquid filler.

We introduce the concept of the porosity of the medium.

The porosity of the medium will be denoted by a quantity determined by the formula:

$$K_0 = V_p / V_s, \tag{1}$$

where - the volume of the second component relative to the first medium in a volume element as a whole, V_s - the total volume of the elementary sample.

We'll assume, that $K_0 = const$.

The first type of two-component medium consisting of two elastic components is characterized by different conditions of adhesion between them and in the general case of the stress-strain relationship, we write in the form:

$$\sigma_{ij}^{(1)} = \sigma_{ij}^{(1)}\left(\varepsilon_x^{(1)}, \varepsilon_y^{(1)}, \varepsilon_z^{(1)}; \varepsilon_x^{(2)}, \varepsilon_y^{(2)}, \varepsilon_z^{(2)}; \varepsilon_{xy}^{(1)}, \varepsilon_{yz}^{(1)}, \varepsilon_{xz}^{(1)}; \varepsilon_{xy}^{(2)}, \varepsilon_{yz}^{(2)}, \varepsilon_{xz}^{(2)}\right) \qquad (2)$$

$$\sigma_{ij}^{(2)} = \sigma_{ij}^{(2)}\left(\varepsilon_x^{(1)}, \varepsilon_y^{(1)}, \varepsilon_z^{(1)}; \varepsilon_x^{(2)}, \varepsilon_y^{(2)}, \varepsilon_z^{(2)}; \varepsilon_{xy}^{(1)}, \varepsilon_{yz}^{(1)}, \varepsilon_{xz}^{(1)}; \varepsilon_{xy}^{(2)}, \varepsilon_{yz}^{(2)}, \varepsilon_{xz}^{(2)}\right),$$

where the index " 1 " refers to the first component, the index " 2 " - to the second.

The second type of two-component medium requires some explanation.

The term "pore" refers to a medium with openly communicating pores.

The connection between the constituent components of the medium will be considered imperfect, i.e. The liquid component can not flow out of the medium.

Let the continuous deformable medium consist of two elastic continua with different mechanical characteristics whose densities will be denoted by $\rho_j\,(j=1,2)$, the displacement vectors of the points.

$$\vec{U}^{(j)} \quad (j=1,2).$$

Depending on the deformation stresses are [1]

$$\sigma_{\alpha\alpha}^{(1)} = -\alpha_2 + \lambda_1 e^{(1)} + 2\mu_1 \varepsilon_{\alpha\alpha}^{(1)} + \lambda_3 e^{(2)} + 2\mu_3 \varepsilon_{\alpha\alpha}^{(2)};$$

$$\frac{1}{2}\left(\sigma_{\alpha\beta}^{(1)} + \sigma_{\beta\alpha}^{(1)}\right) = 2\mu_1 \varepsilon_{\alpha\beta}^{(1)} + 2\mu_3 \varepsilon_{\alpha\beta}^{(2)}; \qquad (3)$$

$$\frac{1}{2}\left(\sigma_{\alpha\beta}^{(1)} - \sigma_{\beta\alpha}^{(1)}\right) = -\lambda_5\left(h_{\beta\alpha} - h_{\alpha\beta}\right),$$

for the first component,

$$\sigma_{\alpha\alpha}^{(2)} = \alpha_2 + \lambda_2 e^{(2)} + 2\mu_2 \varepsilon_{\alpha\alpha}^{(2)} + \lambda_4 e^{(1)} + 2\mu_4 \varepsilon_{\alpha\alpha}^{(1)};$$

$$\frac{1}{2}\left(\sigma_{\alpha\beta}^{(2)} + \sigma_{\beta\alpha}^{(2)}\right) = 2\mu_2 \varepsilon_{\alpha\beta}^{(2)} + 2\mu_3 \varepsilon_{\alpha\beta}^{(1)}; \qquad (4)$$

$$\frac{1}{2}\left(\sigma_{\alpha\beta}^{(2)} - \sigma_{\beta\alpha}^{(2)}\right) = \lambda_5\left(h_{\beta\alpha} - h_{\alpha\beta}\right),$$

