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Research motion of mechanism of variable structure

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Key words mechanism, variable, structure, dynamic, generalized functions

Abstract We research the dynamics of the reduced link of mechanism of variable structure. Dynamics mechanism of variable structure is described by the differential equations with discontinuous coefficients. For the solution of differential equations of motion mechanism of variable structure with finite discontinuous coefficients is proposed using the generalized functions and the method of variable time scale.

Исследование динамики звена приведения механизма переменной структуры

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Ключевые слова: механизм, переменный, структура, обобщенные функции

В работе исследуется динамика звена приведения механизма переменной структуры. Динамика механизма переменной структуры описывается дифференциальным уравнением с разрывными коэффициентами. Для решения дифференциальных уравнений движения механизма переменной структуры с конечно-разрывными коэффициентами, предлагается использовать обобщенные функции и метод переменного масштаба времени.

Introduction. During the movement mechanisms of variable structure the number of moving links, degrees of freedom, types and classes of mechanism are changed. Mechanisms of variable structure are used to execute of the complex technology processes. The appearance of additional impulses at the time of changing the structure of mechanism of is a negative side of mechanisms of variable structure [1-5]. Dynamics mechanism of variable structure is described by differential equations with discontinuous coefficients, in particular, the reduced moment of inertia is a piecewise continuous and a positive definite function of the position. Research of the structure, kinematics and dynamics of the cam-lever mechanisms of variable structure which are used in the mining and textile industries is given in [1-5]. In these mechanisms due to the elastic links and connections and stopping some links, the structure is changed.

Definition of the law of motion of mechanism of variable structure is connected with the solution of the differential equations of motion with the finite discontinuous coefficients. Basically in mechanisms of variable structure the right-hand side of the equation is a continuous function of the position and time.

The differential equations of motion of mechanisms of variable structure.

In linkage mechanism of variable structure (see figure 1) as a result instantly imposes additional connection as the fixed stop, there is a break prestressed level 2 (see figure 1). The transfer functions of mechanisms of variable structure are interrupted (See figure 2). If the input link 1 has a moment of inertia, and the links 2 and 3 (see figure 1) are without inertia after contact with the catch in the mechanism is a "soft" impulse. In the event that some of the moving links of mechanism have a mass, except of the input link 1, then in during the restructuring occurs the "hard" impulse.

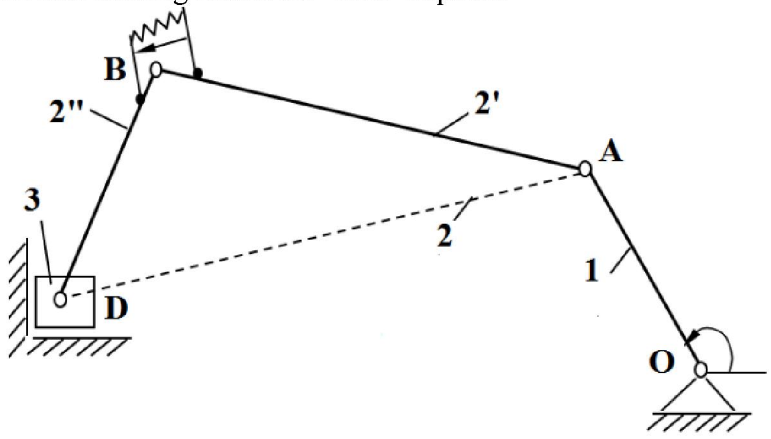

Figure 1 - Scheme of linkage mechanism of variable structure

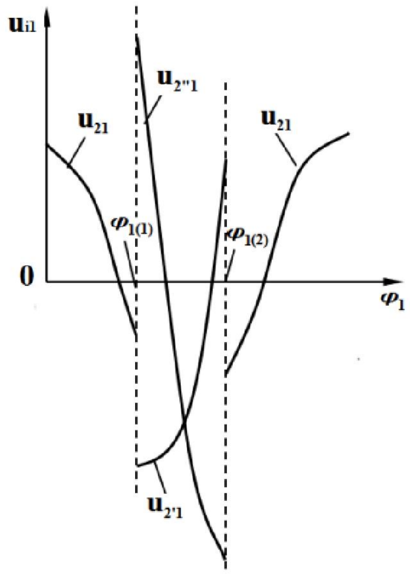

Figure 2 - The transfer functions of linkage mechanism of variable structure

We define the law of motion of the reduced link 1 of mechanism of variable structure (see figure 1) in the locality of the point of discontinuity of the inertial parameters. The movement of the reduced link of mechanism of variable structure with one degree of freedom is described by the equation

$$J_n(\varphi)\ddot{\varphi}(t) + 0{,}5J'_n(\varphi)\dot{\varphi}^2(t) = M_n(\varphi), \qquad (1)$$

here $J_n(\varphi)$, $J'_n(\varphi)$ - the reduced moment of inertia and its derivative in the generalized coordinate φ, and $M_n(\varphi)$ - the reduced moment of driving forces and the resistance forces.

