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NUMERICAL ANALYSIS OF THE SOLUTION OF SOME OSCILLATION PROBLEMS BY THE DECOMPOSITION METHOD

Abstract: Rectangular flat plates are one of the main elements of building structures and constructions. While solving applied problems of oscillation of rectangular flat elements then a wide class of oscillation problems occur related to various boundary-value problems: approximate oscillation equations, various boundary conditions at the edges of a flat element and initial conditions. In the theory of oscillation, an important point is to determine the frequencies of intrinsic variations, to solve problems on forced variations of a plane element, and to study the dissemination of harmonic waves in them. In this paper, we present the results on the investigation of natural and forced oscillations of flat elements taking into account the stratification of element's material of rheological viscous properties, the influence of the environment a deformable base, anisotropy, etc. The influence of these factors makes it much more difficult to study the problems of natural and forced oscillations of a flat element on dissemination of harmonic waves in them.

Key words: natural oscillations, forced oscillations, frequency equations, transcendental equations, decomposition method, relaxation time, voltage, plate.

In the study of harmonic waves in deformable bodies, there is introduced a concept of phase velocity as the rate of change of the environmental state, while the phase velocity is expressed in terms of the natural oscillation frequencies, and therefore the study of harmonic wave dissemination is directly related to the problems of determining natural shapes and frequencies of oscillation concerning flat elements.

In this paper, we present the results on the investigation of the natural and forced oscillations of flat elements taking into account the stratification of the element's material, rheological viscous properties, the influence of the environment, a deformable base, anisotropy, etc. The influence of these factors makes it much more difficult to study the problems of natural and forced oscillations of a flat element on dissemination of harmonic waves in them.

Therefore, the work is devoted to the formulation of various boundary-value problems of rectangular flat element oscillations taking into account the viscosity as well as the abovementioned factors of geometric and mechanical nature.

First of all we consider the frequency equation

$$\alpha_0 \cos(\alpha_0 l_1)\sin(\alpha_1 l_1) - \alpha_1 \sin(\alpha_0 l_1)\cos(\alpha_1 l_1) = 0. \tag{1}$$

and its equivalent equation

$$\alpha_0\alpha_1 \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(-1)^{i+j}\frac{\alpha_1^{2i}\alpha_0^{2j} - \alpha_0^{2i}\alpha_1^{2j}}{(2i+1)!(2j)!} l^{2(i+j)} = 0 \tag{2}$$

One of these frequency equations follows from the condition $a = 0$ that leads to the frequency equation

$$\xi^4 - \frac{8\left[(2-\nu)\gamma + \frac{3}{2}(1-\nu)\right]}{(7-8\nu)}\xi^2 + \frac{8\gamma^2}{(7-8\nu)} = 0; \tag{3}$$

The frequency equation (3) also follows from the equation

$$\xi^4 + \frac{2}{\tau_0}\xi^3 + \frac{8}{(7-8\nu)}\left[(2-\nu)\gamma + \frac{(7-8\nu)}{8\tau_0^2} + \frac{3}{2}(1-\nu)\right]\xi^2 + \frac{12(1-\nu)}{(7-8\nu)\tau_0}\left[1+2(2-\nu)\gamma\right]\xi + \\ + \frac{8}{(7-8\nu)}\gamma^2 = 0, \tag{4}$$

for elastic plate or from equation

$$B_0\xi^4 + \frac{2B_0}{\tau_0}\xi^3 + (1 + \frac{B_0}{\tau_0^2} + B_1\gamma)\xi^2 + \frac{1}{\tau_0}(1 + B_1\gamma)\xi + B_2\gamma^2 = 0 \tag{5}$$

for hinged plate.

If we consider other approximate frequency equations derived from equation (2), for example, the equation

$$\xi^2 = \frac{2\gamma + 10l^{-2}}{(2-\nu)}; \tag{6}$$

the root of which is equal to

$$\xi = \sqrt{\frac{2\gamma + 10l^{-2}}{(2-\nu)}} \tag{7}$$

The conditions of convergence (2), described by inequalities

$$\left|a_0^2 a_1^2\right| \le q_{i,j}^2 = q_{i,j}^2 = q^2 \frac{(2i+3)(2j+2)}{l^2} \tag{8}$$

or

$$D^2 - E \le C_{i,j}^2 \tag{9}$$

also contain the left side of equation (3) and indicate that all the roots of the transcendental equation (2) are between the roots ξ_1 и ξ_2 and which are the lower and upper boundaries of all frequencies of the transcendental equation (1).

