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DARBOUX TRANSFORMATION FOR THE (1+1)-DIMENSIONAL NONLOCAL FOCUSING NONLINEAR SCHRÖDINGER EQUATION

Abstract. In this article, we consider a (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation. This equation is studied in works M.J. Ablowitz and Ziad H. Musslimani, Li-Yuan Ma, Zuo-Nong Zhu, T. A. Gadzhimuradov A. M. Agalarov and other. The nonlocal nonlinear Schrödinger equation is integrable the inverse scattering methods. We constructed Darboux transformation (DT) of this equation. Also, we will derive determinant representation of one-fold, two-fold and n-fold DT of (1+1)-dimensional nonlocal focusing Schrödinger equation. As an application of these conversion of this equation, soliton solutions will get from trivial "seed" solutions.

Key words: (1+1)-dimensional nonlocal focusig nonlinear Schrödinger equation, integrable system, Lax pair, determinant representation of one and two-fold Darboux transformation.

Introduction

Nonlinear Schrödinger equation (NLS) is one classical integrable nonlinear equations. It appears in a variety of physical areas like nonlinear optics, plasma physics, fluid mechanics as well as in purely mathematical context like differential geometry of curve. Nonlinearity is the fascinating subjects which has many applications in almost all areas of science. Usually nonlinear phenomena are modeled by nonlinear ordinary or partial differential equations. Many of these nonlinear differential equations (NDE) are completely integrable. This means that these integrable NDS have some classes of interesting exact solutions such as solitons, dromions,rogue waves, similaritions and so on. They are of great mathematical as well as physical interest and the investigation of the solitons become one of the most exciting and extremely active areas of research in modern science and technology in the past several decades. In particular, many of the completely integrable NDE have been found and studied [1]-[5].

Lax pair of the (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation

In the section we consider the (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation, which reads as

$$iq_t(x,t)+iq_{xx}(x,t)+2q(x,t)q^*(-x,t)q(x,t)=0, \tag{1}$$

$$iq_t^*(-x,t)+iq_{xx}^*(x,t)-2q^*(-x,t)q(x,t)q^*(-x,t)=0, \tag{2}$$

where $q(x,t)$, $q^*(-x,t)$ are complex functions of the real variables x,t denote partial derivatives with respect to the variables. It is obvious that self-induced potential $\psi(x,t)=q(x,t)q^*(x,t)$ is PT symmetric: $q(x,t)=q^*(-x,t)$.

These equations are studied in works M.J. Ablowitz1 and Ziad H. Musslimani, Li-Yuan Ma, Zuo-Nong Zhu, T. A. Gadzhimuradov A. M. Agalarov [6]-[12] and other.

We will concentrate on the linear eigenvalue problem of the (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation (1)-(2). The linear problem is expressed in the form of the Lax pair A and B as

$$\psi_x = A\psi, \quad (3)$$

$$\psi_t = B\psi, \quad (4)$$

where $\psi = \begin{pmatrix} \psi_1(x,t,\lambda) \\ \psi_2(x,t,\lambda) \end{pmatrix}$ is the vector eigenfunction, λ is a spectral parameter and A, B matrices 2×2 have the form

$$A = -i\lambda\sigma_3 + A_0, \qquad B = \lambda^2 B_2 + \lambda B_1 + B_0 .$$

Here $\sigma_3, A_0, B_2, B_1, B_0$ are 2×2 matrices:

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad A_0 = \begin{pmatrix} 0 & q(x,t) \\ -q^*(-x,t) & 0 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} -2i & 0 \\ 0 & 2i \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0 & 2q(x,t) \\ -2q^*(-x,t) & 0 \end{pmatrix}, \quad B_0 = i\begin{pmatrix} q(x,t)q^*(-x,t) & q_x(x,t) \\ -q^*{}_x(-x,t) & -q(x,t)q^*(-x,t) \end{pmatrix}$$

cross derivatives $\psi_{xt} = \psi_{tx}$ from (3), (4).