For the second component,
in this case take place depending

$$\mu_4 = \mu_3;$$

$$\alpha_2 = \lambda_3 - \lambda_4 \qquad (5)$$

here $\alpha_2, \lambda_2 \mu_2$ elastic constants,

$$\varepsilon_{\alpha\beta}^{(j)} = \frac{1}{2}\left(\frac{\partial U_\alpha^{(j)}}{\partial \beta} + \frac{\partial U_\beta^{(j)}}{\partial \alpha}\right); \quad (j=1,2);$$

$$h_{\alpha\beta} = \frac{\partial U_\alpha^{(2)}}{\partial \beta} + \frac{\partial U_\beta^{(1)}}{\partial \alpha}; \qquad (6)$$

Where $U_{\alpha}^{(j)}, U_{\beta}^{(j)}$ the components of displacement $\vec{U}^{(j)}$ vectors. Elastic mechanical characteristics depend on both the porosity and the adhesion conditions between the grains that make up the medium and are determined experimentally.

As can be seen from the dependences (3) and (4), the law of shear stresses does not hold, due to the mutual influence of the environmental component and other factors.

The equations of motion in stresses are:

$$\frac{\partial\sigma_{\alpha\beta}^{(1)}}{\partial\beta} - N_{\alpha} = \rho_{11}\frac{\partial^2 U_{\alpha}^{(1)}}{\partial t^2} + \rho_{12}\frac{\partial^2 U_{\alpha}^{(2)}}{\partial t^2};$$

$$\frac{\partial\sigma_{\alpha\beta}^{(2)}}{\partial\beta} - N_{\alpha} = \rho_{12}\frac{\partial^2 U_{\alpha}^{(1)}}{\partial t^2} + \rho_{22}\frac{\partial^2 U_{\alpha}^{(2)}}{\partial t^2}, \quad (7)$$

where

$$N_a = \frac{\alpha_2}{\rho}\left[\rho_1\frac{\partial\varepsilon^{(2)}}{\partial\alpha} + \rho_2\frac{\partial\varepsilon^{(1)}}{\partial\alpha}\right] + v\left[\frac{\partial U_{\alpha}^{(1)}}{\partial t} - \frac{\partial U_{\alpha}^{(2)}}{\partial t}\right] \quad (8)$$

v - diffusion coefficient, and take the values (x, y, z) in a Cartesian coordinate system, or other coordinates (cylindrical, spherical, etc).

The quantities have the dimensionality of the density and are equal to:

$$\rho_1 = \rho_{11} + \rho_{12};$$
$$\rho_2 = \rho_{22} + \rho_{12};$$
$$\rho = \rho_1 + \rho_2, \quad (9)$$

wherein

$$\rho_{11}\rho_{22} - \rho_{12}^2 \rangle 0;$$
$$\rho_{12}\langle 0.$$

ρ_{12} - plays the role of an attached mass.

If we add the left and right sides of equations (7) and introduce the notation:

$$\sigma_{\alpha\beta}^{(1)} + \sigma_{\alpha\beta}^{(2)} = \sigma_{\alpha\beta}, \quad (10)$$

then, we can take

$$\frac{\partial\sigma_{\alpha\beta}}{\partial\beta} = \rho_1\frac{\partial^2 U_{\alpha}^{(1)}}{\partial t^2} + \rho_2\frac{\partial^2 U_{\alpha}^{(2)}}{\partial t^2} \quad (11)$$

i.e. the total stress depends on the acceleration of the particles that make up the two-component medium. In the case of continuous one-component medium $U_{\alpha}^{(1)} = U_{\alpha}^{(2)}$, from (11) we obtain the known equations:

$$\frac{\partial\sigma_{\alpha\beta}}{\partial\beta} = \rho\frac{\partial^2 U_{\alpha}}{\partial t^2}; \quad (12)$$
$$(U_{\alpha}^{(1)} = U_{\alpha}^{(2)} = U_{\alpha})$$

Similarly, under conditions (12), the Hooke's law for isotropic homogeneous elastic medium is obtained from relations (3) and (4).