Let us assume that in position $\varphi = \varphi_k$ the mechanism changes the structure, i.e. $\varphi_к$ - is the angle of discontinuity of function $J_n(\varphi)$. The derivative $J'_n(\varphi)$ is understood in the generalized sense [6]. Therefore, the generalized function corresponding to the function $J'_n(\varphi)$ has the form

$$J'_n(\varphi) = \Delta J\delta(\varphi - \varphi_k) \ , \qquad (2)$$

where ΔJ the final discontinuity of the reduced moment of inertia, $\delta(\varphi - \varphi_k)$ - the Dirac function. Now the equation (1) can be written as:

$$\ddot{\varphi}(t) + \frac{\Delta J\delta(\varphi - \varphi_k)}{2J_n(\varphi)}\dot{\varphi}^2(t) + \frac{M_n(\varphi)}{J_n(\varphi)} = 0, \qquad (3)$$

In this paper, the method of variable time scale [7, 8] for the classical second order differential equation is applied. Let us introduce the replacement

$$y(\varphi) = U(z) \ , \qquad (4)$$

where $z = \psi(t)$

$$\dot{\psi} = e^{-\int\limits_0^{\varphi}\frac{\Delta J\delta(\sigma-\varphi_k)}{2J_n(\sigma)}d\sigma} y'(\varphi) \ . \qquad (5)$$

Then the non-linear equation (3) is converted into the linear

$$U''(z) + U(z) = 0 \ . \qquad (6)$$

Suppose that $U(z)$ and $\psi(z)$ twice continuously differentiable functions, then we have

$$U''(z) = \frac{1}{\dot{\psi}^2}\left\{ y''(\varphi)\dot{\varphi}^2 + y'(\varphi)\left[\ddot{\varphi} - \frac{\dot{\varphi}\,\ddot{\psi}}{\dot{\psi}}\right]\right\}, \qquad (7)$$

inserting (7) into equation (6) we have

$$\ddot{\varphi} + \left[\dot{\varphi}\frac{y''(\varphi)}{y'(\varphi)} - \frac{\ddot{\psi}(t)}{\dot{\psi}(t)}\right]\dot{\varphi} + \frac{y(\varphi)\dot{\psi}^2(t)}{y'(\varphi)} = 0 \ . \qquad (8)$$

Comparing equation (8) and (3) we obtain

$$\frac{dy'}{y'} - \frac{d\dot{\psi}}{\dot{\psi}} = \Delta J\frac{\delta(\varphi - \varphi_k)}{J_n(\varphi)}d\varphi, \qquad (9)$$

$$\frac{y(\varphi)\cdot\dot{\psi}^2(t)}{y'(\varphi)} = \frac{M_n(\varphi)}{J_n(\varphi)} \ . \qquad (10)$$

Integrating (9) and assuming that the constant of integrated is zero, we obtain

$$y'(\varphi) = \dot{\psi}\, e^{\frac{\Delta J\theta(\varphi-\varphi_k)}{J_n(\varphi_k+0)}} \ , \qquad (11)$$

where θ - the Heaviside function

$$\theta(\varphi - \varphi_k) = \begin{cases} 1 \text{ при } \varphi \geq \varphi_K \\ 0 \text{ при } \varphi \angle \varphi_K \end{cases},$$

Inserting

$$\dot{\psi}(t) = y'(\phi)\, e^{-\frac{\Delta J\theta(\phi-\phi_k)}{J_n(\phi_k+0)}} \ , \qquad (12)$$

in equation (10), we have

$$y(\varphi)\,y'(\varphi) = \frac{M_n(\varphi)}{J_n(\varphi)} e^{2\frac{\Delta J\theta(\varphi-\varphi_k)}{J_n(\varphi_k+0)}} \ . \qquad (13)$$

Integrating equation (12) and assuming that $y(o) = 0$, we obtain

$$y(\varphi) = \left[2\int_0^{\varphi} \frac{M_n(\sigma)}{J_n(\sigma)} e^{2\frac{\Delta J\theta\ (\varphi-\varphi_k)}{J_n\ (\varphi_k+0)}} d\sigma \right]^{\frac{1}{2}} . \qquad (14)$$