This equation (1) - (2) is integrable by the inverse scattering method [1]. The inverse scattering method implies that the partial differential equation can be represented as a Lax pair, a linear system of two operators A and B from (3) - (4) whose compatibility condition is the system (1) - (2) under consideration

$$A_t - B_x + AB - BA = 0 .$$

In this next section, we construct one-fold DT of the (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation [6]-[15].

One-fold Darboux transformation for the (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation

Darboux transformations constitute one of the most fruitful approaches to the construction of soliton type solutions of integrable nonlinear equations. DT transform solutions of partial differential equations into solutions of same class partial differential equations.

We consider the following transformation of the system of equation (3)-(4)

$$\psi^{[1]} = T\psi = (\lambda I - M)\psi, \tag{5}$$

such that the new function $\psi^{[1]}$ satisfies system

$$\begin{aligned} \psi_x^{[1]} &= A^{[1]}\psi^{[1]'}, \\ \psi_t^{[1]} &= B^{[1]}\psi^{[1]} \end{aligned} \tag{6}$$

where matrices $A^{[1]}$ and $B^{[1]}$ depend on functions $q^{[1]}(x,t), q^{*[1]}(-x,t)$ and λ. Here M and I are matrices have the form

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The relation between $q^{[1]}(x,t), q^{*[1]}(-x,t)$ and $A^{[1]} - B^{[1]}$ is the same as the relation between functions $q(x,t)$, $q^*(-x,t)$ and $A - B$. In order the equations (6) to hold , Darboux matrix T must satisfy the following equations

$$\lambda^0: \quad M_x = A_0^{[1]}M - MA_0, \tag{7}$$

$$\lambda^1: \quad A_0^{[1]} = A_0 + i[M, \sigma_3], \tag{8}$$

$$\lambda^2: \quad iI\sigma_3 = iI\sigma_3. \tag{9}$$

Finally, from identities (7)-(9) we obtain solutions of equations (1)-(2)

$$q^{[1]} = q - 2im_{12}, \quad q^{*[1]} = q^* - 2im_{21}, \tag{10}$$

Hence we get conclusion $m_{21} = -m_{12}^*$ in our attractive interaction case. Then, comparing the coefficients of λ^i $(i = 0, 1, 2)$ of the two sides of the equation (6) gives us of the following identities

$$\lambda^0: \quad M_t = B_0^{[1]}M - MB_0, \tag{11}$$

$$\lambda^1: \quad B_0 - MB_1 = B_0^{[1]} - B_1^{[1]}M, \tag{12}$$

$$\lambda^2: \quad B_1 - MB_2 = B_1^{[1]} - B_2^{[1]}M. \tag{13}$$

Then the system of equations (11)-(13) gives

$$iq^{[1]}q^{*[1]} - 2q^{[1]}m_{21} - 2q^*m_{12} - iqq^* = 0,$$
$$iq^{[1]}q^{*[1]} - 2q^{[1]}m_{12} - 2qm_{21} - iqq^* = 0,$$
$$iq_x^{[1]} - 2q^{[1]}m_{22} + 2qm_{11} - iq_x = 0,$$
$$iq_1^{*[1]} - 2q^{*[1]}m_{11} + 2q^*m_{22} - iq_x^* = 0.$$

These equation (7)-(13) give one-fold transformation of the (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation (1)-(2).

