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ON THE INITIAL-BOUNDARY VALUE PROBLEM FOR SYSTEM OF THE PARTIAL DIFFERENTIAL EQUATIONS OF FOURTH ORDER

Abstract. A initial-boundary value problem for system of the partial differential equations of fourth order is considered. We study the existence of classical solutions to the initial-boundary value problem for system of the partial differential equations of fourth order and offer the methods for finding its approximate solutions. Sufficient conditions for the existence and uniqueness of a classical solution to the initial-boundary value problem for system of the partial differential equations of fourth order are set. By introducing of a new unknown functions, we reduce the considered problem to an equivalent problem consisting of a nonlocal problem for the system of hyperbolic equations of second order with functional parameters and the integral relations. We offer the algorithm for finding an approximate solution to the investigated problem and prove its convergence. Sufficient conditions for the existence of unique solution to the equivalent problem with parameters are established. Conditions of unique solvability to the initial-boundary value problem for system of the partial differential equations of fourth order are obtained in the terms of initial data. Separately, the result is given for the initial-periodic in time boundary value problem.

Keywords: system of the partial differential equations of fourth order, initial-boundary value problem, nonlocal problem, system of the hyperbolic equations of second order, solvability, algorithm.

1. Introduction. Currently, the problems of mathematical physics connected with the description of wave motion of liquids of different nature are drawn by great attention. This interest is caused not only by big applied importance of these problems, but their new theoretical and mathematical content often do not have analogues in the classical mathematical physics. One of the important classes of such problems are the initial-boundary value problems for fourth order partial differential equations. To date, various methods for researching and solving the initial-boundary value problems for fourth order partial differential equations of hyperbolic and composite types are developed in [1-12]. In order to investigate various boundary value problems for fourth order partial differential equations along with the classical methods of mathematical physics (the Fourier method, the method of Green's functions, Poincare's metric concept) we apply the method of differential inequalities and other methods of qualitative theory of ordinary differential equations. Based on them, the conditions for solvability of considered boundary value problems are obtained, and the ways for finding their solutions are offered. Fourth order system of partial differential equations began to be studied relatively recently.

In the present work we consider system of the partial differential equations of fourth order at the rectangular domain. Boundary condition for time variable are specified as a combination of values from the partial derivatives of required solution on third orders by spatial variable. We investigate the questions of existence and uniqueness of the classical solution to initial-boundary value problem for system of the partial differential equations of fourth order and its applications.

2. Methods. For solve to considered problem we use a method of introduction additional functional parameters [13-29]. The original problem is reduced to an equivalent problem consisting of nonlocal problem for system of the hyperbolic equations of second order with functional parameters and integral relations. Sufficient conditions for the unique solvability to investigated problem are established in the terms of initial data. Algorithms for finding solution to the equivalent problem are constructed. Conditions of unique solvability to initial-boundary value problem for system of partial differential equations of fourth order are established in the terms of system's coefficients and boundary matrices. Separately, the result is given for the initial-periodic in time boundary value problem.

Note that, in [30, 31] a similar approach has been applied to the initial-boundary value problem and nonlocal problem for the system of partial differential equations of third order.

2. Statement of problem. At the domain $\Omega = [0,T]\times[0,\omega]$ we consider the initial-periodic boundary value problem for system of the partial differential equations of fourth order in the following form

$$\frac{\partial^4 u}{\partial t\partial x^3} = A_1(t,x)\frac{\partial^3 u}{\partial x^3} + A_2(t,x)\frac{\partial^3 u}{\partial t\partial x^2} + A_3(t,x)\frac{\partial^2 u}{\partial x^2} + A_4(t,x)\frac{\partial^2 u}{\partial t\partial x} + A_5(t,x)\frac{\partial u}{\partial x} +$$
$$+ A_6(t,x)\frac{\partial u}{\partial t} + A_7(t,x)u + f(t,x), \qquad (t,x)\in\Omega, \tag{1}$$

