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SOLUTION OF THE PROBLEM OF LOWERING OF MATERIALS OF VISCOUS LAYER DOWN THE HILLSLOPE

Abstract. The article deals with the problem of a modeling investigation of the mechanism of the landslides origin of sedimentary rocks. It is assumed that a viscous layer of rock, located on the surface of a stable elevation, with a decrease in the coefficient of viscosity, material flows down the slope under the influence of gravity. To study this process, a mathematical model method was used, as a result of which a mathematical model of the given process was obtained and a mathematical problem on the solution of a quasilinear equation of parabolic type was formulated. To solve the mathematical problem, a finite-difference method was used; a nonlinear implicit calculation scheme was chosen on the basis of which an algorithm for solving the problem was formulated and a computer program was developed. A numerical experiment was performed for various possible variants; the results are presented in the form of graphs and tables.

Keywords: sedimentary rocks, rheological properties, creep, mechanism of landslides origin, mathematical model, solution algorithm, numerical experiment.

The setting of the problem. One of the causes of catastrophic phenomena occurring in foothill areas or on hillslopes of elevations is the lowering of ground materials down their slope. As a rule, with the preservation of certain conditions, a stable position of ground materials remains. However, under the influence of natural phenomena, for example, prolonged heavy rains that lead to a change in the viscosity properties of the materials composing the upper layers of the ground, creep motions may occur under the influence of gravity. Research in this direction is relevant since the study of the mechanism of origin of one of such phenomena frequently occurring on the Earth is considered important for the prevention of catastrophes associated with them [1,2].

It is known [3,4,5] that sedimentary rocks, which cover more than 75% of the surface of the terrestrial land, have the property of creep. "Creep is a phenomenon of gradual growth of strain in time with constant stress and a decrease in strength under long-term loading" [5, p. 36]. Therefore, creep is the cause of such phenomena as landslides, mudflows, glacier flow and others.

The proposed article is devoted to a model investigation of one version of the mechanism of landslides origin when the ground lowering occurs under the influence of its own weight with a change in their rheological properties. In this case, a physical model of "creeping" flows in the viscous layer is used [3,4,7,8], and for the study of the process under consideration - the mathematical model method [6].

Mathematical model and setting of the mathematical problem. Let us consider a certain viscous layer of a certain thickness (power) lying on the surface of a stable hill. It is assumed that at the initial time the viscous layer is in a stable position, i.e. there is no movement in it. Then, because of the decrease in the coefficient of viscosity of the layer, it moves down the hillslope under the influence of its own weight. It is required to compile a mathematical model of this problem and set its mathematical formulation.

To solve the problem, it is necessary to introduce the notations for the main parameters describing the process under consideration.

Let it be assumed that there is a rectangular coordinate system, in which x and y - horizontal coordinates, and z - vertical coordinate; z axis is directed upwards, backwards to the direction of the gravitational vector \vec{g} (Figure 1).

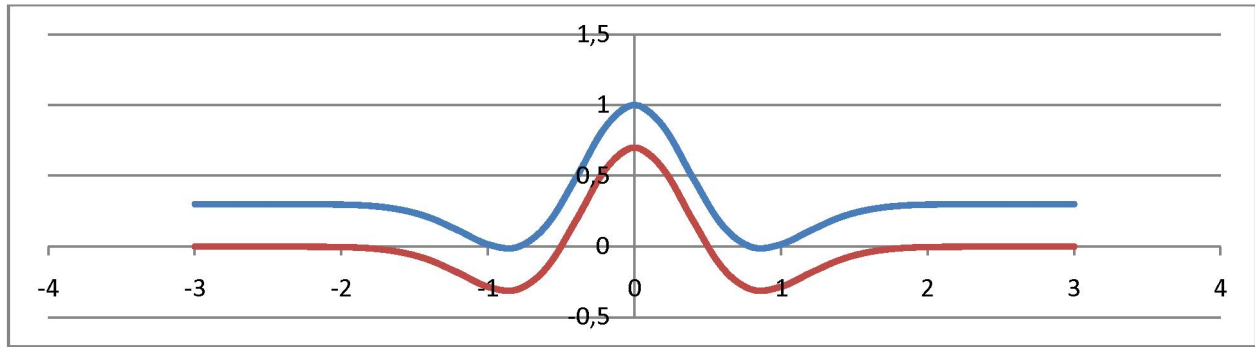


Figure 1 - Initial position of a viscous layer resting on a hill

It is assumed that the center (vertex) of the hill coincides with the origin of coordinates ($x = 0$) and z axis is a line of symmetry. The boundary between the moving part (the viscous layer) and the fixed part of the hill is determined by the function $z = \xi(x, y)$, and the free surface of the viscous layer - $z = u(x, y, t)$. Here t - time. The required function is $z = u(x, y, t)$.