The equations of motion (7) are simplified by introducing potentials and longitudinal Φ_j and $\vec{\Psi}_j$ transverse waves.

$$\vec{U}^{(j)} = grad\Phi_j + rot\vec{\Psi}_j; \quad \vec{\Psi}_j = \vec{\Psi}_j\left(\vec{\Psi}_j^{(1)}, \vec{\Psi}_j^{(2)}, \vec{\Psi}_j^{(3)}\right) \tag{13}$$

the solenoidal condition must be satisfied

$$div\vec{\Psi}_j = 0 \ (j = 1,2) \tag{14}$$

In potentials Φ_j и $\vec{\Psi}_j$ the equations of motion (7) are reduced to the form:

$$A_1\Delta\Phi_1 + B_1\Delta\Phi_2 = \rho_{11}\frac{\partial^2\Phi_1}{\partial t^2} + \rho_{12}\frac{\partial^2\Phi_2}{\partial t^2} + v\left(\frac{\partial\Phi_1}{\partial t} - \frac{\partial\Phi_2}{\partial t}\right); \tag{15}$$

$$A_2\Delta\Phi_2 + B_2\Delta\Phi_1 = \rho_{12}\frac{\partial^2\Phi_1}{\partial t^2} + \rho_{22}\frac{\partial^2\Phi_2}{\partial t^2} - v\left(\frac{\partial\Phi_1}{\partial t} - \frac{\partial\Phi_2}{\partial t}\right); \tag{16}$$

$$(\mu_1 - \lambda_5)\Delta\vec{\Psi}_1 + (\mu_1 + \lambda_5)\Delta\vec{\Psi}_2 = \rho_{11}\frac{\partial^2\vec{\Psi}_1}{\partial t^2} + \rho_{12}\frac{\partial^2\vec{\Psi}_2}{\partial t^2} - v\left(\frac{\partial\vec{\Psi}_1}{\partial t} - \frac{\partial\vec{\Psi}_2}{\partial t}\right); \tag{17}$$

$$(\mu_1 - \lambda_5)\Delta\vec{\Psi}_2 + (\mu_1 + \lambda_5)\Delta\vec{\Psi}_1 = \rho_{12}\frac{\partial^2\vec{\Psi}_1}{\partial t^2} + \rho_{22}\frac{\partial^2\vec{\Psi}_2}{\partial t^2} + v\left(\frac{\partial\vec{\Psi}_1}{\partial t} - \frac{\partial\vec{\Psi}_2}{\partial t}\right); \tag{18}$$

where Δ - three-dimensional Laplace operator,

$$A_j = \left[\lambda_j + 2\mu_j + (-1)^j\frac{\rho_2\alpha_2}{\rho}\right]; \quad B_j = \left[\lambda_{j+1} + 2\mu_{j+1} + (-1)^j\frac{\rho_1\alpha_2}{\rho}\right]; \tag{19}$$

and constants λ_j, μ_j must satisfy the inequalities [3]:

$$A_1; A_2 \neq \left(B_1^2; B_2^2\right),$$

$$(A_1 + \mu_1)(A_2 + \mu_2) - (B_{1,2} + \mu_3)^2 \neq 0$$

$$\mu_1\mu_2 \neq \mu_3^2;$$

$$(\lambda_1 + \mu_1)(\lambda_2 + \mu_2) \neq (\lambda_3 + \mu_3)(\lambda_4 + \mu_4).$$

In the absence of diffusion, i.e. for v = 0, in the equations (15), (16) and (17), (18) we set

$$\Phi_1 = \varphi; \ \Phi_2 = \gamma\varphi; \quad \vec{\Psi}_1 = \vec{\psi}_1; \ \vec{\Psi}_2 = \delta\psi \tag{20}$$

Substituting (20) into equations (15.) - (18.), to determine and obtain algebraic equations;

$$(B_1\rho_{22} - A_2\rho_{12})\gamma^2 + [(B_1 - B_2)\rho_{12} + (A_1\rho_{22} - A_2\rho_{11})]\gamma - (-A_1\rho_{12} + B_2\rho_{11}) = 0; \tag{21}$$

$$[(\mu_3 + \lambda_5)\rho_{22} - (\mu_2 - \lambda_5)\rho_{12}]\delta^2 + [(\mu_1 + \lambda_5)\rho_{22} - (\mu_2 - \lambda_5)\rho_{11}]\delta - [(\mu_3 + \lambda_5)\rho_{11} - (\mu_1 - \lambda_5)\rho_{12}] = 0 \tag{22}$$

As it can be seen from equations (21) and (22), they have two real roots, which we denote by (γ_1, γ_2) and (δ_1, δ_2).

