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**NOVEL APPROACHES TO MODELING
AND CALCULATING FLOWS PARAMETERS
OF DENSE POLYDISPERSED SUSPENSIONS**

Abstract. Modeling the sediments of dense polydisperse suspensions and the calculation of the corresponding equipment have some features that are often not taken into account in engineering calculations. This article provides an overview and analysis of these features, and also outlines ways to solve the problems arising in the calculation of the deposition of polydisperse suspensions in relation to specific structures of industrial apparatuses. New approaches to the mathematical description of the flow of dense suspensions and sediments taking into account the experimentally observed characteristics of the flow of such systems have been offered. The modern model that allows describing this phenomenon and eliminating the corresponding limitations of known models has been submitted. The submitted model describes the fluidity of the medium in a wide range of the content of dispersed solid phase even at its high concentrations. Such result has been achieved thanks to a new heuristic expression for calculating the relative viscosity of the suspension as a function of the concentration of the solid phase.

The formula for calculating the outflow rate from buffer tanks with accounting to the energy dissipation has been submitted. Such an approach allows to propose a calculation scheme that can be adapted to different rheological models.

Keywords: polydisperse, suspensions, buffer tanks, sedimentation, mathematical description.

Introduction. Modeling the sedimentation of polydisperse suspensions and the calculation of the corresponding equipment parameters have some features that are often not taken into account in engineering calculation methods. This paper provides a brief overview and analysis of these features, and also outlines ways to solve the problems arising in the calculation of the deposition of polydisperse suspensions in relation to specific structures of industrial apparatuses.

An important role in the deposition of polydisperse suspensions is played by the phenomena of an increase in the viscosity of concentrated suspensions of solid particles in a liquid when shear stresses are applied to them. This effect is associated with the transition from a flat layered structure of the arrangement of particles in suspension to a chaotic, three-dimensional distribution during deformations [1, 2]. During the transition, the viscosity grows continuously and quickly reaches a maximum; with a further increase in shear rate, it may fall.

The following parameters have an impact on this process: particle size distributions, their volume fraction, shape, interaction with other particles which can be accompanied with aggregation processes, the viscosity of the continuous phase, as well as type, velocity and time of deformation. The best means of avoiding an increase in viscosity in practical situations is to reduce particle sizes or increase their volume fraction. Due to the narrowness of the range of shear rates in which the viscosity increases, to characterize such suspensions, one can use the power dependences of viscosity on shear rate [3].

In theoretical analysis of convective flows, which are set up in undiluted suspensions, during sedimentation of particles near the wall of an inclined sump, one can use the equations of momentum averaged over the ensemble neglecting Reynolds stresses at small Reynolds numbers.

When considering the inhomogeneous system of particles of the dispersed phase, which have a non-spherical shape, a procedure was developed within the framework of the hydrodynamic approach, which allows expressing the force and moment of forces acting on each particle of the dispersed phase in the form of linear combinations of perturbations (temperature gradient, tensor external voltages, etc.), the coefficients at which are the tensors of the corresponding ranks [5].

For the case when the particles of the dispersed phase have the form of weakly deformed spheres, a method has been developed that makes it possible to calculate the specific form of these tensors taking into account the interaction of any number of particles [6]. In the case when the mixture consists of Newtonian liquid and from the solid spherical particles of the same sizes suspended in it, the sources of stress arising in the dispersed phase are:

- 1) particle collisions between themselves,
- 2) chaotic particle motion,
- 3) hydrodynamic interaction of particles and liquids.

Usually the focus is on the consideration of hydrodynamic interaction. The resulting expression for the stress tensor due to the presence of solid particles coincides with the expression obtained in [6] for the case of a slow flow of a dilute suspension of solid particles using the concept of averaging over volume.