$$\frac{\partial^3 u(0,x)}{\partial x^3} = K(x)\frac{\partial^3 u(T,x)}{\partial x^3} + \varphi(x), \qquad x\in[0,\omega], \tag{2}$$

$$u(t,0) = \psi_0(t), \qquad t\in[0,T], \tag{3}$$

$$\frac{\partial u(t,x)}{\partial x}\Big|_{x=0} = \psi_1(t), \qquad t\in[0,T], \tag{4}$$

$$\frac{\partial^2 u(t,x)}{\partial x^2}\Big|_{x=0} = \psi_2(t), \qquad t\in[0,T], \tag{5}$$

where $u(t,x) = col(u_1(t,x), u_2(t,x), \ldots, u_n(t,x))$ is unknown function, the $n\times n$-matrices $A_i(t,x)$, $i=\overline{1,7}$, and n-vector function $f(t,x)$ are continuous on Ω, the $n\times n$-matrix $K(x)$ and n-vector-function $\varphi(x)$ are continuous on $[0,\omega]$, the n-vector-functions $\psi_0(t)$, $\psi_1(t)$ and $\psi_2(t)$ are continuously differentiable on $[0,T]$. The initial data satisfy the condition of approval.

A function $u(t,x)\in C(\Omega,R^n)$ having partial derivatives $\frac{\partial u(t,x)}{\partial x}\in C(\Omega,R^n)$, $\frac{\partial u(t,x)}{\partial t}\in C(\Omega,R^n)$, $\frac{\partial^2 u(t,x)}{\partial x^2}\in C(\Omega,R^n)$, $\frac{\partial^2 u(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$, $\frac{\partial^3 u(t,x)}{\partial x^3}\in C(\Omega,R^n)$, $\frac{\partial^3 u(t,x)}{\partial t\partial x^2}\in C(\Omega,R^n)$, $\frac{\partial^4 u(t,x)}{\partial t\partial x^3}\in C(\Omega,R^n)$, is called a classical solution to problem (1)--(5) if it satisfies system (1) for all $(t,x)\in\Omega$, and boundary conditions (2)--(5).

We will investigate the questions of existence and uniqueness of the classical solutions to the initial-boundary value problem for system of the partial differential equations of fourth order (1)--(5) and the approaches of constructing its approximate solutions. For this goals, we applied the method of introduction additional functional parameters proposed in [13-31] for solving of various nonlocal problems for systems of hyperbolic equations with mixed derivatives. Considered problem is provided to nonlocal problem for the system of hyperbolic equations of second order including additional functions and integral relation. The algorithm for finding the approximate solution of the investigated problem is proposed and its convergence proved. Sufficient conditions of the existence unique classical solution to problem (1)--(5) are obtained in the terms of initial data.

3. Scheme of the method and reduction to equivalent problem. We introduce a new unknown functions $v(t,x)=\dfrac{\partial u(t,x)}{\partial x}$, $w(t,x)=\dfrac{\partial^2 u(t,x)}{\partial x^2}$ and rewrite the problem (1)--(5) in the following form

$$\frac{\partial^2 w}{\partial t\partial x}=A_1(t,x)\frac{\partial w}{\partial x}+A_2(t,x)\frac{\partial w}{\partial t}+A_3(t,x)w+f(t,x)+$$

$$+A_4(t,x)\frac{\partial v(t,x)}{\partial t}+A_5(t,x)v(t,x)+A_6(t,x)\frac{\partial u(t,x)}{\partial t}+A_7(t,x)u(t,x),\qquad (t,x)\in\Omega, \qquad (6)$$

$$\frac{\partial w(0,x)}{\partial x}=K(x)\frac{\partial w(T,x)}{\partial x}+\varphi(x),\qquad x\in[0,\omega], \qquad (7)$$

$$w(t,0)=\psi_2(t),\qquad t\in[0,T], \qquad (8)$$

$$v(t,x)=\psi_0(t)+\int_0^x w(t,\xi)d\xi,\quad u(t,x)=\psi_0(t)+\psi_1(t)x+\int_0^x\int_0^\xi w(t,\xi_1)d\xi_1 d\xi,\quad (t,x)\in\Omega. \qquad (9)$$

Here the conditions (3) and (4) are taken into account in (9).