Hence, by the principle of superposition can be put;

$$\begin{aligned}&\Phi_1 = \varphi_1 + \varphi_2;\\&\Phi_2 = \lambda_1\varphi_1 + \lambda_2\varphi_2;\\&\vec{\Psi}_1 = \vec{\psi}_1 + \vec{\psi}_2;\\&\vec{\Psi}_1 = \delta_1\vec{\psi}_1 + \delta_2\vec{\psi}_2;\end{aligned} \tag{23}$$

And for φ_1 и $\vec{\psi}_1$ we obtain separate wave equations:

$$\Delta\varphi_j = \frac{1}{a_j^2}\frac{\partial^2\varphi_j}{\partial t^2}; \qquad (j=1,2) \tag{24.}$$

$$\Delta\vec{\psi} = -\frac{1}{b_j^2}\frac{\partial^2\vec{\psi}_o}{\partial t^2}; \qquad (j=1,2) \tag{25}$$

while generalized velocities a_j, b_j longitudinal and transverse waves are equal:

$$\begin{aligned}&a_j^2 = \frac{A_1 + \gamma_j B_1}{\rho_{11} + \gamma_j\rho_{12}} = \frac{A_2\gamma_j + B_2}{\rho_{12} + \gamma_j\rho_{22}}; \quad (j=1,2)\\&b_j^2 = \frac{(\mu_1 - \lambda_5) + (\mu_3 + \lambda_5)\delta_j}{\rho_{11} + \gamma_j\rho_{12}} = \frac{(\mu_2 - \lambda_5)\delta_j + (\mu_3 + \lambda_5)}{\rho_{12} + \gamma_j\rho_{22}};\\&(j=1,2)\end{aligned} \tag{26}$$

In the presence of diffusion, the system of equations (15) - (18) does not reduce to separate equations of the type (24) - (25).

Let us consider a porous elastic medium with liquid filler with an imperfect bond between the elastic skeleton and the reservoir.

Dependencies between strains and stresses are more conveniently written in the form

$$\sigma_{\alpha\beta} = 2\mu\varepsilon_{\alpha\beta} + \delta_{\alpha\beta}(\lambda\varepsilon + Q\varepsilon_0); \tag{27}$$

$$\sigma = Q\varepsilon + R\varepsilon_0; \quad \sigma = -RK_0$$

where $\varepsilon_{\alpha\beta}$ - deformation of the skeleton.

$$\varepsilon = \varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz}$$

ε_0 – volumetric deformation of liquid filler; λ, μ, Q, R – mechanical characteristics of a two-component medium; P – pressure in the liquid component.

There are types of the equations of motion in stresses:

$$\begin{aligned}&\frac{\partial\sigma_{\alpha\beta}}{\partial\beta} = \rho_{11}\text{-}\frac{\partial^2 U_\alpha^{(1)}}{\partial t^2} + \rho_{22}\frac{\partial^2 U_\alpha^{(2)}}{\partial t^2};\\&\frac{\partial\sigma}{\partial\beta} = \rho_{12}\frac{\partial^2 U_\alpha^{(1)}}{\partial t^2} + \rho_{22}\frac{\partial^2 U_\alpha^{(2)}}{\partial t^2}.\end{aligned} \tag{28}$$

wherein α and β run through values in the Cartesian coordinate system and dependencies (x, y, z) and dependencies must be fulfilled (18).

Equations of motion (28) are simplified by introducing potentials $\Phi_j, \vec{\Psi}_j$ by formulas (13), wherein $\vec{\Psi}_j$ must satisfy condition (14).

In the potentials of equation $\Phi_j, \vec{\Psi}_j$ motion (28) are reduced to the form:

$$(\lambda+2\mu)\Delta\Phi_1+Q\Phi_2=\rho_{11}\frac{\partial^2\Phi_1}{\partial t^2}+\rho_{12}\frac{\partial^2\Phi_2}{\partial t^2};$$
$$Q\Delta\Phi_1+R\Delta\Phi_2=\rho_{12}\frac{\partial^2\Phi_1}{\partial t^2}+\rho_{22}\frac{\partial^2\Phi_2}{\partial t^2}; \quad (29)$$
$$\mu\Delta\vec{\Psi}_1=\frac{\left(\rho_{11}\vartheta_{22}-\rho_{12}^2\right)}{\rho_{22}}\frac{\partial^2\vec{\Psi}_1}{\partial t^2}+\rho_{22}\frac{\partial^2\vec{\Psi}_2}{\partial t^2};$$
$$\frac{\partial^2\vec{\Psi}_2}{\partial t^2}=-\frac{\rho_{12}}{\rho_{22}}\frac{\partial^2\vec{\Psi}_1}{\partial t^2};$$

