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PHONON-PHONON INTERACTION
IN THE CRYSTAL LATTICE OF NEUTRON STAR

Abstract. The aim of this paper is to calculate the oscillation frequency of the phonons in the crystal lattice of neutron stars. Also, we calculated density of states of phonon and density of local phonon of impurity atoms. The frequency of the atoms and impurities is compared. We have built graphics on the basis of the frequency depending on their densities.

Keywords: phonon, neutron star, impurity, crystal lattice, frequency.

Introduction

Compact stars have extremely high-density and astronomical objects which appear after evolution of normal stars. Consequently, compact body means that their excess hydrogen energy expended, and converted to another reactions [1]. The difference between normal and high-density stars are little size and density. The supernova stars are smaller than the normal stars [2].

A neutron star is a type of compact star. Neutron stars are the smallest and densest stars in the Universe. With a radius of only about 11-11.5 km (7 miles), they can, however, have a mass of about twice that of the Sun. They can result from the gravitational collapse of a massive star that produces a supernova. Neutron stars are composed almost entirely of neutrons, which are subatomic particles with no net electrical charge and with slightly larger mass than protons.

The outer layer of a neutron star has a complex structure which depends strongly on the nuclear density. In the inner crust of the star, due to the high density and pressure, a large fraction of neutrons occupies unbound states. Nuclei which are still present are therefore immersed in a neutron gas. The structure of this part of the star has been the subject of a considerable theoretical effort.
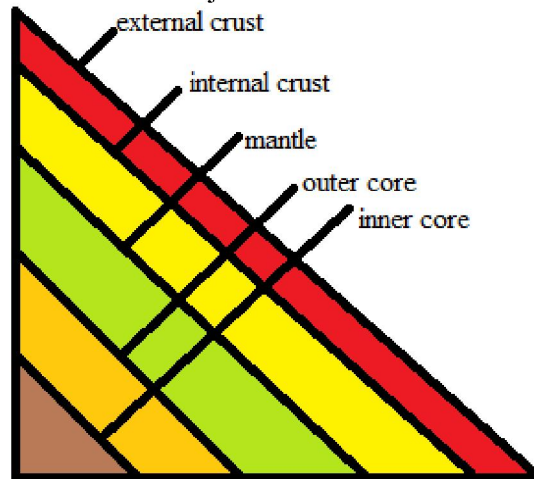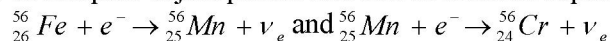

Figure 1 – Structure of neutron star

Here, in crystal lattice of compact object passes reaction of electron capture:

$$^{56}_{26}Fe + e^- \rightarrow\ ^{56}_{25}Mn + v_e \text{ and } ^{56}_{25}Mn + e^- \rightarrow\ ^{56}_{24}Cr + v_e$$

Electron capture is a process in which the proton-rich nucleus of neutral atom absorbs an inner atomic electron.

In physics, a phonon is a collective excitation in a periodic, elastic arrangement of atoms or molecules in condensed matter, likes olids and some liquids. Often designated a quasiparticle, it represents an excited state in the quantum mechanical quantization of the modes of vibrations of elastic structures of interacting particles.

Phonons play a major role in many of the physical properties of condensed matter, like thermal conductivity and electrical conductivity. The study of phonons is an important part of condensed matter physics.

Phonon is elementary particle which transfer oscillations in the crystal cell [3]. In this regard, the phonon participate in all processes of the crystal lattice (energy, transport pressure). Therefore, to calculate the frequency of fluctuations phonons and oscillations are very important [4].

The equations in this section do not use axioms of quantum mechanics but instead use relations for which there exists a direct correspondence in classical mechanics.

For example: a rigid regular, crystalline (not amorphous), lattice is composed of N particles. These particles may be atoms or molecules. N is a large number, say $\sim 10^{23}$, or on the order of Avogadro's number for a typical sample of a solid. Since the lattice is rigid, the atoms must be exerting forces on one another to keep each atom near its equilibrium position. These forces may be Van der Waals forces, covalent bonds, electrostatic attractions, and others, all of which are ultimately due to the electric force. Magnetic and gravitational forces are generally negligible. The forces between each pair of atoms may be characterized by a potential energy function V that depends on the distance of separation of the atoms. The potential energy of the entire lattice is the sum of all pair wise potential energies:

$$\sum_{i \neq j} V(r_i - r_j)$$

where r_i is the position of the ith atom, and V is the potential energy between two atoms.