A triple functions $(w(t,x),v(t,x),u(t,x))$, where the function $w(t,x)\in C(\Omega,R^n)$ has partial derivatives $\dfrac{\partial w(t,x)}{\partial x}\in C(\Omega,R^n)$, $\dfrac{\partial w(t,x)}{\partial t}\in C(\Omega,R^n)$, $\dfrac{\partial^2 w(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$, the functions $v(t,x)\in C(\Omega,R^n)$ and $u(t,x)\in C(\Omega,R^n)$ have partial derivatives $\dfrac{\partial v(t,x)}{\partial x}\in C(\Omega,R^n)$, $\dfrac{\partial v(t,x)}{\partial t}\in C(\Omega,R^n)$, $\dfrac{\partial^2 v(t,x)}{\partial x^2}\in C(\Omega,R^n)$, $\dfrac{\partial^2 v(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$, $\dfrac{\partial^3 v(t,x)}{\partial t\partial x^2}\in C(\Omega,R^n)$, $\dfrac{\partial u(t,x)}{\partial x}\in C(\Omega,R^n)$, $\dfrac{\partial u(t,x)}{\partial t}\in C(\Omega,R^n)$, $\dfrac{\partial^2 u(t,x)}{\partial x^2}\in C(\Omega,R^n)$, $\dfrac{\partial^2 u(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$, $\dfrac{\partial^3 u(t,x)}{\partial t\partial x^2}\in C(\Omega,R^n)$, $\dfrac{\partial^3 u(t,x)}{\partial x^3}\in C(\Omega,R^n)$, $\dfrac{\partial^4 u(t,x)}{\partial t\partial x^3}\in C(\Omega,R^n)$, is called a solution to problem (6)--(9) if it satisfies of the system of hyperbolic equations of second order (6) for all $(t,x)\in\Omega$, the boundary conditions (7), (8), and the integral relations (9).

At fixed $v(t,x)$ and $u(t,x)$ the problem (6)--(8) is a nonlocal problem for the system of hyperbolic equations with respect to $w(t,x)$ on Ω. The integral relations (9) allow us to determine the unknown functions $v(t,x)$ and $u(t,x)$ for all $(t,x)\in\Omega$.

4. Algorithm. The unknown function $w(t,x)$ will be determined from nonlocal problem for the system of hyperbolic equations (6)--(8). The unknown functions $v(t,x)$ and $u(t,x)$ will be found from integral relations (9).

If we know the functions $v(t,x)$ and $u(t,x)$, then from nonlocal problem (6)--(8) find the function $w(t,x)$. Conversely, if we known the functions $v(t,x)$ and $u(t,x)$, then from nonlocal problem (6)--(8) we find the function $w(t,x)$. Since the functions $v(t,x)$, $u(t,x)$ and $v(t,x)$ are unknowns together, for finding of the solution to problem (6)--(9) we use an iterative method. The solution to problem (6)--(9) is the triple functions $(w^*(t,x),v^*(t,x),u^*(t,x))$ we defined as a limit of sequence of triples $(w^{(k)}(t,x),v^{(k)}(t,x),u^{(k)}(t,x))$, $k=0,1,2,\ldots$, according to the following algorithm:

Step 0. 1) Suppose in the right-hand part of the system (6) $\frac{\partial v(t,x)}{\partial t} = \dot{\psi}_0(t)$, $v(t,x) = \psi_0(t)$, $\frac{\partial u(t,x)}{\partial t} = \dot{\psi}_0(t) + \dot{\psi}_1(t)x$, and $u(t,x) = \psi_0(t) + \psi_1(t)x$, from nonlocal problem (6)--(8) we find the initial approximation $w^{(0)}(t,x)$ for all $(t,x) \in \Omega$;

2) From the integral relations (9) under $w(t,x) = w^{(0)}(t,x)$, we find the functions $v^{(0)}(t,x)$ and $u^{(0)}(t,x)$ for all $(t,x) \in \Omega$.