The diluted suspension of identical non-deformable solid particles in incompressible fluid is quite often considered, where using the averaging procedure by volume and time based on the equation of motion of a single particle in a turbulent flow, the equations of conservation of momentum and moment of momentum for the solid phase are derived. The main attention is paid, as a rule, to the derivation of the relations necessary for the closure of these equations for the fluxes of the quantity of motion and angular momentum, which are caused by turbulent flow pulsations.

It is assumed that the inertia of particles is sufficiently large, as a result of which the flow turbulence has only a weak effect on their movement, determined by the balance of gravity, aerodynamic force and momentum transfer during collisions of particles with solid walls bounding the current suspension. In this case, it is possible to obtain a closing relation of the second order [7, 8].

In many theoretical and experimental studies of the sedimentation process under the action of gravity of a narrow fraction of polydisperse suspensions, the time variation of the thickness of the zone in the upper part of the precipitation column separating the pure liquid from the slurry layer containing particles of all sizes is traced. It is assumed that the deposited particles, the dimensions of which obey the normal law of distribution, move in the Stokes mode. Inertial forces and Brownian motion are not taken into account. The distribution of the concentration and size of particles over the time and frequency of the precipitation column is calculated by numerically solving the mass conservation equation, supplemented by a relation determining the relationship between the deposition rates and the local particle concentrations [9].

In the case when the sedimentation of a suspension occurs in an arbitrary container, in the approximation of a creeping flow, the problem of precipitating a diluted monodisperse suspension of solid spherical particles inside the container is considered. Particles have a statistically uniform distribution in space. In the expression for the dimensionless average particle velocity on the axis of an arbitrary container (as characteristic, the Stokes velocity of a single sphere in an unbounded fluid is taken) in the limiting case, when its walls are removed to infinity, it can be shown that the coefficient of volume concentration depends only on the asymptotics the tendency to infinity of the linear dimensions of the container in the averaging procedure. This allows obtaining the value of the sedimentation rate of the suspension inside a container with infinitely remote walls of almost arbitrary shape [10].

In this paper, new approaches to the mathematical description of the flow of dense suspensions and sediments are proposed taking into account the real physical flow regularities of such systems.

General mathematical model the flow of dense suspensions and sediments.

The flow of thick suspensions, as a rule, occurs at low Reynolds numbers. Thus, in particular, the outflow of viscous sediments from tanks and bunkers is carried out in a creeping regime. In the case of fine solid particles contained in the non-stratified suspension as a dispersed phase, the suspension can be regarded as a homogeneous liquid with some effective viscosity [11, 12].

Therefore, it is correct to use Nusselt approximation for describing the flows of thin layers of dense suspensions [13, 14].

The impulse equations of a thin layer of a viscous fluid in the Nusselt approximation can be written in the following form.

$$\frac{\partial}{\partial y} \left(\mu_s \frac{\partial U}{\partial y} \right) + \rho_s g \cos \gamma = 0. \quad (1)$$

Here U is the longitudinal component of liquid flow velocity; g is the gravity acceleration and γ is an angle of the support surface inclination.

The effective viscosity of the suspension, taking into account the effect of solid particles suspended in the liquid, is determined from the relationship [12, 15]:

$$\mu_s = \mu_l \mu_r, \quad (2)$$

where μ_l is a viscosity of the pure liquid and μ_r is a relative viscosity depending on the solid phase content.

The relative viscosity of the suspension is proposed to be calculated by the formula [12, 14]:

$$\mu_r = \left(1 - \frac{\phi}{\phi_m} \right)^{-\alpha}, \quad (3)$$

where ϕ_m is certain maximum content of dispersed solid phase; and α is empirical indicator.

It is known [14] that for a wide class of liquids with hydrophilic inclusions of a finely dispersed solid phase over a wide range of variation of the regime parameters, the following estimates are valid:

$$\phi_m = 1.68; \quad \alpha \approx 1.82 \quad (4)$$

At the same time, as can be seen from formula (3), for, the effective viscosity calculated by formula (3) tends to infinity when $\phi \rightarrow \phi_m$.