Analysis of results

$$R = \left(\frac{3}{4\pi} \cdot M_A \right)^{1/3}$$

(1)

The potential energy of nucleus:

$$U(r) = -\frac{3 \cdot Z^2 \cdot e^2}{2 \cdot R} + \frac{Z^2 \cdot e^2}{2 \cdot R^3} \cdot r^2$$

(2)

Next, we consider the reaction of electron capture. In this case we have to calculate with back way, because with right way we can not solve analogical results:

$$\omega^2 = \frac{4C_1}{M} \sin^2 \frac{Ka}{2}$$

(3)

Here, K-vector of ossilation, $K = \pm \pi / a$; C-force constant. This constant we define:

$$C = \left(\frac{d^2 U}{dR^2} \right)_{R_0}$$

4)

In this case,

$$C = \left(\frac{d^2 U}{dr^2} \right)_{r=R} = \frac{Z^2 \cdot e^2}{R^3} \qquad (5)$$

On (3)- form we substitute (5) and K, we get:

$$\omega_{A,\max} = 2\sqrt{\frac{Z^2 e^2}{R^3 M_A}} \qquad (6)$$

If we compare this expression with Kirzhnis calculation:

$$\omega_{\max} = 2\sqrt{\frac{\beta_1}{m}} \qquad (7)$$

Here, $\beta_1 = \dfrac{Z^2 e^2}{R^3}$, $R = d/2$, d - distance of nucleus.

Next, we can do

$$C = \beta_1 = \frac{Z^2 e^2}{R^3} \qquad (8)$$

Now, we define the distance between two nucleus:

$$\rho = \frac{m}{V}, \quad V = \frac{4}{3}\pi R^3 \qquad (9)$$

$$d = a = 2R$$

$$\rho = \frac{M}{\dfrac{4\pi}{2}\left(\dfrac{a}{2}\right)^2} = 8 \cdot \frac{3M}{4\pi a^2}$$

$$d^3 = a^3 = \frac{6M}{\rho\pi}$$

$$R^3 = \frac{d^3}{8} = \frac{6M}{\rho\pi} \qquad (10)$$

This formula-connection between distance and density. Next, on (4.8) form we substitute (10) and (8):

$$\omega_{\max} = \frac{4Ze}{M}\sqrt{\frac{\pi\rho}{3}} \qquad (11)$$

$$\omega_{\max}\left(\frac{\beta_Z}{m}\right) = \omega_{\max}(\rho) \qquad (12)$$

(12) we get graph. Here, $\rho = (10^6 \pm 10^{14})g/sm^3$

Next, we write the expression for impurity atoms. For impurity atoms we denote coordinates as a $\xi_n = \xi_0$, force constant β_1.

For impurity atoms equation of motion:

$$M^* \frac{\partial \xi_0^2}{\partial t^2} = \beta_1(\xi_1 + \xi_{-1} - 2\xi_0)$$ (13)

But, for neighbour atoms:

$$m \frac{\partial \xi_1^2}{\partial t^2} = \beta_1(\xi_2 + \xi_0 - 2\xi_1)$$ (14)

Using the expression we can define ω_{max} for impurity atoms, when n = 0:

$$\xi_n = \xi(-1)^n \exp(i\omega_{max}^* t)\exp(-\alpha \mid n \mid a)$$ (15)

For local oscillation we consider 2 parameters ω_{max}^* and α:

(13)→(15)

$$(\omega_{max}^*)^2 = 2\frac{\beta_1}{M^*}[1 + \exp(-\alpha a)]$$ (16)

Next, (13)→ (14):

$$(\omega_{max}^*)^2 = \frac{\beta_1}{m}[2 + \exp(-\alpha a) + \exp(\alpha a)]$$ (17)

$$\omega_{max}^2 = 4\frac{\beta_1}{m}$$ (18)

Here, $\beta_1 = \frac{Z^2 e^2}{R^2}$

$$\rho = \frac{m}{V} \rightarrow \left| V = \frac{4}{3}\pi R^3 \right| = \frac{3m}{4\pi R^3}$$ (19)

