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ЧИСЛЕННОЕ ИССЛЕДОВАНИЕ ПРОЦЕССА ПАЛЬЦЕОБРАЗОВАНИЯ ПРИ ТЕЧЕНИИ ДВУХ НЕ СМЕШИВАЮЩИХСЯ ЖИДКОСТЕЙ В КАНАЛЕ

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Ключевые слова: течение двух несмешивающихся жидкостей, контактная линия, контактный угол, капиллярное число, пальцеобразование, граничное условие проскальзывания, метод объема жидкости.

Аннотация. В данной работе численно исследован процесс пальцеобразования при течении двух несмешивающихся жидкостей в канале, а именно исследована связь между шириной пальцеобразования и капиллярным числом и результаты сравнены с работой [6]. Также в данной работе исследованы процессы перехода из ручейного вида в клиновидный вид пальцеобразования для различных значений коэффициента поверхностного натяжения. Все численные расчеты проводились с помощью программы Gerris [7].

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NUMERICAL INVESTIGATION OF FINGERING PATTERN FORMATION DURING THE FLOW OF TWO IMMISCIBLE FLUIDS IN A CHANNEL

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Keywords: flow of two immiscible fluids, contact line, contact angle, capillary number, fingering pattern, slip boundary condition, volume-of-fluid (VOF) method.

Abstract. In this paper we numerically investigated the fingering pattern formation in the unstable interface between two immiscible fluids during the flow in the channel. The relation between finger width and capillary number is investigated in this paper, and compared with work [6]. Also in this work the transition from rivulet-type finger to wedge-shaped finger is investigated for different values of surface tension. All numerical calculations are performed using Gerris program [7].

Introduction. Two immiscible fluids flow can be found in many different situations; some cases in which it plays a central role are the spreading of adhesives, the flowing of lubricants into inaccessible locations, the coating of solid surfaces with a thin uniform layer of liquid, the displacement of oil by water through a porous medium, etc. One of the phenomena which occurs in the two immiscible fluids flow is the fingering pattern formation in the interface between fluids. This phenomenon can be observed in the pressure-driven flow of two immiscible fluids in a channel (see fig. 1). The experiment [1] shows that even if the interface between fluids is initially straight, it quickly deforms, resulting in the formation of finger-like structures. The instability leading to this pattern is referred to as fingering instability. Two

types of fingering shape can be observed in the experiment: wedge-shaped finger and rivulet-type finger (see fig. 1). Formation of fingering pattern can result in poor quality of coating or reducing oil production by water displacement in porous medium, so investigation of fingering instability is important for practical applications. There are exist many quantitative models of this phenomenon and one of the most popular is the thin-film flow down an inclined plane [6]. In this model the linear stability analysis is employed for lubrication-type equations to describe evolution of small perturbations in the interface between two immiscible fluids and this model can't be used for large deformation of the interface. Nevertheless, this model can estimate the finger width of rivulet-type finger (see fig. 1). The finger width depends on the capillary number:

$$h_{layer} = h - h_{finger} \sim Ca^{-\frac{1}{3}}, \tag{1}$$

where Ca is the capillary number. It's the ratio between viscous force and surface tension force:

$$Ca = \frac{\mu U_{CL}}{\sigma}, \tag{2}$$

where U_{CL} is the contact line velocity. In this paper we numerically investigated the fingering pattern formation in the unstable interface between two immiscible fluids for flow in the channel. The relation between finger width and capillary number is investigated in this paper, and compared with (1). Also in this work the transition from rivulet-type finger to wedge-shaped finger is investigated for different values of surface tension, and showed that the rivulet-type finger occurs only when [5]:

$$C = Ca^{-\frac{1}{3}} tg\theta > 1. \tag{3}$$

All numerical calculations are performed using Gerris program [7].
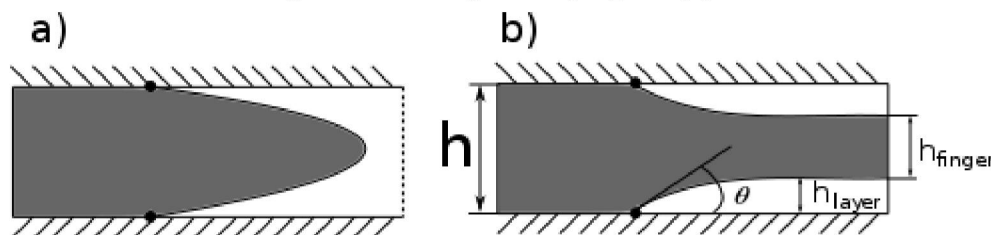

Fig. 1 – Fingering patterns in the interface between two immiscible fluids: a) wedge-shaped finger, b) rivulet-type finger.