This circumstance contradicts the data of experimental studies [15, 16]. In fact, even very thick suspensions with the maximum solids content can be yet attributed to dense sediments that have fluidity. For example, industrial slimes, as well as natural mudflows, save the fluidity up to very high concentrations of the solid phase [17, 18]. Therefore, we propose a somewhat different model that eliminates the mentioned contradiction.

First, for small parameter ϕ values, the model must be consistent with formula (3).

Second, for $\phi \rightarrow \phi_m$ the asymptotic behavior $\mu_r \rightarrow \mu_m$, where μ_m is a certain limiting value of the relative viscosity, must be realized.

Let us introduce the special parameter:

$$\beta = \frac{\phi}{\phi_m - \phi} \quad (5)$$

In accordance with our assumptions, the function must satisfy the following conditions:

$$\frac{d\mu_r}{d\beta}(0) = \alpha, \quad (6)$$

$$\lim_{\beta \rightarrow \infty} \mu_r = \mu_m, \quad (7)$$

$$\mu_r(0) = 1. \quad (8)$$

The simplest approximation for the desired function reads

$$\mu_r = \frac{\frac{\alpha \mu_m}{\mu_m - 1} \beta + 1}{\frac{\alpha}{\mu_m - 1} \beta + 1}. \quad (9)$$

Figure 1 shows some results of calculations using formulas (3) and (9) for different values of the limiting relative viscosity.

It can be seen from the graphs that in the range of solids concentration in the suspension less than 0.12, the difference in the calculated values of the relative viscosity by formulas (3) and (9) does not exceed 18% at $\mu_m \geq 20$.

However, in the concentration range from 0.2 to 0.5, the calculated values already differ by more than 50%, even for $\mu_m = 1000$.

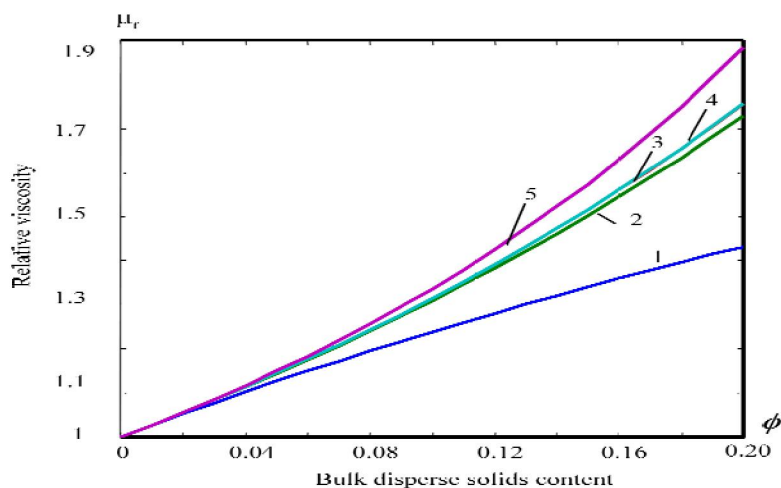


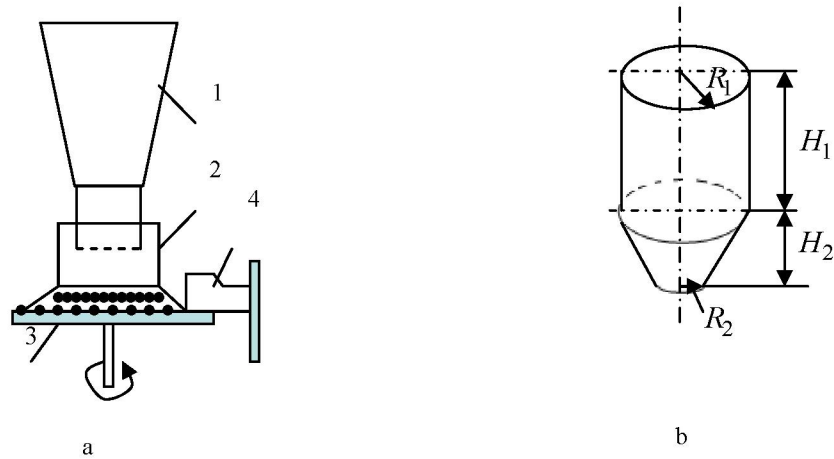
Figure 1 - Calculation using formula (9): 1- $\mu_m = 2$,
2- $\mu_m = 20$, 3- $\mu_m = 100$, 4- $\mu_m = 1000$; 5- calculation using (3)