It is assumed that the thickness of the viscous layer is little in comparison with the horizontal dimensions, which makes it possible to use a similar mathematical model of the problem considered in the work of the author. The transition to dimensionless parameters and the simplifying transformations associated with it allow to write down the corresponding mathematical dependences. In this case, the free surface of the viscous layer under consideration is described by the following differential equation in dimensionless variables [10,11]:

$$\frac{\partial u}{\partial t} = \frac{ER}{3} \cdot \left[\frac{\partial}{\partial x} (u - \xi)^3 \frac{\partial u}{\partial x} + \frac{\partial}{\partial y} (u - \xi) \frac{\partial u}{\partial y} \right]. \tag{1}$$

Here, $z = \xi(x, y)$ is a function that determines the boundary surface between the viscous layer and the underlying fixed base. Let it be given in the following form [10]:

$$\xi(x, y) = (1 - f) \cdot e^{-\frac{x^2 + y^2}{b}} \cdot \left[1 - \frac{2(x^2 + y^2)}{b} \right],$$

where f – initial thickness of the viscous layer. In the equation there is a single parameter $ER = \frac{\rho g H^3}{\eta U L}$, which is dimensionless, depending on the physical and geometric properties of the viscous layer under consideration; where ρ – the density of the material and η – the dynamic coefficient of the viscosity layer, g – acceleration of gravity, U, H, L – characteristic values: speed, vertical and horizontal dimensions respectively.

The received equation (1) is a quasilinear equation of parabolic type with regard to the function $u(x, y, t)$.

The solution of the equation (1) makes it possible to obtain a picture of the change in the free surface of the structure under consideration, to calculate the values of the moving velocities of the materials in the layer by the following formulas [11]:

$$\begin{aligned} u_x &= \frac{ER}{2} \cdot \frac{\partial z}{\partial x} \cdot [(z - u)^2 - (u - \xi)^2]; \\ u_y &= \frac{ER}{2} \cdot \frac{\partial z}{\partial y} \cdot [(z - u)^2 - (u - \xi)^2]. \end{aligned} \tag{2}$$

It follows from the setting of the problem that the initial condition (at $t = 0$) for solving equation (1) has the following form:

$$u(x, y, 0) = e^{-\frac{x^2 + y^2}{b}} \cdot \left[1 - \frac{2(x^2 + y^2)}{b} \right]. \tag{3}$$

This function (3) determines the initial position of the free surface of the viscous layer under consideration. It is obtained as a result of the transition to dimensionless parameters by means of the following substitution [10]:

$$b = \frac{B}{L}, \quad x = \frac{x}{L}, \quad y = \frac{y}{L}, \quad Z_0 = \frac{Z_0}{H}.$$

Since we consider a layer, the characteristic horizontal dimension (L) of which is large enough in comparison with its vertical dimensions (H), it can be assumed, that:

$$u \rightarrow 0 \text{ on condition } x \rightarrow \pm\infty, y \rightarrow \pm\infty. \quad (4)$$

This means that at points away from the center of the hill ($x = 0$), the free surface of the layer very closely coincides with the horizon ($z = 0$). For numerical calculation, we can consider the boundary conditions as follows:

$$x = \pm d, u(\pm d, y, t) = 0; \quad y = \pm d, u(x, \pm d, t) = 0. \quad (5)$$

In this case, the value d can be chosen several times more than the dimensionless value of the maximum hill height. This is done for the purpose of approximate replacement of the condition at infinity.

The setting of the mathematical problem. Thus, the resulting set of formulas (1) - (5) forms the mathematical model of the problem set here, which allows us to formulate the following mathematical problem: *it is required to solve the equation (1) for the initial condition (3) and the boundary conditions (5).*

On the method of solving the mathematical problem. Obviously, because of the presence of nonlinearity with respect to the desired function, the equation (1) cannot be solved by an analytical method, therefore, a finite-difference method is used here. According to academicians Tikhonov A.N. and Samarskiy A.A. [9, p. 593], "At present, the finite-difference method is the only method that allows to find effectively a solution of quasilinear equations".