Formulation of the problem. We numerically solved the Navier-Stokes equations for incompressible, two immiscible, viscous fluids flow in 2D channel:

$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\rho \vec{u}\vec{u}) = -\nabla p + \nabla \cdot (2\mu E), \tag{4}$$

$$E = \frac{1}{2}(\nabla \vec{u} + \nabla \vec{u}^{T}), \tag{5}$$

$$\nabla \cdot \vec{u} = 0, \tag{6}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0, \tag{7}$$

$$\rho = F\rho_1 + (1 - F)\rho_2, \tag{8}$$

$$\mu = F\mu_1 + (1 - F)\mu_2, \tag{9}$$

where F is the parameter that identify a given fluid i (i =1 or 2) is present at a particular location x :

$$F = \begin{cases} 1, & if \ x \ is \ in \ fluid \ i \\ 0, & if \ x \ is \ not \ in \ fluid \ i \end{cases} \tag{10}$$

If we substitute the equation (6) into the equation (5), we have that:

$$\frac{\partial F}{\partial t} + \vec{u} \cdot \nabla F = 0. \tag{11}$$

In order to find the shape and location of the interface between the two fluids, we use the volume-of-fluid method [3] and advect this interface using equation (11). Equations (4, 6 and 11) are numerically solved using the projection method on non-staggered grid [4] and the following boundary conditions were used (see fig. 1):

1) Inlet boundary condition:

$$\frac{\partial u_{in}}{\partial x} = 0, \tag{12}$$

$$v_{in} = 0, \tag{13}$$

$$p_{in} = 1. \tag{14}$$

2) At the walls of the channel:

$$u_w = \lambda \frac{\partial u}{\partial \vec{n}}, \tag{15}$$

$$v_w = 0, \tag{16}$$

$$\frac{\partial p_w}{\partial y} = 0, \tag{17}$$

where λ is the slip length and $\vec{n}$ is the normal vector to the wall. Here we used the Navier slip boundary condition instead of no-slip boundary condition to avoid viscous stress singularity at the contact line [2].

3) At the interface between two fluids – S:

$$\left[\vec{u}\right]_S = 0, \tag{18}$$

$$-\left[-p + 2\mu \vec{n} \cdot E \cdot \vec{n}\right]_S = \sigma k, \tag{19}$$

$$k = -\nabla \cdot \vec{n}, \tag{20}$$

$$-[2\mu \vec{t} \cdot E \cdot \vec{n}]_S = \vec{t} \cdot \nabla_S \sigma, \tag{21}$$

where σ is the surface tension, k is the curvature of the interface - S, $\vec{n}$ is the normal to the interface - S, and $\vec{t}$ is the tangent vector to the interface - S.

4) Outlet boundary condition:

$$\frac{\partial u_{out}}{\partial x} = 0, \tag{22}$$

$$v_{out} = 0, \tag{23}$$

$$p_{out} = 0. \tag{24}$$

Results. The steady state solution of the equations (4, 6) with boundary conditions at the walls of the channel (15 - 17) and with boundary conditions at the interface between the two fluids (18 - 21) can be obtained by neglecting the viscous force on the interface between the two fluids (21):

$$u = \frac{p_{in} - p_{out} - p_c}{\mu L} \left(\frac{h^2 - y^2}{2} + \lambda h\right), \tag{25}$$

where p_c is the capillary pressure. The value of capillary pressure can be obtained from Young-Laplace equation and for 2D case (see fig. 1):

$$p_c = \frac{\sigma \cos\theta}{h}.$$ (26)

The average value of (25):

$$\bar{u} = \frac{p_0 - p - p_c}{\mu L}(\frac{h^2}{3} + \lambda h).$$ (27)

In this work we numerically validated the relation (1) for rivulet-type finger. To validate this relation, we consider two cases: in the first case, the pressure difference between the ends of the channel is constant, but the surface tension of the interface between two immiscible fluids is changed, and in the second case, the surface tension is constant, but the pressure difference is changed. In the first case, the velocity along the center line of the channel is linear depends on the surface tension (25, 26). Since the contact angle - θ (see fig. 1) depends on the capillary number [2]:

$$\theta \sim Ca^{\frac{1}{3}},$$ (28)

therefore the contact angle - θ almost doesn't change. As shown in fig. 2, the contact line velocity almost doesn't change too. In the second case, the velocity along the center line of the channel is linear depends on the pressure difference between the ends of the channel (25), and as shown in fig. 3, the contact line velocity also linear depends on the pressure difference between the ends of the channel. In fig. 4 is shown the relation between the finger width and the capillary number, and this relation is almost matched with the relation (1). Also in this work the transition from rivulet-type finger to wedge-shaped finger is investigated for different values of surface tension. For surface tension $\sigma = 1$, the value $C = 2{,}84$ (3), and for $\sigma = 2$, the value $C = 1{,}59$. The relation between finger width and contact angle is shown in the fig. 5.