Modeling the outflow of dense suspensions from reservoirs

The discharge of sediment (or sludge) is usually carried out in a batch mode. In this case, the outlet sizes of the bunker should be sufficient to provide the required throughput [12, 19]. At the same time, they must exclude the accumulation (hanging) of the loose cargo in the bunker. In order to avoid the sudden collapse of large masses of the loose cargo through the holes, as well as to avoid need in high weight of the gates, the discharge outlet must not be excessively large [19, 20].

The main geometrical parameters of the buffer tanks are shown in Figure 2. The volume of the bunker of cylindrical shape, consisting of a cylinder in the upper part and a truncated cone in the bottom, is found by the following formula:

$$\Omega = \left(\frac{1}{3} \pi H_1 (R_1^2 + R_1 R_2 + R_2^2) \right) + \pi R_1^2 H_1. \quad (10)$$



1- the bunker housing; 2) a receiving vessel; 3- feeder; 4- cutting knife

Figure 2 - Scheme of bunkers for discharging sediments from reactors

The most important work parameters of tanks are their throughput Q and the velocity of the sediment outflow V_f [16, 20]. The rate of the sediment outflow can be calculated from the following considerations. Because the flow of a viscous precipitate occurs in a creeping regime [21, 22], then the total pressure drop along the height of the sediment layer can be determined on the base of the average energy dissipation according to the formula:

$$\Delta P = \frac{\lambda}{\text{Re}} \frac{(H_0 - H) V_f^2 \rho}{d_e^2} \quad (11)$$

After obvious rearrangements the formula for pressure drop at the low Reynolds number reads

$$\Delta P = \frac{\lambda}{2} \mu \frac{(H_0 - H) V_f}{d_e^2} \quad (12)$$

Let us write the Bernoulli equation with respect to the sediment layer surface in the conical hopper and at the outlet:

$$\rho g H + \rho \frac{1}{2} \left(\frac{R_2^2}{R_1^2} \right)^2 V_f^2 = \rho \frac{1}{2} V_f^2 + \Delta P \quad (13)$$

After substituting formula (12) into relation (13) the equation looks as following

$$\rho g H + \rho \frac{1}{2} \left(\frac{R_2^2}{R_1^2} \right)^2 V_f^2 = \rho \frac{1}{2} V_f^2 + \frac{\lambda}{2} \mu \frac{(H_0 - H) V_f}{d_e^2} \quad (14)$$

As a result the formula for outflow rate reads

$$V_f = -\gamma(H_0 - H) + \sqrt{\gamma^2(H_0 - H)^2 + 2gH} \quad (15)$$

where the control parameters are

$$G = \frac{\lambda}{\rho d_e^2}, \beta = \frac{R_2^4}{R_1^4}, \gamma = \frac{G\mu}{2(1-\beta)}. \quad (16)$$

The formula for calculating the outflow rate can be rewritten in the dimensionless form by using the characteristic time of the tank full emptying T^* :

$$\tilde{V}_f = -S_1(1-h) + \sqrt{S_1^2(1-h)^2 + S_2 h}. \quad (17)$$

Here

$$\tilde{V}_f = V_f \frac{T^*}{H_0}; \quad h = \frac{H}{H_0}; \quad S_1 = \gamma T^*; \quad S_2 = 2g \frac{T^{*2}}{H_0}. \quad (18)$$

Figure 3 depicts some results of numerical study of the outflow rate as a function of the height of thick sediment in the buffer tank under the various values of the control parameter $\gamma = \frac{G\mu}{2(1-\beta)}$ for $H_0 = 2$ meters.