In the quasilinear equation (1), the coefficient of the highest derivative of the desired function is a power function with respect to the same function. For the stability of the solution of this type of equations, it is expedient to use an implicit calculation scheme that is nonlinear with respect to the values of the required function [9].

The calculation scheme. In order to reduce the amount of computational work, we can confine ourselves to solving the two-dimensional problem. Selecting the steps h and τ by independent variables x and t respectively, we can get the following calculation scheme:

$$\frac{u_i^{j+1} - u_i^j}{\tau} = \frac{ER}{3 \cdot h} \cdot [(u_{i+1}^{j+1} - \xi_{i+1})^3 \cdot \frac{u_{i+1}^{j+1} - u_i^{j+1}}{h} - (u_i^{j+1} - \xi_i)^3 \cdot \frac{u_i^{j+1} - u_{i-1}^{j+1}}{h}], \quad (6)$$

$i = 1, 2, 3, \dots, n; \quad j = 1, 2, 3, \dots, m; \quad n$ – the number of points of division by x , and m – by t .

It is clear that this scheme is nonlinear with respect to the values of the required function u_i^{j+1} ; so to solve this system of algebraic equations, it is necessary to use the iteration method. To transform the equations (6) we introduce the following notations:

$u_i^{j+1} = u_i$ – value of the function on the new layer and the new iteration;

$u_i^{j+1} = w_i$ – value of the function on the new layer, for the previous iteration;

$u_i^j = v_i$ – the value of the function on the previous layer.

Taking these notations into account, and also after the simplest transformations from the formulas (6), we can obtain the following equation:

$$A_i u_{i-1} - (1 + A_i + A_{i+1}) \cdot u_i + A_{i+1} \cdot u_{i+1} = -v_i, \quad (7)$$

where

$$A_i = \frac{ER \cdot \tau}{3h^2} \cdot \left(\frac{w_i - \xi_i + w_{i-1} - \xi_{i-1}}{2} \right)^3, \quad i = 1, 2, 3, \dots, n-1 \quad (8)$$

Formula (7) is a system of algebraic equations, the main matrix of which has a special form - tridiagonality. To solve this system of equations, a sweep method is used for each iteration. The use of the iteration method allows to obtain a solution of the problem with a specified accuracy, and also the stability of the solution will be ensured.

Now conditions on the boundaries should be added to the system of equations (7); from the formulas (5), the following conditions can be obtained:

- the condition on the left border, at $x = 0$, where the maximum of the desired function is reached and the first derivative is zero, which implies $u_0 \approx u_1$;

- the condition on the right boundary, away from the center, at $x = 3$. It can be assumed that the value of the required function is zero; i.e. $u_n = 0$. Because of the symmetry for the left side of the domain $-3 \leq x \leq 3$ the results of the solution will be the same as for the right side of the domain. Therefore, we can confine ourselves to solving the problem for one, the right-hand side, of the domain.

Algorithm for solving the problem. For each iteration, the system of equations (7) is solved by the sweep method. As the zero approximation, the value of the desired function on the previous layer is used.

Within the iteration, the following operations will be performed:

1⁰. First, the values of the coefficients of the system of equations (7) should be determined by the formulas (8).

2⁰. In the direct run, unknown coefficients are determined using the following formulas:

$$\alpha_1 = 1; \quad \beta_1 = 0; \quad (9)$$

$$\alpha_{i+1} = \frac{A_{i+1}}{1 + A_i + A_{i+1}}; \quad \beta_{i+1} = \frac{v_i + A_i \cdot \beta_i}{1 + A_i + A_{i+1}}, \quad i = 1, 2, 3, \dots, n-1. \quad (10)$$

3⁰. Then, in the reverse run, the values of the desired function are determined using the following formulas:

$$u_n = 0; \quad u_i = \alpha_{i+1} \cdot u_{i+1} + \beta_{i+1}, \quad i = n-1, n-2, \dots, 1. \quad (11)$$

4⁰. The iterative process continues until the accuracy condition is satisfied; it is given by the following inequality:

$$\max\{|u[i] - w[i]|\} < \varepsilon, \quad (12)$$

where ε – a sufficiently small positive number, which must be given in advance. Inequality (12) determines the largest deviation between the values of the desired function for two iterations.