Conclusion. In this paper is numerically investigated the fingering pattern formation in the unstable interface between two immiscible fluids during the flow in the channel. The relation between finger width and capillary number is investigated in this paper, and this relation is reasonably good matched with [6]. Also in this work the transition from rivulet-type finger to wedge-shaped finger is investigated for different values of surface tension, and is showed that this transition occurs only for $C > 1$ [5].
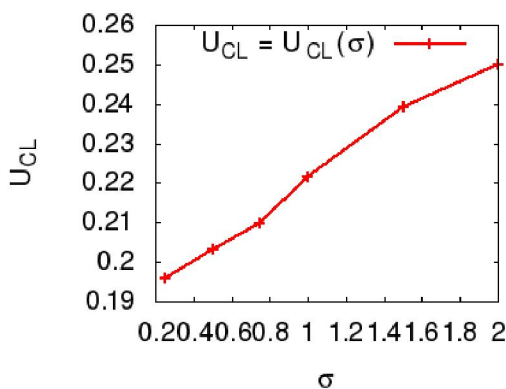

Fig. 2 – The relation between contact line velocity and surface tension
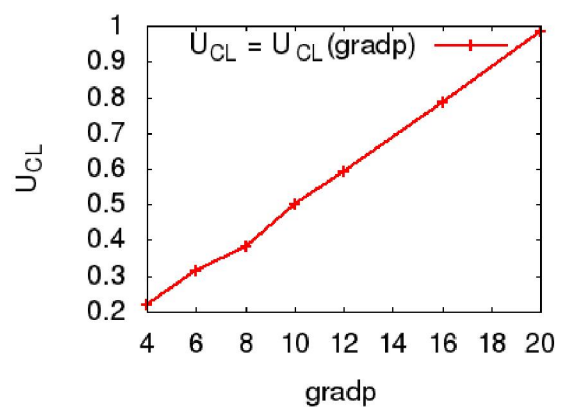

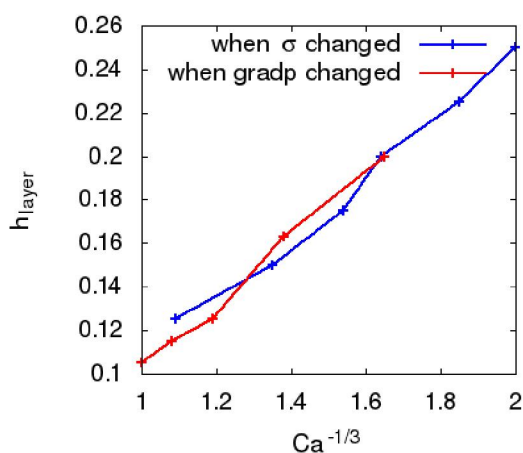Fig. 3 – The relation between contact line velocity and pressure difference

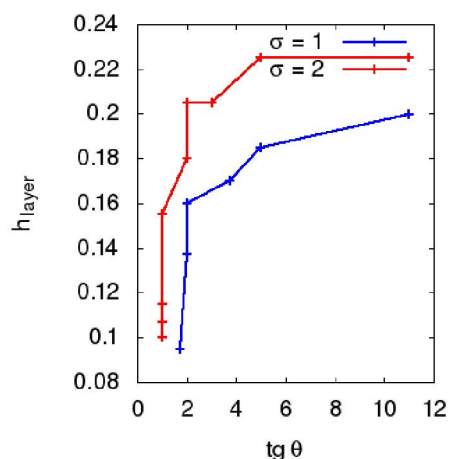Fig. 4 – The relation between finger width
and capillary number

Fig. 5 – The relation between finger width
and contact angle

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ҚАНАЛДАҒЫ ЕКІ АРАЛАСПАЙТЫН СҰЙЫҚТАРДЫҢ АҒЫСТА САУСАҚ ТӘРІЗДІ АҒЫС ФОРМАСЫНЫҢ ПАЙДА БОЛУ ПРОЦЕСІНІҢ САНДЫҚ ЗЕРТТЕУІ

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Түйін сөздер: екі араласпайтын сұйықтардың ағыны, түйіскен сызық, түйіскен бұрыш, капиллярлық сан, саусақ тәрізді ағыс формасының пайда болуы, сырғанақ шекаралық шарт, сұйық көлем әдісі.

Аннотация. Осы жұмыста қаналдағы екі араласпайтын сұйықтардың ағыста саусақ тәрізді ағыс формасының пайда болуы сандық зерттелген, атап айтқанда саусақ формасының ені мен капиллярлық сан арасындағы байланыс зерттелінген, және алынған нәтижелер басқа жұмыстармен салыстырылған [6]. Барлық есептеулер Gerris бағдарламасы арқылы жасалған [7].