We now assume that matrix M in the canonical form

$$M = H\Lambda H^{-1}, \tag{14}$$

where $\det H \neq 0$ and H, Λ has form

$$H = \begin{pmatrix} \psi_1(\lambda_1, x, t) & \psi_1(\lambda_2, x, t) \\ \psi_2(\lambda_1, x, t) & \psi_2(\lambda_2, x, t) \end{pmatrix} = \begin{pmatrix} \psi_{1,1} & \psi_{1,2} \\ \psi_{2,1} & \psi_{2,2} \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \tag{15}$$

and λ_1, λ_2 are complex constants. In order to satisfy the constants of A_0 as mentioned above, then

$$\psi^+ = \psi^{-1}, \quad A_0^+ = -A_0,$$

$$\lambda_2 = -\lambda_1^{-1}, \quad H = \begin{pmatrix} \psi_1(\lambda_1, x, t) & -\psi_2^*(\lambda_1, x, t) \\ \psi_2(\lambda_1, x, t) & \psi_1^*(\lambda_1, x, t) \end{pmatrix} = \begin{pmatrix} \psi_1 & -\psi_2^* \\ \psi_2 & \psi_1^* \end{pmatrix}, \quad H^{-1} = \frac{1}{\Delta}\begin{pmatrix} \psi_1^* & \psi_2^* \\ \psi_2 & \psi_1 \end{pmatrix}, \tag{16}$$

where $\Delta = |\psi_1|^2 + |\psi_2|^2$. Also matrix H satisfy Lax pair (3), (4)

$$H_x = -i\sigma_3 H\Lambda + A_0 H,$$
$$H_t = -2\sigma_3 H\Lambda^2 + B_1 H\Lambda + B_0 H.$$

From (14) use formula (15), (16) we get the values of the matrix M

$$M=\frac{1}{\Delta}\begin{pmatrix}\lambda_1|\psi_1|^2-\lambda_1^*|\psi_2|^2 & (\lambda_1+\lambda_1^*)\psi_1\psi_2^* \\ (\lambda_1+\lambda_1^*)\psi_2\psi_1^* & \lambda_1|\psi_2|^2-\lambda_1^*|\psi_1|^2\end{pmatrix}.$$

In the following section we give the determinant representation of the one-fold and two-fold DT for the (1+1)-dimensional nonlocal nonlinear Schrödinger equation.

Determinant representation of Darboux transformation for the (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation

Here the determinant representation is constructed for the one-fold and two-fold DT of the (1+1)-dimensional nonlocal nonlinear Schrödinger equation. The reduction condition on the eigenfunctions are $\psi_{2,2i}=\psi^*_{1,2i-1}$, $\psi_{2,2i-1}=-\psi^*_{1,2i}$ and for the eigenvalues are $\lambda_{2i}=\lambda^*_{2i-1}$.

The determinant representation of the one-fold DT of the (1+1)-dimensional nonlocal nonlinear Schrödinger equation formulate the following theorem (as papers [13]-[16]).

Theorem 1. The one-fold DT of the (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation is

$$T_1(\lambda,\lambda_1,\lambda_2)=\lambda I-M=\lambda I+t_0^{[1]}=\frac{1}{\Delta_1}\begin{pmatrix}(T_1)_{11} & (T_1)_{12}\\ (T_1)_{21} & (T_1)_{22}\end{pmatrix}, \quad (17)$$

where

$$t_0^{[1]}=\frac{1}{\Delta_1}\begin{pmatrix}\begin{vmatrix}\psi_{2,1} & \lambda_1\psi_{1,1}\\ \psi_{2,2} & \lambda_2\psi_{1,2}\end{vmatrix} & -\begin{vmatrix}\psi_{1,1} & \lambda_1\psi_{1,1}\\ \psi_{1,2} & \lambda_2\psi_{1,2}\end{vmatrix}\\ \begin{vmatrix}\psi_{2,1} & \lambda_1\psi_{2,1}\\ \psi_{2,2} & \lambda_2\psi_{2,2}\end{vmatrix} & -\begin{vmatrix}\psi_{1,1} & \lambda_1\psi_{2,1}\\ \psi_{1,2} & \lambda_2\psi_{2,2}\end{vmatrix}\end{pmatrix},\quad \Delta_1=\begin{vmatrix}\psi_{1,1} & \psi_{1,2}\\ \psi_{2,1} & \psi_{2,2}\end{vmatrix}, \quad (18)$$