thinking

$$\Phi_1=\varphi; \qquad \Phi_2=\gamma\varphi \quad (30)$$

and substituting (30) in the first two equations (29), we obtain γ the following equation for the determination:

$$\gamma^2+\frac{(\lambda+2\mu)\rho_{22}-R\rho_{11}}{\rho_{22}Q-\rho_{12}R}\gamma-\frac{\rho_{11}Q-(\lambda+2\mu)\rho_{12}}{\rho_{22}Q-\rho_{12}R}=0 \quad (31)$$

which has two real roots γ_1 и γ_2. Consequently,

$$\Phi_1=\varphi_1+\varphi_2; \qquad \Phi_2=\gamma_1\varphi_1+\gamma_2\varphi_2 \quad (32)$$

and potentials φ_1,φ_2 satisfy the wave equations:

$$\Delta\varphi_j=\frac{1}{a_j^2}\frac{\partial^2\varphi_j}{\partial t^2} \qquad (j=1,2) \quad (33)$$

where the generalized velocities a_j are:

$$a_j^2=\frac{(\lambda+2\mu)Q\gamma_j}{\rho_{11}+\rho_{12}\gamma_j}; \quad (34)$$

The last two of equations (29) can conveniently be reduced to the form:

$$\Delta\vec{\Psi}_1=\frac{1}{b^2}\frac{\partial^2\vec{\Psi}_1}{\partial t^2}; \qquad \vec{\Psi}_2=-\frac{\rho_{12}}{\rho_{22}}\vec{\Psi}_1 \quad (35)$$

wherein

$$b^2=\frac{\mu\rho_{22}}{\rho_{11}\rho_{22}-\rho_{12}^2}; \quad (36)$$

In the generalized potentials, the values of displacements will be written as:

$$U_x^{(1)}=\frac{\partial}{\partial x}(\varphi_1+\varphi_2)+\frac{\partial\Psi_3^{(1)}}{\partial y}-\frac{\partial\Psi_2^{(1)}}{\partial z};$$
$$U_x^{(2)}=\frac{\partial}{\partial x}(\gamma_1\varphi_1+\gamma_2\varphi_2)+\frac{\partial\Psi_3^{(3)}}{\partial y}-\frac{\partial\Psi_2^{(2)}}{\partial z};$$
$$U_y^{(1)}=\frac{\partial}{\partial y}(\varphi_1+\varphi_2)+\frac{\partial\Psi_1^{(1)}}{\partial z}-\frac{\partial\Psi_3^{(1)}}{\partial x};$$

$$U_y^{(1)} = \frac{\partial}{\partial y}(\gamma_1\varphi_1 + \gamma_1\varphi_2) + \frac{\partial \Psi_1^{(2)}}{\partial z} - \frac{\partial \Psi_3^{(2)}}{\partial x};$$
$$U_z^{(1)} = \frac{\partial}{\partial z}(\varphi_1 + \varphi_2) + \frac{\partial \Psi_2^{(1)}}{\partial x} - \frac{\partial \Psi_1^{(1)}}{\partial y};$$
$$U_z^{(1)} = \frac{\partial}{\partial z}(\gamma_1\varphi_1 + \gamma_2\varphi_2) + \frac{\partial \Psi_2^{(2)}}{\partial x} - \frac{\partial \Psi_1^{(2)}}{\partial y}; \quad (37)$$

For a two-component elastic medium with $\nu = 0$:

$$\Psi_i^{(j)} = \delta_1 \Psi_i^{(j)} + \delta_2 \Psi_i^{(j)} \quad (38)$$

and for two-component porous mediuma

$$\Psi_i^{(2)} = -\frac{\rho_{12}}{\rho_{22}} \Psi_i^{(1)} \quad (39)$$

The deformation and stress of two-component media are determined through the displacement (37) according to the known formulas and generalized Hooke's laws (3), (4) and (27).