Here, we define the R^3:

$$R^3 = \frac{3m}{4\pi\rho}$$ (20)

$$\beta_1 = \frac{Z^2 e^2}{3M} 4\pi\rho$$ (21)

(21)→(18)

$$(\omega_{max})^2 = \frac{4}{m}\frac{Z^2 e^2}{3M} 4\pi\rho = \frac{16}{3}\pi \frac{Z^2 e^2}{mM}\rho$$

In our case we consider the impurity atoms, that is why Z+1;

$$\beta^*_1 = \frac{(Z+1)^2 e^2}{3M} 4\pi\rho \qquad (22)$$

$$\omega^* = \sqrt{\frac{16}{3}\pi \frac{Z^2 e^2}{mM}\rho} = \frac{4}{M}(Z+1)e\sqrt{\frac{\pi\rho}{3}} \qquad (23)$$

For finding the ω_{max}, (23) expression we have to do transformation:

$$(\omega^*_{max})^2 = 2\frac{\beta_1}{M^*}[1+\exp(-\alpha a)] \rightarrow \left|\exp(-\alpha a) = x\right| \rightarrow \frac{1+x}{M}2\beta^*_1 \rightarrow 2(1+x)\frac{Z+1}{M} \qquad (24)$$

We equate the (23) and (24) and find the x:

$$x = \frac{\dfrac{ZM}{2} - \dfrac{M}{Z+1}}{\dfrac{M}{Z+1}} = 2\frac{Z+1}{Z} - 1 = 1 + 2\frac{1}{Z} \qquad (25)$$

(25)→ (24):

$$(\omega^*_{max})^2 = -4\frac{(Z+1)^2}{ZM} \qquad (26)$$

Using the expression (26) we find the maximum point of quazilocal oscillation.
Using the expression (27):

$$\omega^*_{max}\left(\frac{\beta_1}{m}\right) = \omega^*_{max}(\rho) \qquad (27)$$

draw a graph.
The red line is not impurity, the blue line is with impurity:
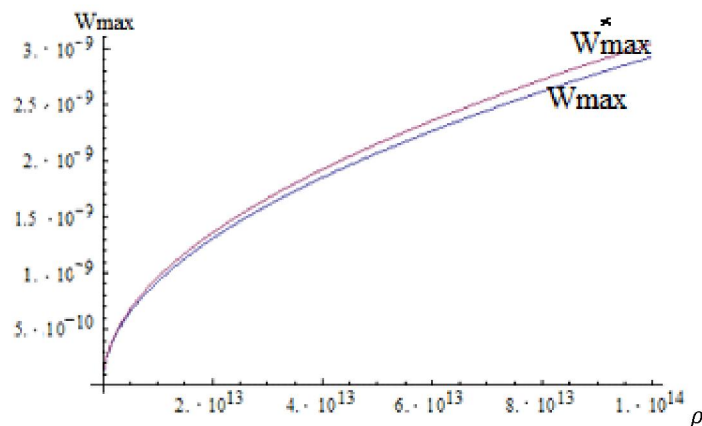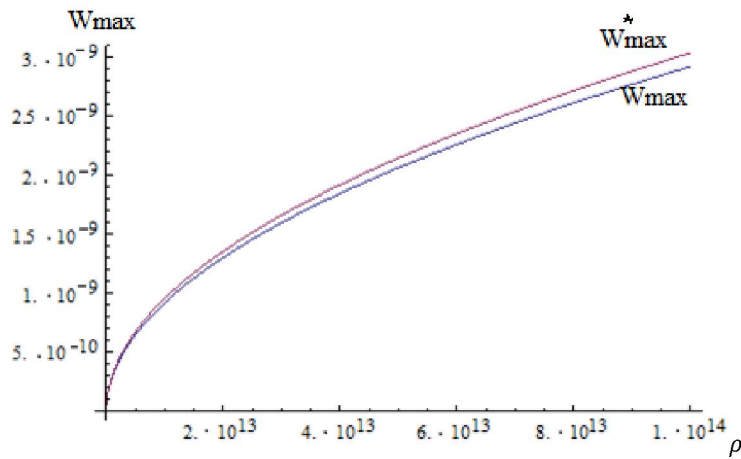