Step 1. 1) Suppose in the right-hand part of system (6) $\frac{\partial v(t,x)}{\partial t} = \frac{\partial v^{(0)}(t,x)}{\partial t}$, $v(t,x) = v^{(0)}(t,x)$, $\frac{\partial u(t,x)}{\partial t} = \frac{\partial u^{(0)}(t,x)}{\partial t}$, and $u(t,x) = u^{(0)}(t,x)$, from nonlocal problem (6)--(8) we find the first approximation $w^{(1)}(t,x)$ for all $(t,x) \in \Omega$.

2) From the integral relations (9) under $w(t,x) = w^{(1)}(t,x)$, we find the functions $v^{(1)}(t,x)$ and $u^{(1)}(t,x)$ for all $(t,x) \in \Omega$.

And so on.

Step k. 1) Suppose in the right-hand part of system (6) $\frac{\partial v(t,x)}{\partial t} = \frac{\partial v^{(k-1)}(t,x)}{\partial t}$, $v(t,x) = v^{(k-1)}(t,x)$, $\frac{\partial u(t,x)}{\partial t} = \frac{\partial u^{(k-1)}(t,x)}{\partial t}$, and $u(t,x) = u^{(k-1)}(t,x)$, from nonlocal problem (6)--(8) we find the k-th approximation $w^{(k)}(t,x)$ for all $(t,x) \in \Omega$:

$$\frac{\partial^2 w^{(k)}}{\partial t \partial x} = A_1(t,x)\frac{\partial w^{(k)}}{\partial x} + A_2(t,x)\frac{\partial w^{(k)}}{\partial t} + A_3(t,x)w^{(k)} + f(t,x) +$$

$$+ A_4(t,x)\frac{\partial v^{(k-1)}(t,x)}{\partial t} + A_5(t,x)v^{(k-1)}(t,x) + A_6(t,x)\frac{\partial u^{(k-1)}(t,x)}{\partial t} + A_7(t,x)u^{(k-1)}(t,x), \quad (t,x) \in \Omega, \quad (10)$$

$$\frac{\partial w^{(k)}(0,x)}{\partial x} = K(x)\frac{\partial w^{(k)}(T,x)}{\partial x} + \varphi(x), \qquad x \in [0,\omega], \quad (11)$$

$$w^{(k)}(t,0) = \psi_2(t), \qquad t \in [0,T]. \quad (12)$$

2) From the integral relations (9) under $w(t,x) = w^{(k)}(t,x)$, we find the functions $v^{(k)}(t,x)$ and $u^{(k)}(t,x)$ for all $(t,x) \in \Omega$:

$$v^{(k)}(t,x) = \psi_0(t) + \int_0^x w^{(k)}(t,\xi)d\xi, \quad u^{(k)}(t,x) = \psi_0(t) + \psi_1(t)x + \int_0^x\int_0^\xi w^{(k)}(t,\xi_1)d\xi_1 d\xi, \quad (t,x) \in \Omega. \quad (13)$$

Here $k = 1,2,3,\ldots$.

5. The main results. The following theorem gives conditions of feasibility and convergence of the constructed algorithm and the conditions of the existence unique solution to problem (6)--(9).

Theorem 1. *Suppose that*

i) the $n \times n$*-matrices* $A_i(t,x)$, $i = \overline{1,7}$, *and* n*-vector function* $f(t,x)$ *are continuous on* Ω;

ii) the $n \times n$*-matrix* $K(x)$ *and* n*-vector-function* $\varphi(x)$ *are continuous on* $[0,\omega]$;

iii) the n*-vector-functions* $\psi_0(t)$, $\psi_1(t)$ *and* $\psi_2(t)$ *are continuously differentiable on* $[0,T]$;

iv) the $n \times n$*-matrix* $Q(x) = I - K(x)\left[I + \int_0^T A_1(\tau,x)d\tau\right]$ *is invertible for all* $x \in [0,\omega]$, *where* I *is unit matrix on dimension* n.