Numerical implementation of the algorithm. The development of a computer program for solving this problem is not very difficult. The computer program was developed [12,13], with the help of which a numerical experiment was carried out. The following specific data are included in the numerical experiment plan:

- for a dimensionless quantity ER the following four values were accepted: $ER = 0,01$; $ER = 0,1$; $ER = 1,0$; $ER = 10$;

- steps on independent variables: $h = 0,02$; $\tau = 0,0001$;

- the initial thickness of the layer is assumed to be constant and equal to $f = 0,3$;

- to determine the accuracy of calculations it is assumed $\varepsilon = 0,0001$;

- the calculations were carried out for the instants of time $0 \leq t \leq 10$;

- The interval over the horizontal variable was $-3 \leq x \leq 3$.

Results of the numerical solution of the problem. It should be noted that the solution of this problem depends only on one dimensionless parameter ER . Therefore, the numerical experiment was carried out for different values of this parameter. This parameter is determined by the physical and geometric characteristics of the viscous layer under consideration, therefore the initial numerical data are taken from the special literature on the physical and mechanical properties of rocks [3,4,5]. Elementary calculations have shown that for most sedimentary rocks, including clay ones, covering a significant part of the earth's surface, the order of the values of the dimensionless parameter ER may be within 0.01; 0.1; 1.0; 10. For these values of this parameter, calculations were made.

As a result of the numerical implementation of the algorithm for solving this problem, results are obtained, which are presented in the form of graphs and tables. There was determined the positions of the viscous layer for different instants of time in the range $0 \leq t \leq 10$ for different values of the dimensionless parameter ER . Because of the fact that at $ER = 0,01$ the change in the initial position of the viscous layer turned out to be insignificant, the graph for this case is not presented here. Figures 2 - 4 show the positions of the viscous layer at the time point $t = 10$ for parameter values $ER : 0,1; 1,0; 10$.

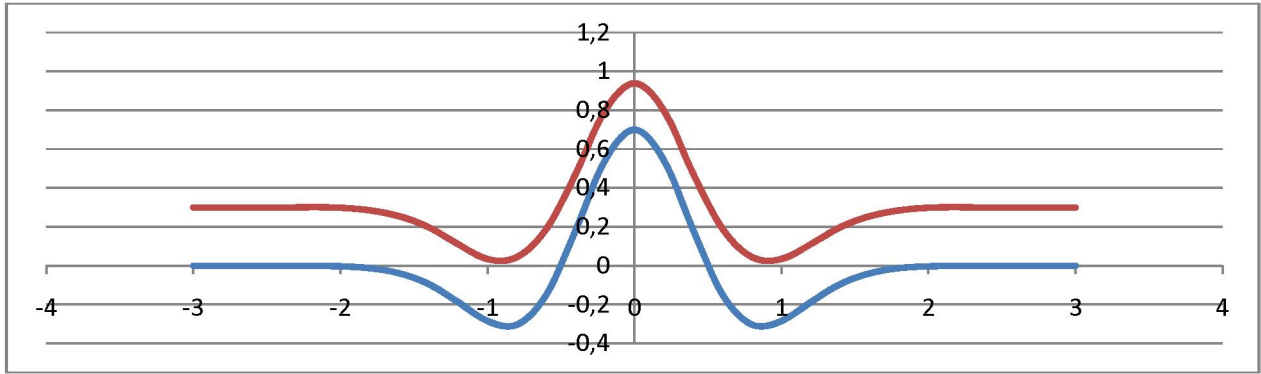


Figure 2 - The position of the viscous layer at $t = 10$ for $ER = 0,1$.

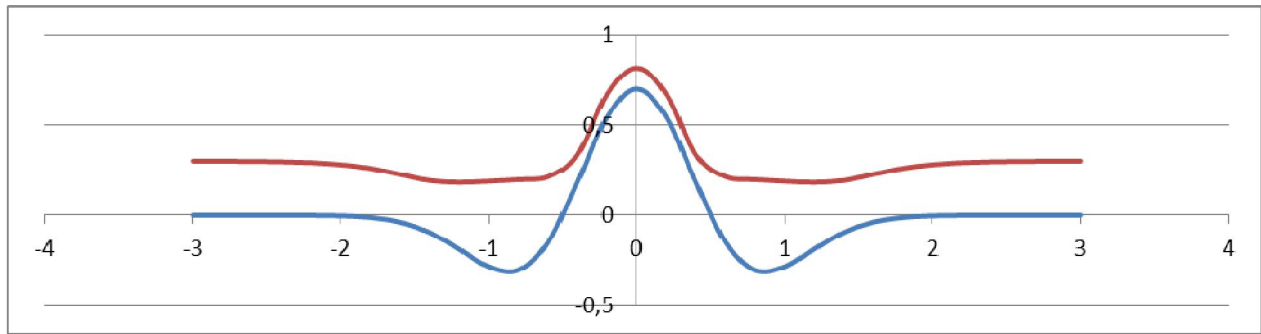


Figure 3 - The position of the viscous layer at $t = 10$ for $ER = 1$.