It is known that the solution of equation (6) has the form

$$y(\varphi) = c_1 \cos\psi\ (t) + c_2 \sin\psi\ (t) . \qquad (15)$$

We show that (15) satisfies the equation (3). Indeed, from (15) we obtain

$$y'(\varphi)\dot{\varphi}\ (t) = \left[-c_1 \sin\psi\ (t) + c_2 \cos\psi\ (t)\right] \dot{\psi}\ (t) \qquad (16)$$

Taking into account (12) we have

$$\dot{\varphi}\ e^{\frac{\Delta J\theta\ (\varphi-\varphi_k)}{J_n(\varphi_k+0)}} = c_2 \cos\psi(t) - c_1 \sin\psi(t) . \qquad (17)$$

We differentiate (17) to t and we obtain

$$\left\{ \ddot{\varphi} + \left[\frac{\Delta J\theta(\varphi - \varphi_k)}{J_n(\varphi_k + 0)} \right]' \dot{\varphi}^2 \right\} e^{\frac{\Delta J\theta\ (\varphi-\varphi_k)}{J_n(\varphi_k+0)}} = -y(\varphi)\dot{\psi}(t) =$$

$$= -y(\varphi)y'(\varphi)e^{-\frac{\Delta J\theta\ (\varphi-\varphi_k)}{J_n(\varphi_k+0)}}$$

or

$$\ddot{\varphi} + \left[\frac{\Delta J\theta\ \ (\varphi - \varphi_k)}{J_n(\varphi_k + 0)} \right]' \dot{\varphi}^2 = -yy'e^{\frac{-2\Delta J\theta\ (\varphi-\varphi_k)}{J_n(\varphi_k+0)}} = -\frac{M_n(\varphi)}{J_n(\varphi)} .$$

By the property of the generalized function [6] we have

$$\left[\frac{\Delta J\theta\ \ (\varphi-\varphi_k)}{J_n\ (\varphi)} \right]' = \frac{\Delta J\delta\ \ (\varphi-\varphi_k)}{J_n\ (\varphi)} ,$$

hence, we have the equation (3)

$$\dot{\varphi}(t) = e^{-\frac{\Delta J\theta\ (\varphi-\varphi_k)}{J_n\ (\varphi_k+0)}} \left[c_2 \cos\psi(t) - c_1 \sin\psi(t)\right], \qquad (18)$$

here $\psi(t)$ is determined from equations (5) and $y(\varphi)$ from (14). Arbitrary constants determined from the initial conditions.

Thus, in a position when the mechanism changes its structure, i.e. in the locality point of discontinuity of the inertial characteristics, the angular velocity of link driving can be calculated by the expression.

$$\dot{\varphi}(t) = 2\int_0^t \left[\dot{\varphi}(t_0)\cos\psi(t)\text{-}y(\varphi_0)e^{\int_0^{\varphi} \frac{\Delta J\delta(\varphi-\varphi_k)}{J_n(\varphi)} d\varphi} \sin\psi(t) \right] dt ,$$

where

$$\psi = 2\ \mathrm{arctg}\ e^{-\dot{\varphi}(t_0)\frac{\Delta J\delta(\varphi-\varphi_k)}{J_n(\varphi)}t} ,$$

$$y(\varphi)=\left[2\int_0^{\varphi}\frac{M_n(\varphi)}{J_n(\varphi)}e^{\frac{\Delta J_n\theta(\varphi-\varphi_k)}{J_n(\varphi_k+0)}}d\varphi\right]^{\frac{1}{2}}.$$

Conchusion Research of the dynamics of mechanism of variable structure which is described by the differential equations with discontinuous coefficients, can be carried out using the method of generalized functions and the variable time scale for the classical second order differential equation.

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Айнымалы құрылымды механизмнің келтіру звеносының динамикасын зерттеу

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Кілт сөздер: механизм, айнамалы, құрылым, кеңейтілген функциялар

Жұмыста айнымалы құрылымды механизмнің келтіру звеносының динамикасы зерттелген. Айнымалы құрылымды механизмнің динамикасы үзілісті еселіктері бар дифференциалдық теңдеулермен сипатталған. Айнымалы құрылымды механизмнің қозғалысының үзімді-үзілісті еселіктері бар дифференциалдық теңдеулерін шешу үшін кеңейтілген функциялар мен уақыттың айнымалы масштабы әдісін қолдану ұсынылған.

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