Plane generalized stress state is used in the study of wave processes in bounded media such as plates. It is approximately formulated under the condition that the unknown quantities are independent of the transverse coordinates z , i.e. when

$$U_{1z} = U_{2z} \approx 0 \quad (40)$$

In this case from (27)

$$\varepsilon_{zz} = -\left[\frac{\lambda}{\lambda + 2\mu}(\varepsilon_{xx} + \varepsilon_{yy}) + \frac{Q}{\lambda + 2\mu}\varepsilon_0\right] \quad (41)$$

and relations (17) have type:

$$\begin{aligned}
&\sigma_{xx} \frac{4\mu(\lambda + \mu)}{(\lambda + 2\mu)}\varepsilon_{xx} + \frac{2\mu\lambda}{(\lambda + 2\mu)}\varepsilon_{yy} + \frac{2\mu Q}{(\lambda + 2\mu)}\varepsilon_0;\\
&\sigma_{xx} \frac{2\mu\lambda}{(\lambda + 2\mu)}\varepsilon_{xx} + \frac{4\mu(\lambda + \mu)}{(\lambda + 2\mu)}\varepsilon_{yy} + \frac{2\mu Q}{(\lambda + 2\mu)}\varepsilon_0;\\
&\sigma_{xy} = \mu\varepsilon_{xy};\\
&\sigma = Q\varepsilon + R\varepsilon_0;
\end{aligned} \quad (42)$$

wherein:

$$\begin{aligned}
&\varepsilon = \varepsilon_{xx} + \varepsilon_{yy}; \quad \varepsilon_0 = \varepsilon_{xx}^{(0)} + \varepsilon_{yy}^{(0)};\\
&\varepsilon_{xx} = \frac{\partial U_1}{\partial x}; \quad \varepsilon_{xy} = \frac{\partial U_1}{\partial y} + \frac{\partial U_1}{\partial x};\\
&\varepsilon_{xx}^{(0)} = \frac{\partial U_2}{\partial x}; \quad \varepsilon_{yy}^{(0)} = \frac{\partial U_2}{\partial y};
\end{aligned} \quad (43)$$

We write the equations of motion in the form:

$$\begin{aligned}
&\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}=\rho_{11}\frac{\partial^2 U_1}{\partial t^2}+\rho_{12}\frac{\partial^2 U_2}{\partial t^2};\\
&\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}=\rho_{11}\frac{\partial^2 U_1}{\partial t^2}+\rho_{22}\frac{\partial^2 U_2}{\partial t^2};\\
&\frac{\partial\sigma}{\partial x}=\rho_{12}\frac{\partial^2 U_1}{\partial t^2}+\rho_{22}\frac{\partial^2 U_2}{\partial t^2};\\
&\frac{\partial\sigma}{\partial y}=\rho_{12}\frac{\partial^2 U_1}{\partial t^2}+\rho_{22}\frac{\partial^2 U_2}{\partial t^2};
\end{aligned} \tag{44}$$

Equations (44) can be reduced to a system of wave equations, assuming:

$$\begin{aligned}
&U_1=\frac{\partial\varphi_1}{\partial x}+\frac{\partial\Psi_1}{\partial y}; \qquad U_2=\frac{\partial\varphi_2}{\partial x}+\frac{\partial\Psi_2}{\partial y}\\
&V_1=\frac{\partial\varphi_1}{\partial y}-\frac{\partial\Psi_1}{\partial x}; \qquad V_2=\frac{\partial\varphi_2}{\partial y}-\frac{\partial\Psi_2}{\partial x};
\end{aligned} \tag{45}$$

The model of the simultaneous generalized stress state is used in the study of one-dimensional waves in bounded media such as rods of rectangular cross-section.

The theory of transversally isotropic prestressed porous two-component medium is formulated for an infinite layer of finite thickness that is in space (x, y, z), and the medium is bounded on the coordinate.

On the basis of the mathematical theory of oscillations of elastic or viscoelastic plate under nonstationary external actions, exact equations of longitudinal and transverse vibrations of viscoelastic plates are derived with and without allowance for initial displacements and stresses, approximate equations for the physical nonlinearity of the material. For such problems, expressions are obtained for all displacements and stresses along the thickness of the plate and basic boundary problems are formulated, leading to longitudinal or transverse vibrations of the plate. On the basis of exact equations some approximate equations that follow from them with some degree of accuracy are analyzed and approximate boundary value problems are formulated for them.