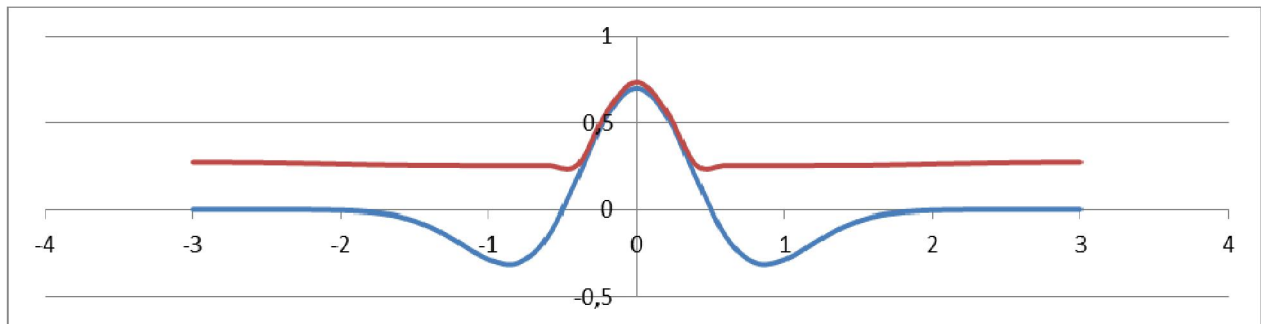


Figure 4 - The position of the viscous layer at $t = 10$ for $ER = 10$.

Conclusions. The main parameter affecting the process under consideration is the dynamic coefficient of viscosity of the layer. Therefore, when studying this process, it is necessary to take into account the change in the viscosity of the layer materials as the main factor. The dimensionless parameter depends inversely on the dynamic coefficient of viscosity of the layer under consideration. At low values of the coefficient of viscosity η the parameter value ER will be great, and conversely, when the viscosity coefficient is of great importance, this parameter will have a small value.

From an analysis of the obtained results, it follows that for a sufficiently large value of the dynamic coefficient of viscosity of the layer under consideration ($ER = 0,1$ and $ER = 0,01$) the change in the

initial state of the layer will be insignificant. In fact, the lowering of the maximum point (the vertex) of the outer surface of the layer during the period of time $t = 10$ is for the case when $ER = 0,1$, only by 6.15% (decrease from 1 to 0.9385), and for the case when $ER = 0,01$, only by 1.09% (the same, from 1 to 0.9891). For comparison, there can be given the data for $ER = 1$ and $ER = 10$. In the last two cases, the viscosity coefficient will have relatively small values. The lowering of the viscous layer materials will be significant at this; the descent of the top of the layer is: for the case at $ER = 1$ about 18%, and for $ER = 10$ - 26%. Figure 5 shows the graphs showing the changes in the vertex for three cases discussed above.

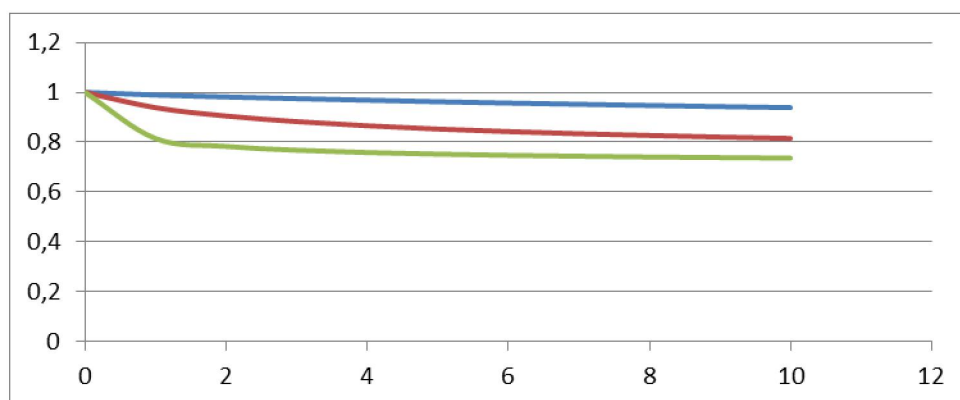


Figure 5 - Graphs for changing the maximum value of the function $u(0, t)$ in time for different values of the parameter ER :

- top line for $ER = 0,1$;
- middle line for $ER = 1$;
- lower line for $ER = 10$.

