REPORTS OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
ISSN 2224-5227
Volume 3, Number 307 (2016), 10 – 16

ИДЕНТИФИКАЦИЯ СИСТЕМ С ПОМОЩЬЮ МОДИФИЦИРОВАННОГО РАЗРЕЖЕННОГО LMS АЛГОРИТМА С УТЕЧКОЙ

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Ключевые слова: адаптивные алгоритмы, идентификация систем, разреженные системы.

Аннотация. В данной статье предлагается новый LMS алгоритм с утечкой (LLMS), который улучшает алгоритм ZA-LLMS (Zero-Attracting Leaky-LMS) используемый для идентификации разреженной системы. Предложенный алгоритм использует разреженность системы с преимуществами переменности размера шага и штрафа l_0-нормы. Мы сравнили производительность предложенного алгоритма с LLMS и ZA-LLMS с точки зрения скорости сходимости и среднеквадратичного отклонения (MSD). Эксперименты проводились в среде MATLAB. Моделирование показало, что предложенный алгоритм имеет превосходство над другими алгоритмами для обоих типов входных сигналов: аддитивного белого Гауссовского шума (AWGN) и аддитивного коррелированного Гауссовского шума (ACGN).

A MODIFIED SPARSE LEAKY-LMS ALGORITHM FOR SYSTEM IDENTIFICATION

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Keywords. Adaptive algorithms, system identification, sparse systems.

Abstract. In this paper, we propose a new Leaky-LMS (LLMS) algorithm that improves the Zero-Attracting Leaky-LMS (ZA-LLMS) for sparse system identification. The proposed algorithm exploits the sparsity of the system with the advantages of the variable step-size and l_0-norm penalty. We compared the performance of our proposed algorithm with the LLMS and ZA-LLMS in terms of the convergence rate and mean-square-deviation (MSD). Experiments were performed in MATLAB. Simulations showed that the proposed algorithm has superiority over the other algorithms for both types of input signals of additive white Gaussian noise (AWGN) and additive correlated Gaussian noise (ACGN).

I. INTRODUCTION

The least-mean-square (LMS) algorithm is a well-known algorithm and has been successfully used for system identification model (see Fig. 1) in adaptive filtering technology [1]. Many researchers studied to improve the performance of the conventional LMS algorithm for different environments. Thus, many different LMS-type algorithms were proposed.

Leaky-LMS-type algorithms were proposed [2,3] to overcome the issues when the input signal is highly correlated, by using shrinkage in its update equation. Another LMS based algorithm VSSLMS uses a variable step-size in update equation of the standard LMS to increase the convergence speed at the beginning stages of the iterations and decrease MSD at later iterations [4,5]. In order to improve the performance of the LMS algorithm when the system is sparse (most of the system coefficients are zero), ZA-LMS algorithm was proposed in [6].

In [7], the author proposed ZA-LLMS algorithm which combines the LLMS algorithm and ZA-LMS algorithm for sparse system identification. A better performance was obtained for AWGN and ACGN input signals. In [8], a high performance algorithm called zero-attracting function-controlled variable step-size LMS (ZAFC-VSSLMS) was proposed by using the advantages of variable step-size and l_0-norm penalty. We were motivated by the inspiration of the combination of these two algorithms. So in this paper, we proposed a new algorithm that combines the ZA-LLMS and ZAFC-VSSLMS algorithms. In the next section, a brief review of the LLMS and ZA-LLMS algorithms is provided. We derived the proposed algorithm in Section III. In Section IV, the simulations are presented and the performances of the algorithms are compared. Conclusions are drawn in the last section.

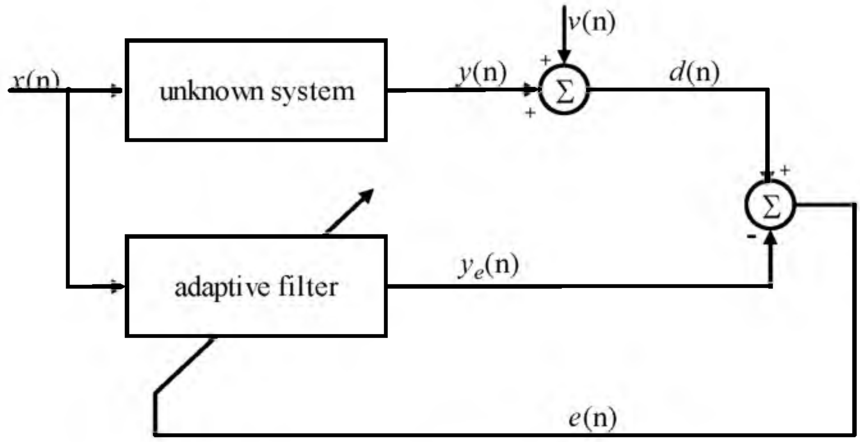

Fig. 1 – Block diagram of the system identification process.