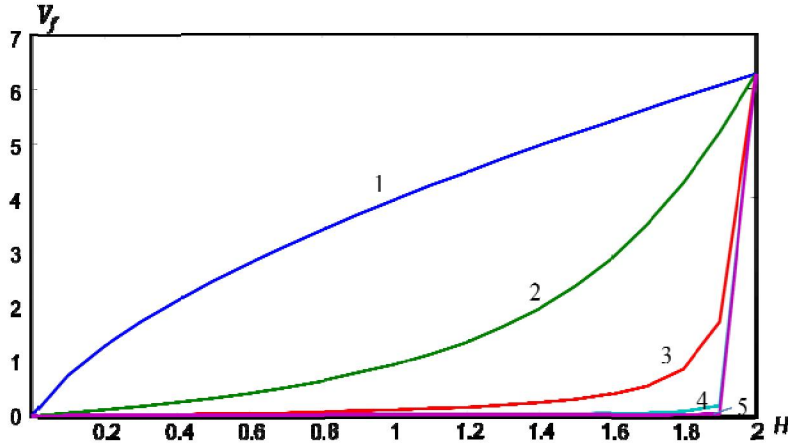


Figure 3 - Outflow rate as a function of the parameter γ . 1- $\gamma = 10$, 2- $\gamma = 10^2$, 3- $\gamma = 10^3$, 4- $\gamma = 10^5$

It can be seen from the graphs that while increasing the control parameter γ , the behavior of the dependence of outflow rate on the height of a sediment layer is radically changed. Namely, under the certain viscosity of the thick sediment, the emptying of the reservoir after a while occurs at such a low rate that it practically ceases. This phenomenon predicted by our simple model was indeed observed in experiments and in engineering practice [18, 21]. The analysis of known models and calculation methods does not lead us nevertheless to other models that describe this phenomenon convincingly [22, 23].

Creation of reliable high-performance machines specifies the use of new materials for manufacture of their parts. At the same time, manufacturers do not have time to introduce new processing technologies using durable cutting tools that require a lot of time and financial resources [24].

Conclusions. As a result of the work, an approach to the problem of describing the flow of dense suspensions and deposits has been developed. The submitted model allows taking into account the fluidity of the medium up to high concentrations of the dispersed solid phase in the suspension, and also describing the features of the flow of thick suspensions near a solid wall.

The novel model demonstrates good qualitative agreement with experimental observations, but it requires a more detailed analysis of the array of experimental data in order to clarify a number of control parameters applied to specific physicochemical systems. After such an analysis, the proposed approach and the corresponding model can be useful as a basis for the engineering calculation technique.

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ҚОЮ ПОЛИДИСПЕРСТІ СУСПЕНЗИЯЛАР АҒЫНДАРЫНЫҢ ПАРАМЕТРЛЕРІН МОДЕЛЬДЕУГЕ ЖӘНЕ ЕСЕПТЕУГЕ ЖАҢА АМАЛДАР

Аннотация. Полидисперсті суспензиялардың шөгуін модельдеу мен тиісті аппаратураны есептеудің кейбір ерекшеліктері бар, олар инженерлік есептеулер кезінде бөліктей ескерілмейді. Осы мақалада осы ерекшеліктерге шолу және талдау жасалынған, сондай-ақ полидисперсті суспензиялардың шөгуін есептеу барысында туындаған мәселелерді, өнеркәсіптік аппараттардың нақты құрылымына қолданымды шешу жолдары атап өтілген.