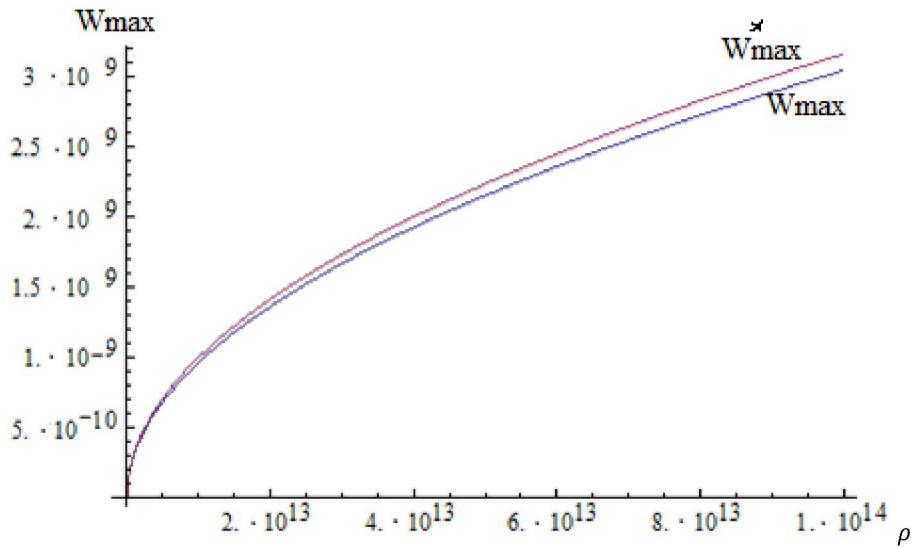
Figure 16 – For Ch

Figure 17 – For Mn

Figure 18 – For Fe

For finding the maximal frequency:

$$\frac{\omega_{max}^{*}}{\omega_{max}} = \frac{4\dfrac{(Z+1)^2}{ZM}}{\dfrac{4}{M}(Z+1)e\sqrt{\dfrac{\pi\rho}{2}}} = \frac{Z+1}{Ze}\sqrt{\frac{3}{\pi\rho}} \qquad (28)$$

$$\frac{\omega_{max}^{*}}{\omega_{max}} = \frac{4\dfrac{(Z+1)^2}{ZM}}{\dfrac{4}{M}Ze\sqrt{\dfrac{\pi\rho}{2}}} = \frac{(Z+1)^2}{Z^2e}\sqrt{\frac{3}{\pi\rho}} \qquad (29)$$