II. REVIEW OF THE RELATED ALGORITHMS

a) Leaky-LMS (LLMS) Algorithm

In a system identification process, the desired signal is defined as,

$$d(n) = \mathbf{h}^T \mathbf{x}(n) + v(n) \tag{1}$$

where $\mathbf{h} = [\mathrm{h}_0, ..., h_{N-1}]^T$ is the unknown system coefficients with length *N*, $\mathbf{x}(n) = [x_0, ..., x_{N-1}]^T$ is the input-tap vector and $v(n)$ is the additive noise. In addition to being independent of the noise sample $v(n)$ with zero mean and variance of σ_υ^2, the input data sequence $\mathbf{x}(n)$ and the additive noise sample $v(n)$ are also assumed to be independent.

The cost function of the LLMS algorithm is given by,

$$J_1(n) = \frac{1}{2} e^2(n) + \gamma \mathbf{w}^T(n)\mathbf{w}(n) \tag{2}$$

where $\mathbf{w}(n)$ is the filter-tap vector at time n, γ is a positive constant called ‘leakage factor’ and $e(n)$ is the instantaneous error and given by,

$$e(n) = d(n) - \mathbf{w}^T(n)\mathbf{x}(n) \tag{3}$$

The update equation of the LLMS algorithm can be derived by using the gradient method as,

$$\begin{aligned}\mathbf{w}(n+1) &= \mathbf{w}(n) + \mu \frac{\partial J(n)}{\partial \mathbf{w}(n)} \\ &= (1-\mu\gamma)\mathbf{w}(n) + \mu\,\mathrm{e(n)}\end{aligned} \tag{4}$$

where μ is the step-size parameter of the algorithm.

b) Zero-Attracting Leaky-LMS (ZA-LLMS) Algorithm

The cost function of the LLMS algorithm was modified by adding the log-sum penalty of the filter-tap vector as given below:

$$J_2(n)) = \frac{1}{2}e^2(n) + \gamma\mathbf{w}^T(n)\mathbf{w}(n) + \gamma^{'}\sum_{i=1}^{N}(1+\frac{|w_i|}{\xi^{'}}) \tag{5}$$

where $\gamma^{'}$ and $\xi^{'}$ are positive parameters. Taking the gradient of the cost function and subtracting from the previous filter-tap vector iteratively, then the update equation was derived as follows [7]:

$$\mathbf{w}(n+1) = (1-\mu\gamma)\mathbf{w}(n) + \mu\,\mathrm{e(n)}\mathbf{x}(n) - \rho\frac{\mathrm{sgn}[\mathbf{w}(n)]}{1+\xi|\mathbf{w}(n)|} \tag{6}$$

where $\rho = \frac{\mu\gamma'}{\xi'}$ is the zero-attracting parameter, $\xi = \frac{1}{\xi'}$ and sgn(.) operation is defined as,

$$\mathrm{sgn}(x) = \begin{cases}\frac{x}{|x|} & if \quad x \neq 0 \\ 0 & if \quad x = 0\end{cases} \tag{7}$$

III. THE PROPOSED ALGORITHM

An improved sparse LMS-type algorithm was proposed in [8] by exploiting the advantages of variable step-size and recently proposed [9] l_0-norm which gives an approximate value of $\|\cdot\|_0$. We modify the cost function of that algorithm by adding the weight vector norm penalty as,

$$J_3(n)) = \frac{1}{2}e^2(n) + \gamma\mathbf{w}^T(n)\mathbf{w}(n) + \varepsilon\|\mathbf{w}(n)\|_0 \tag{8}$$

where ε is a small positive constant and $\|\mathbf{w}(n)\|_0$ denotes the l_0-norm of the weight vector given as,

$$\|\mathbf{w}(n)\|_0 \,\square\, \sum_{k=0}^{N-1}(1-e^{-\lambda|\mathbf{w}(n)|}) \tag{9}$$

where λ is a positive parameter. Deriving (8) with respect to $\mathbf{w}(n)$ and substituting in the update equation we get,

$$\mathbf{w}(n+1) = (1-\mu(n)\gamma)\mathbf{w}(n) + \mu(n)e(n)\mathbf{x}(n) - \rho(n)\,\mathrm{sgn}[\mathbf{w}(n)]e^{-\lambda|\mathbf{w}(n)|} \tag{10}$$

where $\rho(n) = \mu(n)\varepsilon\lambda$. It is seen that, the update equation of the ZA-LLMS algorithm has been modified by changing the constant step-size μ with $\mu(n)$ given in [8] and the zero-attractor $\rho\dfrac{\mathrm{sgn}[\mathbf{w}(n)]}{1+\xi|\mathbf{w}(n)|}$ with $\rho(n)\,\mathrm{sgn}[\mathbf{w}(n)]e^{-\lambda|\mathbf{w}(n)|}$.