Then the nonlocal problem for system of the hyperbolic equations with parameters (6)--(9) has a unique solution $(w^*(t,x), v^*(t,x), u^*(t,x))$ *as a limit of sequences* $(w^{(k)}(t,x), v^{(k)}(t,x), u^{(k)}(t,x))$ *defining by the algorithm proposed above for* $k = 0,1,2,\ldots$.

Proof. Let the conditions *i) - iv)* of the Theorem be satisfied. From the 0th step of the above algorithm and Theorem 1 from [21] it follows that the nonlocal problem for system of the hyperbolic equations

$$\frac{\partial^2 w}{\partial t \partial x} = A_1(t,x)\frac{\partial w}{\partial x} + A_2(t,x)\frac{\partial w}{\partial t} + A_3(t,x)w + f(t,x) + A_4(t,x)\dot{\psi}_0(t) + A_5(t,x)\psi_0(t) +$$
$$+ A_6(t,x)[\dot{\psi}_0(t) + \dot{\psi}_1(t)x] + A_7(t,x)[\psi_0(t) + \psi_1(t)x], \qquad (t,x) \in \Omega, \qquad (14)$$

$$\frac{\partial w(0,x)}{\partial x} = K(x)\frac{\partial w(T,x)}{\partial x} + \varphi(x), \qquad x \in [0,\omega], \qquad (15)$$

$$w(t,0) = \psi_2(t), \qquad t \in [0,T] \qquad (16)$$

has a unique classical solution $w^{(0)}(t,x)$ for all $(t,x) \in \Omega$.

Further we determine the functions $v^{(0)}(t,x)$ and $u^{(0)}(t,x)$ from the integral relations

$$v^{(0)}(t,x) = \psi_0(t) + \int_0^x w^{(0)}(t,\xi)d\xi, \quad u^{(0)}(t,x) = \psi_0(t) + \psi_1(t)x + \int_0^x\int_0^\xi w^{(0)}(t,\xi_1)d\xi_1 d\xi$$

for all $(t,x) \in \Omega$. Functions $v^{(0)}(t,x)$ and $u^{(0)}(t,x)$ together with their partial derivatives $\frac{\partial v^{(0)}(t,x)}{\partial t}$ and $\frac{\partial u^{(0)}(t,x)}{\partial t}$, respectively, are continuous on Ω.

Continuing the iterative process according to the above algorithm, we define successive approximations $w^{(k)}(t,x)$, $v^{(k)}(t,x)$ and $u^{(k)}(t,x)$ for all $(t,x) \in \Omega$ and $k = 1,2,\ldots$.

The conditions *i) - iv)* of Theorem provide the uniform convergence on Ω of the sequences $\{w^{(k)}(t,x)\}$, $\{v^{(k)}(t,x)\}$ and $\{u^{(k)}(t,x)\}$ as $k \to \infty$ to functions $w^*(t,x)$, $v^*(t,x)$ and $u^*(t,x)$, respectively, for all $(t,x) \in \Omega$. In addition, there are finite limits of sequences of their partial derivatives as $k \to \infty$.

The triple founded functions $(w^*(t,x), v^*(t,x), u^*(t,x))$ has all the required continuous partial derivatives on Ω and be solution to problem (6)—(9). Uniqueness of solution to problem (6)—(9) is proved by method of contradiction.

Theorem 1 is proved.

From the equivalence of problems (6)—(9) and (1)—(5) it follows

Theorem 2. *Suppose that the conditions i) - iv) of Theorem 1 are fulfilled.*

Then the initial-periodic boundary value problem for system of the partial differential equations of fourth order (1)--(5) has a unique classical solution $u^*(t,x)$.

For $K(x) = I$ and $\varphi(x) = 0$ we obtain the initial-periodic boundary value problem for system of the partial differential equations of fourth order (1), (3)-(5) with condition

$$\frac{\partial^3 u(0,x)}{\partial x^3} = \frac{\partial^3 u(T,x)}{\partial x^3}, \qquad x \in [0,\omega]. \qquad (2')$$

Then the following assertion is true.