$$(T_1)_{11}=\begin{vmatrix}1 & 0 & \lambda\\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1}\\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2}\end{vmatrix},\quad (T_1)_{12}=\begin{vmatrix}0 & 1 & 0\\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1}\\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2}\end{vmatrix}, \quad (19a)$$

$$(T_1)_{21}=\begin{vmatrix}1 & 0 & 0\\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{2,1}\\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{2,2}\end{vmatrix},\quad (T_1)_{22}=\begin{vmatrix}0 & 1 & \lambda\\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{2,1}\\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{2,2}\end{vmatrix}. \quad (19b)$$

The matrix T_1 satisfies the following equations of system

$$T_{1x}+T_1A=A^{[1]}T_1, \quad (20a)$$

$$T_{1t}+T_1B=B^{[1]}T_1. \quad (20b)$$

From (20) we get for $A_0^{[1]}$:

$$A_0^{[1]}=A_0+[\sigma_3,t_0^{[1]}]. \quad (21)$$

Then the solutions of the system (1)-(2) have the form

$$q^{[1]}=q-2im_{12}=q+2i(t_0^{[1]})_{12}=q-2i\frac{(T_1)_{12}}{\Delta_1}, \quad (22a)$$

$$q^{*[1]} = q^* - 2im_{21} = q^* + 2i(t_0^{[1]})_{21} = q^* - 2i\frac{(T_1)_{21}}{\Delta_1}. \quad (22b)$$

We can find the transformation T_1 which has the following property

$$T_1(\lambda, \lambda_1, \lambda_2)\Big|_{\lambda=\lambda_1}\begin{pmatrix}\psi_{1,1}\\ \psi_{2,1}\end{pmatrix} = 0. \quad (23)$$

Now we prove the theorem.
Proof of the theorem.
From the formulae (5), (14), (23) it follows that

$$M = \frac{1}{\Delta_1}\begin{pmatrix}\lambda_1\psi_{1,1}\psi_{2,2} - \lambda_2\psi_{1,2}\psi_{2,1} & (\lambda_2 - \lambda_1)\psi_{1,1}\psi_{1,2}\\ (\lambda_1 - \lambda_2)\psi_{2,1}\psi_{2,2} & \lambda_1\psi_{1,2}\psi_{2,1} + \lambda_2\psi_{1,1}\psi_{2,2}\end{pmatrix}.$$

From equation (17), we have

$$T_1(\lambda, \lambda_1, \lambda_2) = \lambda I - M = \frac{1}{\Delta_1}\begin{pmatrix}\lambda\Delta_1 - \begin{vmatrix}\psi_{2,1} & \lambda_1\psi_{1,1}\\ \psi_{2,2} & \lambda_2\psi_{1,2}\end{vmatrix} & -\begin{vmatrix}\psi_{1,1} & \lambda_1\psi_{1,1}\\ \psi_{1,2} & \lambda_2\psi_{1,2}\end{vmatrix}\\ \begin{vmatrix}\psi_{2,1} & \lambda_1\psi_{2,1}\\ \psi_{2,2} & \lambda_2\psi_{2,2}\end{vmatrix} & \lambda\Delta_1 + \begin{vmatrix}\psi_{1,1} & \lambda_1\psi_{2,1}\\ \psi_{1,2} & \lambda_2\psi_{2,2}\end{vmatrix}\end{pmatrix}$$