IV. SIMULATION RESULTS

In this section, we compare the performance of the proposed algorithm with LLMS and ZA-LLMS algorithms in high-sparse and low-sparse system identification settings. Two different experiments are performed for each of AWGN and ACGN input signals. To increase the reliability of the expected ensemble average, experiments were repeated by 200 independent Monte-Carlo runs. The constant parameters are found by extensive tests of simulations to obtain the optimal performance as follows: For LLMS: μ=0.002 and γ=0.001. For ZA-LLMS: μ=0.002, γ=0.001, ρ=0.0005 and ξ=30. For the proposed algorithm: ρ=0.0005 and λ=8.

In the first experiment, all algorithms are compared for 90% high-sparsity and 50% low-sparsity of the system with 20 coefficients having in the first part, two '1' and 18 '0'; in the second part, ten '1' and ten '0' for 5000 iterations. Signal-to-noise ratio (SNR) is kept at 10 dB by regulating the variances of the input signal and the additive noise. The performances of the of the algorithm are compared in terms of convergence speed and $MSD = E\left\{\|\mathbf{h}-\mathbf{w}(n)\|^2\right\}$. Fig. 2 and Fig. 3 give the MSD vs. iteration number of the three algorithms for 90% sparsity and 50% sparsity levels respectively. They show that, the proposed algorithm has a fairly fast convergence with lower MSD than that of the other algorithms.

In the second experiment, all conditions are kept as same as in the previous experiment except the input signal type. A correlated signal is created by the AR(1) process as $x(n) = 0.4x(n-1) + v_0(n)$ and the normalized. Fig. 4 and Fig. 5 show that, the proposed algorithm has again a faster convergence and lower MSD than the other algorithms for 90% sparsity and 50% sparsity levels respectively.

IV. CONCLUSIONS

In this work, we proposed a modified leaky-LMS algorithm for sparse system identification. It was derived by combining the ZA-LLMS and ZAFC-LMS algorithms. The performance of the proposed algorithm was compared with LLMS and ZA-LLMS algorithms for 90% and 50% sparsity levels of the system with AWGN and ACGN input signals in two different experiments performed in MATLAB. Simulations showed that the proposed algorithm has a very high performance with a quite faster convergence and lower MSD than that of the other algorithms. As a future work, it is recommended that the proposed algorithm can be modified for transform domain or be tested for non-stationary systems.