Осындай жүйелердің ағынының тәжірибелік бақылау сипаттамаларын ескере отырып, тығыз суспензиялардың ағыны мен шөгінділерді математикалық сипаттауға жаңа амалдар ұсынылған. Осы құбылысты сипаттауға және белгілі модельдердің тиісті шектеулерін жоюға мүмкіндік беретін заманауи модель ұсынылған. Ұсынылған модель жоғары концентрацияларда да дисперсті қатты фаза құрамының кең аралығында ортаның аққыштығын сипаттайды. Осындай нәтижеге қатты фазаның концентрациясына тәуелді суспензияның салыстырмалы тұтқырлығын есептеуге арналған жаңа эвристикалық өрнектің арқасында қол жетті.

Энергияның таралуын ескере отырып, резервуарлардан ағу жылдамдығын есептеуге арналған формула ұсынылды. Мұндай амал әр түрлі реологиялық модельдерге бейімделген есептеу сұлбасын ұсынуға мүмкіндік береді.

Түйін сөздер: полидисперсия, суспензиялар, резервуарлар, седиментация, математикалық дискрипторлар.

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НОВЫЕ ПОДХОДЫ К МОДЕЛИРОВАНИЮ И РАСЧЕТУ ПАРАМЕТРОВ ПОТОКОВ ГУСТЫХ ПОЛИДИСПЕРСНЫХ СУСПЕНЗИЙ

Аннотация. Моделирование осаждения полидисперсных суспензий и расчет соответствующей аппаратуры имеют некоторые особенности, которые зачастую не учитываются при инженерных расчетах. В настоящей статье приведен обзор и анализ этих особенностей, а также намечены пути решения возникающих при расчете осаждения полидисперсных суспензий проблем применительно к конкретным конструкциям промышленных аппаратов.

Предложены новые подходы к математическому описанию потока плотных суспензий и осадков с учетом экспериментально наблюдаемых характеристик потока таких систем. Представлена современная модель, позволяющая описать это явление и устранить соответствующие ограничения известных моделей. Представленная модель описывает текучесть среды в широком диапазоне содержания дисперсной твердой фазы даже при ее высоких концентрациях. Такой результат был достигнут благодаря новому эвристическому выражению для расчета относительной вязкости суспензии в зависимости от концентрации твердой фазы.

Предложена формула для расчета скорости истечения из резервуаров с учетом рассеяния энергии. Такой подход позволяет предложить схему расчета, которая может быть адаптирована к различным реологическим моделям.

Ключевые слова: полидисперсия, суспензии, резервуары, седиментация, математические дискрипторы.