Using the (28) and (29) we draw the graph. These graphs show us the frequency, when oscillation of phonons were maximum:
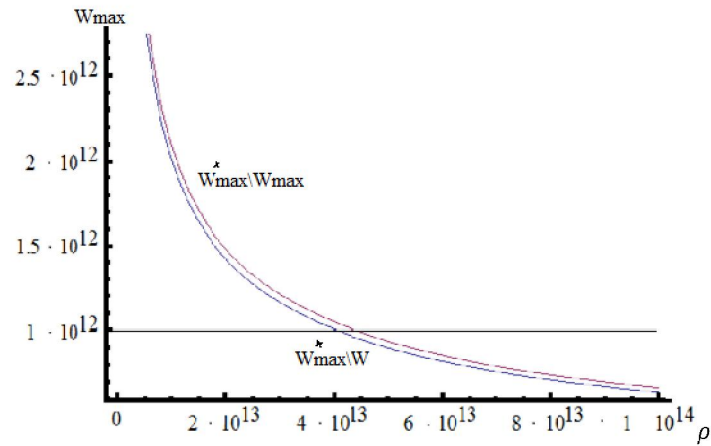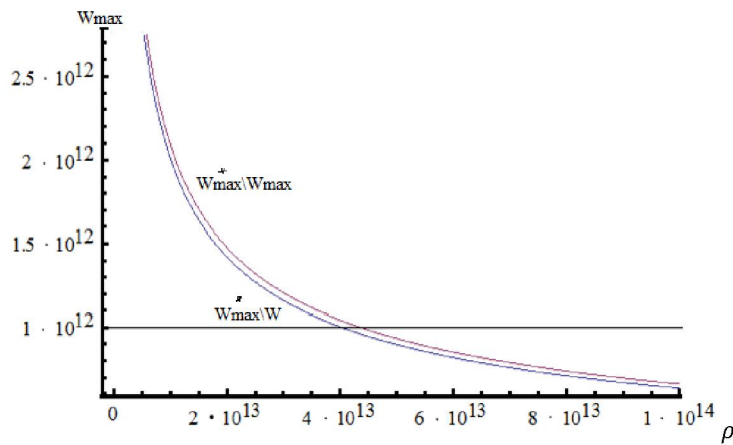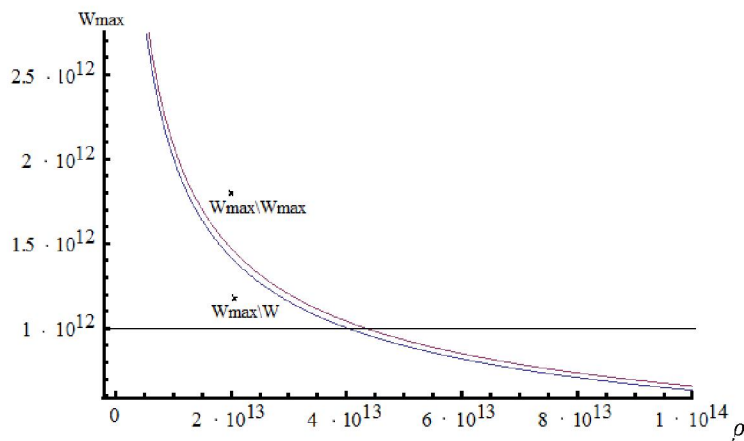
Figure 19– For Ch

Figure 20 – Mn

Figure 21 – Fe

Conclusion

This research work is calculated by frequencies and efficient way. That is, if we solve calculations directly, calculation will be difficult. In this case, we were chosen as an effective way to back. The results of the calculations shown and compared in figures as phonons frequencies.

During the calculations of the research work, the results were as follows: the density of compact objects is $\rho = (10^6 \pm 10^{14})$g/$sm^3$, so we choose on the extent of this frequency. Because neutron stars

have a greater density ρ, that is why phonon effects are very important phenomena. Influence of phonons spread in several directions:

1. Phonons description changes in the crystal cell of different depths (layers). In this case, the frequency changes slowly;

2. $\rho = \rho(z)$ dependence: A(Z+1, N)+e$\rightarrow$ A(Z, N+1)+ν_e

Here, value of the frequencies to be changed rather than the from the first position. In other words, ρ is the critical value of the crystalline lattice and quickly varies depending on the depth.

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НЕЙТРОНДЫ ЖҰЛДЫЗДАРДЫҢ КРИСТАЛДЫҚ ТОРЛАРЫНДАҒЫ ФОНОН-ФОНОНДЫ ӘСЕРЛЕСУЛЕР

Аннотация: Бұл жұмыстың негізгі мақсаты, нейтронды жұлдыздардың кристалдық торларындағы фонондардың тербеліс жиіліктерін есептеу болып табылады. Сонымен қатар, фонон күйлерінің тығыздықтары мен қоспа атомдардың локальды фонондарының тығыздықтарын есептедік, таза күйдегі атом мен қоспа атомдарының жиіліктерін салыстырдық. Жиіліктің тығыздыққа байланысы негізінде график тұрғыздық.

Тірек сөздер: фонон, нейтронды жұлдыз, қоспалар, кристалдық торлар, жиілік және т.б.

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ФОНОН- ФОНОННОЕ ВЗАИМОДЕЙСТВИЕ В КРИСТАЛИЧЕСКИХ РЕШЕТКАХ НЕЙТРОННЫХ ЗВЕЗД

Аннотация: Основа этой работы является, расчет частот колебании фононов в кристаллической решетке нейтронных звезд. Сделан расчет по плотностям фононных состоянии и плотности атомов локальных фононов, сравнивали частоты атомов в чистом виде и в примесях. На основе зависимости частот от их плотности мы построили гравик.

Ключевые слова: фонон, нейтронные звезды, примеси, кристаллическая решетка, частота и т.д.