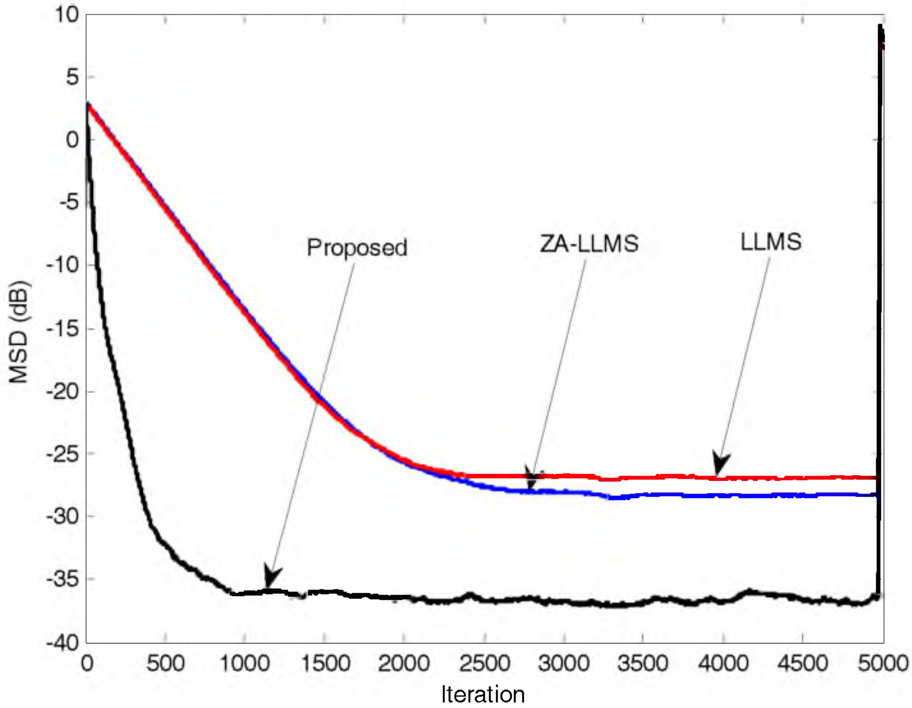

Fig. 2 – Steady state behavior of the LLMS, ZA-LLMS and the proposed algorithm for 90% sparsity with AWGN.

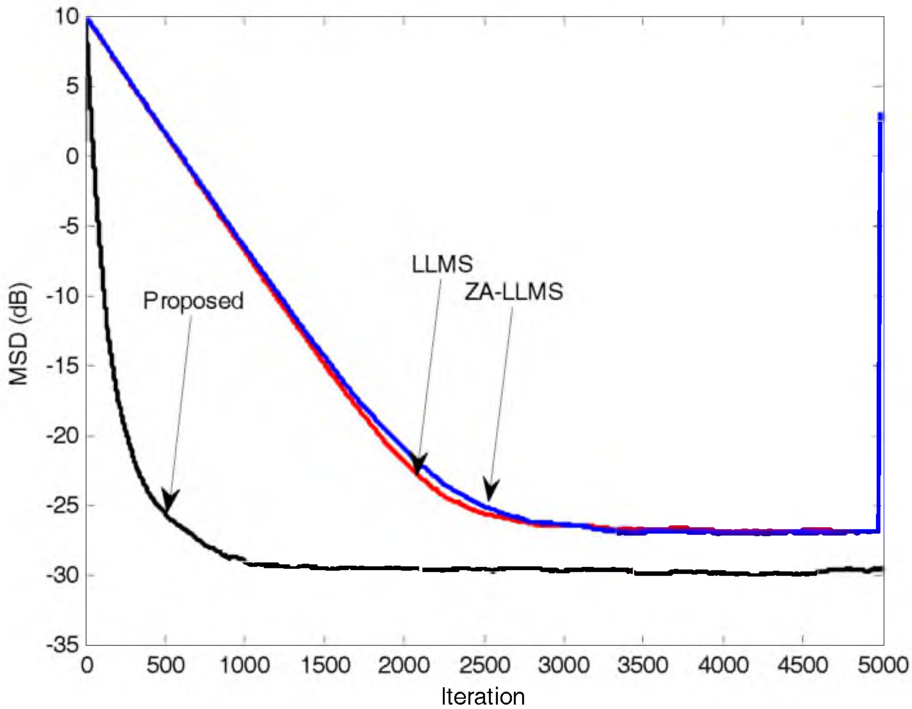

Fig. 3 – Steady state behavior of the LLMS, ZA-LLMS and the proposed algorithm for 50% sparsity with AWGN.