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REFERENCES

- [1] Buevich Ju.A., Isaev A.M. Jelementarnaja teorija psevdoturbulentnosti v melko-dispersnyh suspenzijah. // IZhF. 1989. № 2. P. 239-246. (In Russian).
- [2] Brady John F., Phillips Ronald J., Lester Julia C., Bossis Georges. Dynamic simulation of hydrodynamically interacting suspensions. J. Fluid Mech. 1988. 195. C.257-280. (in Eng)
- [3] Durlafsky Louis J., Brady John F. Dynamic simulation of bounded suspensions of hydrodynamically interacting particles. J. Fluid Mech. 1989. 200. C. 39-67. (in Eng).
- [4] Witheridge Grant M., Wilkinson David L. Density measurement of particle and floc suspensions. J. Hydraul.Eng. 1989. 115, № 3. C. 403-408. (in Eng).
- [5] Volchenok V.F. Modelirovanie svojstv polidispersnyh struktur. Minsk: Nauka i tehnika, 1991. (In Russian).
- [6] Zolotov A.V. Osazhdenie suspenzii v proizvodnom kontejnere. // Trudy XIV konferencii molodyh uchenykh Moskovskogo fiziko-tehnicheskogo instituta. Ch.1. M., 1989. P. 37-41. Dep.v VINITI 11.09.89., № 5761-V89. (In Russian).
- [7] Dorodnycyn A.A. Review of methods for solving the Navier-Stokes equations // Lect. Notes Phys. 1973. Vol. 18. P. 23-47.
- [8] Wagner Norman J., Russel William B. Nonequilibrium statistical mechanics of concentrated colloidal dispersions: hard spheres in weak flows with many-body thermodynamic interactions. Surface Sci. 1989. 209. № 3. P. 475-518.
- [9] Bouhroum Ahsene. Einfluss der Viskositats und Dichteunterschiede zweier mischbarer Flussigkeiten auf den Koeffizienten der longitudinalen hydrodynamischen Dispersion. Dtsch. gewasserk. mitt. 1989. 33, № 1. P. 9-13.
- [10] Noetinger Benoit. A two fluid model for sedimentation phenomena. Physica. A. 1989. 157, № 3. C. 1139-1179.
- [11] O. Seyvet, P. Navard, Collision-induced dispersion of agglomerate suspensions in a shear flow// Journal of Applied Polymer Science, 78, 1130-1133 (2000).
- [12] R.I. Tanner, Fuzhong Q.I., K.D. Housiadas, A differential approach to suspensions with power-law matrices// Rheol. Acta 49, 169-176 (2010).
- [13] J.F. Brady and J.F. Morris, Microstructure of strongly sheared suspensions and its impact on rheology and diffusion// J. Fluid Mech. 348, 1997, 103-139 (1997).
- [14] R.Z. Pen, L.V. Chendylova, I.L. Shapiro, Chemistry of Plant Raw Materials 1, 11-14 (2004). (In Russian).
- [15] A.M. Brenner, V.G. Golubev, Film condensation from a dusty vapor-gas mixture// Theoretical Foundations of Chemical Engineering 36(2), 123-128 (2002).
- [16] M.K. Lyon, D.W. Mead, R.E. Elliott, L.G. Leal, Structure formation in moderately concentrated viscoelastic suspensions in simple shear flow// J. Rheol. 45(4), 881-890 (2001).
- [17] A.W. Jenike, "Storage and flow of solids", (Bulletin No.123, Utah Engineering Experiment Station, 1970), pp. 1-209.
- [18] R.D. Nelson, Jr., "Dispersed Powders in Liquids", in Handbook of Powder Technology, edited by J.C. Williams, T. Allen (Elsevier, Vol.7, Amsterdam, Oxford, New York, Tokyo, 1988), pp. 1-264.
- [19] Yuan Lin, Nhan Phan-Thien, Boo Cheong Khoo, Normal stress differences behavior of polymeric particle suspension in shear flow// J. Rheol. 58(1), 223-235 (2014).
- [20] V. Kumar, B. Nazari, D. Bousfield, M. Toivakka, Rheology of microfibrillated cellulose suspensions in pressure-driven flow// Appl. Rheol. 26, 1-11 (2016).
- [21] N. Ohl, W. Gleissle, The characterization of the steady-state shear and normal stress functions of highly concentrated suspensions formulated with viscoelastic liquids// J. Rheol. 37(2), 381-406 (1993).
- [22] J. Happel, H. Brenner, Low Reynolds Number Hydrodynamics: with Special Applications to Particulate Media. (Martinus Nijhoff Publishers, Hague, Boston, Lancaster, 1983), p. 553.
- [23] S.A. Boronin, A.A. Osipov, Effects of particle migration on suspension flow in a hydraulic fracture// Fluid Dynamics 49(2), 80-94 (2014).
- [24] D. T. Khojibergenov, U. D. Khozhibergenova, A. B. Akhmedov, B. N. Absadykov//News of the National Academy of Sciences of the Republic of Kazakhstan. Series of geology and technical sciences. 2019 Vol. 2, N 434. P.69-78. ISSN 2224-5278. <https://doi.org/10.32014/2019.2518-170X.40>