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А. Ж. Сейтмуратов[1], М. И. Рамазанов[2], Н. К. Медеубаев[2] , Б. К. Калиев[1]

[1]Қорқыт Ата атындағы Қызылорда мемлекеттік университеті, Қызылорда, Қазақстан,
[2]Е. Бөкетов атындағы Қарағанды мемлекеттік университеті, Қарағанды, Қазақстан

СТАЦИОНАРЛЫҚ ЕМЕС ІШКІ ӘСЕР КЕЗІНДЕГІ ҚАТТЫ НЕМЕСЕ СОЗЫЛМАЛЫ-ҚАТТЫ ПЛАСТИНКАЛАРДЫҢ МАТЕМАТИКАЛЫҚ ТЕРБЕЛІС ТЕОРИЯСЫ

Аннотация. Бұл жұмыста автор стационарлы емес сыртқы әсерлер кезінде иілгіш және тұтқыр иілгіш пластиналардың динамикалық қозғалысын анықтау дірілдерін математикалық теория тұрғысынан түсіндіруге талпынған. Осы көзқарасқа негізделген физикалық бейсызықтық материалдар теңдеулеріне жақындатылған, бастапқы ауыстыру мен кернеулерді ескерген және ескерусіз тұтқыр иілгіш пластиналардың бойлық және көлденең дірілдерінің нақты теңдеулері шығарылған. Пластиналардың қалыңдығы бойынша барлық кернеу мен ауыстырулар үшін есептер қарастырылған және де пластиналардың бойлық және көлденең дірілдеріне алып келетін негізгі шектік есептер құрастырылған. Нақты теңдеулер негізінде сол немесе олардан кейінгі дәлдік дәрежесі бар кейбір жуықтау теңдеулері талданған және олар үшін шекаралық есептер құрастырылған.

Түйін сөздер: упругий, вязкоупругий, нестационарный, колебания, нелинейный, деформируемый, продольный, поперечный.

А. Ж. Сейтмуратов[1], М. И. Рамазанов[2], Н. К. Медеубаев[2] , Б. К. Калиев[1]

[1]Кызылординский государственный университет им. Коркыт Ата, Кызылорда, Казахстан,
[2]Карагандинский государственный университет им. Букетова, Караганда, Казахстан

МАТЕМАТИЧЕСКАЯ ТЕОРИЯ КОЛЕБАНИЙ УПРУГИХ ИЛИ ВЯЗКОУПРУГИХ ПЛАСТИН ПРИ НЕСТАЦИОНАРНЫХ ВНЕШНИХ ВОЗДЕЙСТВИЯХ

Аннотация. В настоящей работе предпринята попытка изложения математической теории колебаний упругой или вязкоупругой пластинки для изучения динамического их поведения при нестационарных внешних воздействиях. На основе такого подхода выведены точные уравнения продольных и поперечных колебаний вязкоупругих пластин с учетом и без учета начальных смещений и напряжений, приближенные уравнения физической нелинейности материала. Для всех задач получены выражения для всех смещений и напряжений по толщине пластинки и сформулированы основные краевые задачи, приводящие к продольному или поперечному колебаниям пластинки. На основе точных уравнений проанализированы некоторые вытекающие из них приближенные уравнения с той или иной степенью точности и сформулированы для них приближенные краевые задачи.

Ключевые слова: упругий, вязкоупругий, нестационарный, колебания, нелинейный, деформируемый, продольный, поперечный.

Information about authors:

Seitmuratov Angisin – Doktor of Physical and Matematical Sciences, Professoz, Korkyt Ata. Kyzylorda State University. Kyzylorda

Ramazanov Murat – Doktor of Physical and Matematical Sciences, Professoz, Buketov. Karaganda State University.Karaganda.

Medeubaev Nurbolat – a senior teacher of department is "Algebra, mate.logic andgeometry", Buketov. Karaganda State University.Karaganda.

Kaliev Bakit – manager of department of "Physicist and mathematician", associate professor, Korkyt Ata. Kyzylorda State University. Kyzylorda