$$\lambda I - M = \lambda I + t_0^{[1]} = \frac{1}{\Delta_1}\begin{pmatrix}\lambda\Delta_1 - \begin{vmatrix}\psi_{2,1} & \lambda_1\psi_{1,1}\\ \psi_{2,2} & \lambda_2\psi_{1,2}\end{vmatrix} & -\begin{vmatrix}\psi_{1,1} & \lambda_1\psi_{1,1}\\ \psi_{1,2} & \lambda_2\psi_{1,2}\end{vmatrix}\\ \begin{vmatrix}\psi_{2,1} & \lambda_1\psi_{2,1}\\ \psi_{2,2} & \lambda_2\psi_{2,2}\end{vmatrix} & \lambda\Delta_1 + \begin{vmatrix}\psi_{1,1} & \lambda_1\psi_{2,1}\\ \psi_{1,2} & \lambda_2\psi_{2,2}\end{vmatrix}\end{pmatrix}$$

and the elements of the matrix are as follows:

$$(T_1)_{11} = \begin{vmatrix}\psi_{2,1} & \lambda_1\psi_{1,1}\\ \psi_{2,2} & \lambda_2\psi_{1,2}\end{vmatrix} - \lambda\Delta_1 = \begin{vmatrix}1 & 0 & \lambda\\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1}\\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2}\end{vmatrix},$$

$$(T_1)_{12} = -\begin{vmatrix}\psi_{1,1} & \lambda_1\psi_{1,1}\\ \psi_{1,2} & \lambda_2\psi_{1,2}\end{vmatrix} = \begin{vmatrix}0 & 1 & 0\\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1}\\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2}\end{vmatrix},$$

$$(T_1)_{21} = \begin{vmatrix}\psi_{2,1} & \lambda_1\psi_{2,1}\\ \psi_{2,2} & \lambda_2\psi_{2,2}\end{vmatrix} = \begin{vmatrix}1 & 0 & 0\\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{2,1}\\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{2,2}\end{vmatrix},$$

$$(T_1)_{22} = -\begin{vmatrix}\psi_{1,1} & \lambda_1\psi_{2,1}\\ \psi_{1,2} & \lambda_2\psi_{2,2}\end{vmatrix} - \lambda\Delta_1 = \begin{vmatrix}0 & 1 & \lambda\\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{2,1}\\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{2,2}\end{vmatrix},$$

And we get (19). Further substituting (17) into (20a), we obtain

$$t_{0x}^{[1]} + \lambda A_0 - i\lambda^2\sigma_3 + \lambda t_0^{[1]}\sigma_3 + t_0^{[1]}A_0 = -i\lambda^2\sigma_3 + \lambda A_0^{[1]} - i\lambda\sigma_3 t_0^{[1]} + A_0^{[1]}t_0^{[1]}. \quad (24)$$

Comparing the coefficients of λ^i $(i = 0, 1, 2)$ of the two sides of the equation (24), we get

$$\lambda^0: \qquad t_{0x}^{[1]} + t_0^{[1]}A_0 = A_0^{[1]}t_0^{[1]}. \quad (25a)$$

$$\lambda^1: \qquad A_0 - it_0^{[1]}\sigma_3 = A_0^{[1]} - i\sigma_3 t_0^{[1]}. \quad (25b)$$

$$\lambda^2: \qquad -i\sigma_3 I = -i\sigma_3 I \quad (25c)$$

From (25b) we obtain the first identity (21). Further substituting the elements of the matrices A_0, $t_0^{[1]}$, σ_3, we obtain the solution of equation (1), (2) in the form (22).

Thus Theorem 1 is proved.

In same way theorem 1 we can formulate the next theorem.

Theorem 2. The two-fold DT of the (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation is

$$T_2(\lambda, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = \lambda^2 I + \lambda t_1^{[2]} + t_0^{[2]} = \frac{1}{\Delta_2}\begin{pmatrix} (T_2)_{11} & (T_2)_{12} \\ (T_2)_{21} & (T_2)_{22} \end{pmatrix}, \quad (26)$$