In addition, it should be noted that due to the lowering of the layer materials, the upper parts are thinning (Figure 4), and due to this process, the lower parts of the considered area thicken, where sedimentary rocks accumulate, the thickness of which reaches considerable sizes. For example, the thickening at the lowest level (on the sole) of the hill for the various variants was in the following values:

Table 1- Maximum increase in the layer thickness

ER	0.1	1.0	10
Layer thickness	0.3408	0.4972	0.5527
% of increasing	13.6	65.7	84.2

In conclusion, it should be noted that the results of the solution of this problem allow a theoretical (mathematical) description of the mechanism of the occurrence of landslides lying on elevated areas. An assessment of the changes that occur due to landslides with a decrease in the coefficient of viscosity of sedimentary rocks was made. The obtained results of the study make it possible to estimate the scale of catastrophic consequences due to the occurrence of landslides. For example, it can be seen from Table 1 that there is an increase in the thickness of the layer in the lower parts of the earth's surface by 65-84%, which occurs due to rocks sliding from the upper parts of the hill. This means that these places on the earth's surface are covered by sedimentary rocks of considerable volume. This situation can lead to undesirable consequences.

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ТҰТҚЫРЛЫ ҚАБАТТЫҢ МАТЕРИАЛДАРЫНЫҢ ҚЫРАТ БАУРАЙЫНА ТӨМЕН ТҮСУИ ТУРАЛЫ ЕСЕПТИ ШЕШУ

Аннотация. Мақалада шөгінді тау жыныстарының көшкінінің пайда болуының механизмін модельдік зерттеу туралы мәселе қарастырылған. Тұрақты қырат бетінде орналасқан тау жыныстарының тұтқырлы қабатының тұтқырлық коэффициенті төмендегенде өз салмағының әсерімен қырат баурайымен төмен қарай қозғалады деп ұйғарылған. Осындай процесті зерттеу үшін математикалық модельдеу әдісін қолдану нәтижесінде квазисызықтық парабола типіндегі теңдеуді шешу туралы математикалық есеп қойылған. Есепті шешу үшін шектелген-айырма әдісі қолданылған; сызықтық емес әрі айқын емес есептеу схемасы таңдалынып, есептің алгоритмі мен компьютерлік программасы құрастырылған. Өртүрлі варианттар үшін сандық эксперимент орындалды; есептеу нәтижелері графиктер мен кестелер түрінде берілген.

Түйін сөздер: шөгінді тау жыныстары, реологиялық қасиеттер, жылжу, көшкінің пайда болу механизмі, математикалық модель, есептеу алгоритмі, сандық эксперимент.

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РЕШЕНИЕ ЗАДАЧИ ОБ ОПУСКАНИИ МАТЕРИАЛОВ ВЯЗКОГО СЛОЯ ПО СКЛОНУ ВОЗВЫШЕННОСТИ

Аннотация. В статье рассматривается задача о модельном исследовании механизма возникновения оползней осадочных горных пород. Предполагается, что вязкий слой породы, находящийся на поверхности устойчивой возвышенности, при уменьшении коэффициента вязкости возникает движение материалов вниз по склону под действием силы тяжести. Для исследования данного процесса использован метод математического моделирования, в результате которого получена математическая модель данного процесса и сформулирована математическая задача о решении квазилинейного уравнения параболического типа. Для решения математической задачи использован конечно-разностный метод; была выбрана нелинейная неявная расчетная схема, на основе которой сформулирован алгоритм решения задачи и разработана компьютерная программа. Проведен численный эксперимент для различных возможных вариантов; результаты представлены в виде графиков и таблиц.

Ключевые слова: осадочные породы, реологические свойства, ползучесть, механизм возникновения оползней, математическая модель, алгоритм решения, численный эксперимент.

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