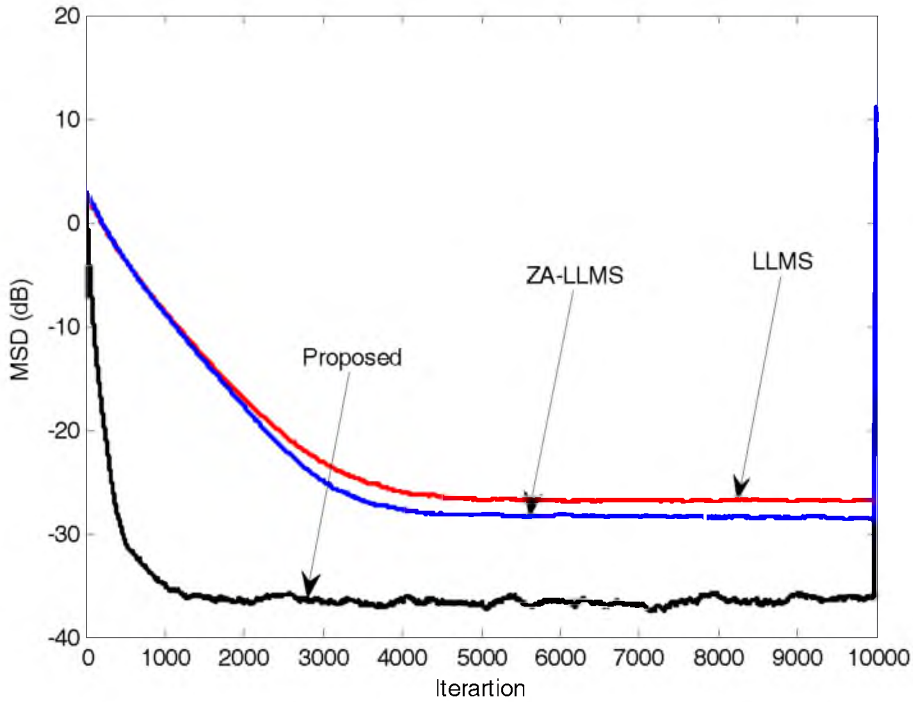

Fig. 4 – Steady state behavior of the LLMS, ZA-LLMS and the proposed algorithm for 90% sparsity with ACGN.

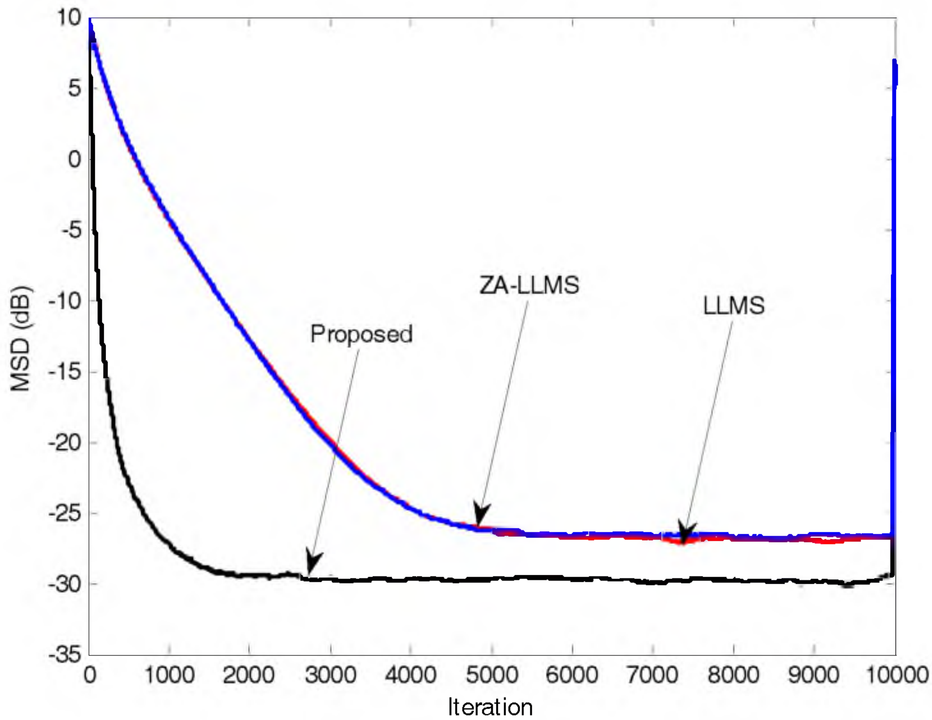

Fig. 5 – Steady state behavior of the LLMS, ZA-LLMS and the proposed algorithm for 50% sparsity with ACGN.

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МОДИФИКАЦИЯЛАНҒАН СИРЕК LMS-АҒАТЫН АЛГОРИТМНІҢ КӨМЕҒІМЕН ЖҮЙЕЛЕРДІ СӘЙКЕСТЕНДІРУ

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Тірек сөздер: адаптивті алгоритмдер, жүйелерді сәйкестендіру, сиректелген жүйелер.

Аңдатпа. Бұл мақалада сиректелген жүйені сәйкестендіру үшін ZA-LLMS (Zero-Attracting Leaky-LMS) алгоритімін жақсартуда қолданылатын жаңа LMS-агатын алгоритмін ұсынылады. Ұсынылган алгоритм қадам мөлшері мен l_0-нормалы айыппұлдың ауытқу артықшылықтарымен сиректелген жүйені қолданады. Өнімділігін салыстыру мақсатында ұсынылып отырган алгоритмді LLMS-пен және ZA-LLMS-пен жинақтылық жылдамдығы мен ортақвадратталган ауытқу (MSD) тұрғысынан салыстырдық. Зерттеулер MATLAB орталығында жүргізілді. Модельдеудің жетістігі, екі типтегі кіріс сигналдарыны үшін: қосымша ақ Гаусс шуы (AWGN) мен қосымша корреляциялық Гаусс шуы (ACGN), ұсынылып отырган алгоритм басқада алгоритмдерден үстемділігінің артықшылығын көрсетеді.

Поступила 16.05.2016 г.