$$(T_2)_{11} = \begin{pmatrix} 1 & 0 & \lambda & 0 & \lambda^2 \\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1} & \lambda_1\psi_{2,1} & \lambda_1^2\psi_{1,1} \\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2} & \lambda_2\psi_{2,2} & \lambda_2^2\psi_{1,2} \\ \psi_{1,3} & \psi_{2,3} & \lambda_3\psi_{1,3} & \lambda_3\psi_{2,3} & \lambda_3^2\psi_{1,3} \\ \psi_{1,4} & \psi_{2,4} & \lambda_4\psi_{1,4} & \lambda_4\psi_{2,4} & \lambda_4^2\psi_{1,4} \end{pmatrix}, \quad (T_2)_{12} = \begin{pmatrix} 0 & 1 & 0 & \lambda & 0 \\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1} & \lambda_1\psi_{2,1} & \lambda_1^2\psi_{1,1} \\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2} & \lambda_2\psi_{2,2} & \lambda_2^2\psi_{1,2} \\ \psi_{1,3} & \psi_{2,3} & \lambda_3\psi_{1,3} & \lambda_3\psi_{2,3} & \lambda_3^2\psi_{1,3} \\ \psi_{1,4} & \psi_{2,4} & \lambda_4\psi_{1,4} & \lambda_4\psi_{2,4} & \lambda_4^2\psi_{1,4} \end{pmatrix},$$

$$(T_2)_{21} = \begin{pmatrix} 1 & 0 & \lambda & 0 & 0 \\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1} & \lambda_1\psi_{2,1} & \lambda_1^2\psi_{2,1} \\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2} & \lambda_2\psi_{2,2} & \lambda_2^2\psi_{2,2} \\ \psi_{1,3} & \psi_{2,3} & \lambda_3\psi_{1,3} & \lambda_3\psi_{2,3} & \lambda_3^2\psi_{2,3} \\ \psi_{1,4} & \psi_{2,4} & \lambda_4\psi_{1,4} & \lambda_4\psi_{2,4} & \lambda_4^2\psi_{2,4} \end{pmatrix}, \quad (T_2)_{22} = \begin{pmatrix} 0 & 1 & 0 & \lambda & \lambda^2 \\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1} & \lambda_1\psi_{2,1} & \lambda_1^2\psi_{2,1} \\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2} & \lambda_2\psi_{2,2} & \lambda_2^2\psi_{2,2} \\ \psi_{1,3} & \psi_{2,3} & \lambda_3\psi_{1,3} & \lambda_3\psi_{2,3} & \lambda_3^2\psi_{2,3} \\ \psi_{1,4} & \psi_{2,4} & \lambda_4\psi_{1,4} & \lambda_4\psi_{2,4} & \lambda_4^2\psi_{2,4} \end{pmatrix},$$

where

$$\Delta_2 = \begin{pmatrix} \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1} & \lambda_1\psi_{2,1} \\ \psi_{1,2} & \psi_{2,,2} & \lambda_2\psi_{1,2} & \lambda_2\psi_{2,2} \\ \psi_{1,3} & \psi_{2,3} & \lambda_3\psi_{1,3} & \lambda_3\psi_{2,3} \\ \psi_{1,4} & \psi_{2,4} & \lambda_4\psi_{1,4} & \lambda_4\psi_{2,4} \end{pmatrix}$$

T_2 satisfies the following equations

$$T_{2x} + T_2 A = A^{[2]}T_2, \quad (27a)$$

$$T_{2t} + T_2 B = B^{[2]}T_2. \quad (27b)$$

From formula (27) we obain

$$A_0^{[2]} = A_0 + [\sigma_3, t_0^{[2]}]. \tag{28}$$

Then the $q^{[2]}$, $q^{*[2]}$ solutions of the system (1)-(2) have the form

$$q^{[2]} = q - 2i\frac{(T_2)_{12}}{\Delta_2}, \quad q^{*[2]} = q^* - 2i\frac{(T_2)_{21}}{\Delta_2}. \tag{29}$$

We can find the transformation T_2 which satisfies the following property

$$T_2(\lambda, \lambda_1, \lambda_2, \lambda_3, \lambda_4)\Big|_{\lambda=\lambda_i}\begin{pmatrix}\psi_{1,i}\\ \psi_{2,i}\end{pmatrix} = 0, \quad i = 1,\ 2,\ 3, 4. \tag{30}$$

The proof of Theorem 2 is analogous to Theorem 1.