Theorem 3. *Suppose that*

1) the $n \times n$ *-matrices* $A_i(t,x)$, $i = \overline{1,7}$, *and* n *-vector function* $f(t,x)$ *are continuous on* Ω;

2) the n *-vector-functions* $\psi_0(t)$, $\psi_1(t)$ *and* $\psi_2(t)$ *are continuously differentiable on* $[0,T]$;

3) *the* $n \times n$ *-matrix* $Q(x) = \int_0^T A_1(\tau,x)d\tau$ *is invertible for all* $x \in [0,\omega]$.

Then the initial-periodic boundary value problem for system of the partial differential equations of fourth order (1), (2'), (3)--(5) has a unique classical solution.

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О НАЧАЛЬНО-КРАЕВОЙ ЗАДАЧЕ ДЛЯ СИСТЕМЫ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ В ЧАСТНЫХ ПРОИЗВОДНЫХ ЧЕТВЕРТОГО ПОРЯДКА

Аннотация. Рассматривается начально-краевая задача для системы дифференциальных уравнений в частных производных четвертого порядка. Исследуются вопросы существования классического решения начально-краевой задачи для системы дифференциальных уравнений в частных производных четвертого порядка и предлагаются методы нахождения их приближенных решений. Установлены достаточные условия существования и единственности классического решения начально-краевой задачи для системы дифференциальных уравнений в частных производных четвертого порядка. Путем введения новых неизвестных функций исследуемая задача сведена к эквивалентной задаче, состоящей из нелокальной задачи для системы гиперболических уравнений второго порядка с функциональными параметрами и интегральных соотношений. Предложены алгоритмы нахождения приближенного решения исследуемой задачи и доказана их сходимость. Установлены достаточные условия существования единственного решения эквивалентной задачи с параметрами. Условия однозначной разрешимости начально-краевой задачи для системы дифференциальных уравнений в частных производных четвертого порядка получены в терминах исходных данных. Отдельно приводится результат для начально-периодической по времени краевой задачи.

Ключевые слова: система дифференциальных уравнений в частных производных четвертого порядка, начально-краевая задача, нелокальная задача, система гиперболических уравнений второго порядка, разрешимость, алгоритм.

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ТӨРТІНШІ РЕТТІ ДЕРБЕС ТУЫНДЫЛЫ ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕР ЖҮЙЕСІ ҮШІН БАСТАПҚЫ - ШЕТТІК ЕСЕП ТУРАЛЫ

Аннотация. Төртінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі үшін бастапқы-шеттік есеп қарастырылады. Төртінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі үшін бастапқы-шеттік есептің классикалық шешімінің бар болуы мәселелері мен олардың жуық шешімдерін табу әдістері зерттелген. Төртінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі үшін бастапқы-шеттік есептің классикалық шешімінің бар болуы мен жалғыздығының жеткілікті шарттары тағайындалған. Жаңа белгісіз функциялар енгізу жолымен зерттеліп отырған есеп гиперболалық теңдеулер жүйесі үшін параметрлері бар бейлокал есептен және интегралдық қатынастардан тұратын пара-пар есепке келтірілген.

Зерттеліп отырған есептің жуық шешімін табу алгоритмдері ұсынылған және олардың жинақтылығы дәлелденген. Параметрлері бар пара-пар есептің жалғыз шешімінің бар болуының жеткілікті шарттары тағайындалған. Төртінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі үшін бастапқы-шеттік есептің бірмәнді шешілімділігінің шарттары бастапқы берілімдер терминінде алынған. Бастапқы-уақыт бойынша периодты шеттік есеп үшін нәтиже жеке келтірілген.

Түйін сөздер: Төртінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі, бастапқы-шеттік есеп, бейлокал есеп, екінші ретті гиперболалық теңдеулер жүйесі, шешілімділік, алгоритм.

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