Thus we gave determinant representations are given for one-fold and two-fold DT for the (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation (1)-(2).

Conclusion

In this paper, we have constructed the DT for the (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation. Using the derived DT, some exact solutions like, the soliton solution will obtained. The determinant representations are given for one-fold and two-fold DT for the (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation. It is interesting to note that the rogue wave soliton of nonlinear equations are currently one of the hottest topics in nonlinear physics and mathematics. The application of the obtained solutions in physics is an interesting subject. In particular, wave hope that the presented solutions may be used in experiments or optical fibre communication. Also we will study some important generalizations of the (1+1)-dimensional nonlocal focusing nonlinear Schrödinger equation in future [10]-[16].

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(1+1)-ӨЛШЕМДІ ЛОКАЛДЫ ЕМЕС ФОКУСТАЛҒАН СЫЗЫҚТЫ ЕМЕС ШРЕДИНГЕР ТЕҢДЕУІ ҮШІН ДАРБУ ТҮРЛЕНДІРУІ

Аннотация. Осы мақалада біз (1+1)-өлшемді локалды емес фокусталған сызықты емес Шредингер теңдеуін қарастырдық. Бұл теңдеулерді M.J. Ablowitz1 and Ziad H. Musslimani, Li-Yuan Ma, Zuo-Nong Zhu, T. A. Gadzhimuradov A. M. Agalarov және т.б. ғалымдар зерттеді. Локалды емес фокусталған сызықты емес Шредингер теңдеуі кері шашырау әдісімен интегралданады. Біз осы теңдеуге Дарбу түрлендіруін құрдық. Сонымен қатар, біз (1+1)-өлшемді локалды емес фокусталған сызықты емес Шредингер теңдеуі үшін бір және екі ретті анықтауыш түріндегі Дарбу түрлендіруін алдық. Осы теңдеудің трансформациясын қолдану ретінде, солитондық түрдегі шешімдерді тривиалды «seed» шешімнен аламыз.

Тірек сөздер: (1+1)-өлшемді локалды емес фокусталған сызықты емес Шредингер теңдеуі, интегралданатын жүйе, Лакс жұбы, Дарбу түрлендіруі, анықтауыш түріндегі бір және екі-еселі Дарбу түрлендіруі.

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ПРЕОБРАЗОВАНИЯ ДАРБУ ДЛЯ (1+1)-МЕРНОГО НЕЛОКАЛЬНОГО ФОКУСИРОВАННОГО НЕЛИНЕЙНОГО УРАВНЕНИЯ ШРЕДИНГЕРА

Аннотация. В этой статье мы рассмотрим (1+1)-мерное нелокальное фокусированное нелинейное уравнение Шредингера. Эти уравнения изучаются в работах M.J. Ablowitz and Ziad H. Musslimani, Li-Yuan Ma, Zuo-Nong Zhu, T. A. Gadzhimuradov A. M. Agalarov и др. Нелокальное нелинейное уравнение Шредингера интегрируется с методами обратные задачи рассеяния. Мы построили преобразование Дарбу этого уравнения. Кроме того, мы получим представление детерминанта однократного и двукратного преобразования Дарбу для (1+1)-мерного нелокального фокусированного уравнения Шредингера. В качестве применения Дарбу преобразования этого уравнения типа солитонного решения получаются из тривиальных «seed» решений.

Ключевые слова: (1+1)–мерное нелокальное фокусированное нелинейное уравнение Шредингера, интегрируемые системы, пара Лакса, представление детерминанта одно и двух-кратного преобразования Дарбу.