A similar conclusion follows from the transcendental equations

$$2 - \frac{a_0^2 + a_1^2}{a_0 a_1}\sin(a_0 l_1)\sin(a_1 l_1) - 2\cos(a_0 l_1)\cos(a_1 l_1) = 0 \tag{10}$$

Of other transcendental equations.

Thus, the natural oscillation frequencies of a rectangular hinged plate ξ_1 и ξ_2, based on an approximate equation of fourth order oscillations in derivatives are the lower and upper boundary of natural oscillation frequencies of a rectangular plate under more difficult conditions for fixing its edges.

других трансцендентных уравнений.

The obtained results belonged to the class of boundary-value problems, when two of the opposite edges of a rectangular plate are hinged, and the other two edges have different fixation conditions or are free from stresses.

If all four edges are arbitrarily fixed, then it is not possible to obtain exact frequency equations as described above.

For such problems, you can successfully apply an approximate method of obtaining frequency equations based on the decomposition method developed in the works of Professor G.I.Pshenichniy [74] for static problems.

Let us consider a number of problems of oscillation of flat rectangular elements under arbitrary boundary conditions along the edges of an element in order to determine the natural oscillation frequencies by the decomposition method.

We present the formulation of the method in the case of a flat element, when the material of the element is elastic. In the future, the method will be applied to elements of a high elastic material.

In the case of a flat element made of an elastic material, an approximate fourth order transverse-oscillation equation is written as

$$\Delta^2 W - D_0 \frac{\partial^2}{\partial t^2}\Delta W + D_1 \frac{\partial^4 W}{\partial t^4} + D_2 \frac{\partial^2 W}{\partial t^2} = 0, \tag{11}$$

where the coefficients are determined by the geometry and material properties of the flat element.

The solution of equation (11) will be sought in the form

$$W = \exp\left(i\frac{b}{h}\right) W_0(x,y) \tag{12}$$

Substituting (4.6.2) into equations (4.6.1), for W_0 we obtain the equation

$$\Delta^2 W_0 + D_0\left(\frac{b}{h}\right)^2 \xi^2 \Delta W_0 + \xi^2\left(\frac{b}{h}\right)^2\left[D_1\left(\frac{b}{h}\right)^2\xi^2 - D_2\right]W_0 = 0 \tag{13}$$

To use the decomposition method, it is more convenient to introduce new independent and dependent variables.

$$\alpha = \frac{\pi}{l_1}x; \qquad \beta = \frac{\pi}{l_2}y; \qquad W_0 = \frac{l_1^4}{\pi^4}v;$$
$$\lambda = \frac{l_1}{l_2}; \qquad \lambda_1 = \frac{l_1}{\pi h} \tag{14}$$

In variables (14), equation (13) takes the form

$$\left[\frac{\partial^4 v}{\partial\alpha^4} + 2\lambda^2\frac{\partial^4 v}{\partial\alpha^2\partial\beta^2} + \lambda^4\frac{\partial^4 v}{\partial\beta^4}\right] + \lambda_1^2 D_0\left(\frac{b}{h}\right)^2\xi^2\left[\frac{\partial^2 v}{\partial\alpha^2} + \lambda^2\frac{\partial^2 v}{\partial\beta^2}\right] + \lambda_1^4\left(\frac{b}{h}\right)^2\xi^2 \times$$
$$\times\left[D_1\left(\frac{b}{h}\right)^2\xi^2 - D_2\right]v = 0 \tag{15}$$

The method of decomposition in the theory of oscillations in general formulation reduces to the following.

The formulation of auxiliary problems is formulated.

1. Find a solution to the equation

$$\frac{\partial^4 v_1}{\partial \alpha^4} = f^{(1)}(\alpha, \beta) \quad (16)$$

under boundary conditions

$$L_1(\alpha, \beta) = 0; \qquad L_2(\alpha, \beta) = 0; \qquad (\alpha = 0; \pi) \quad (17)$$

2. Find a solution to the equation

$$\lambda^4 \frac{\partial^4 v_2}{\partial \beta^4} = f^{(2)}(\alpha, \beta) \quad (18)$$

under boundary conditions

$$L_3(\alpha, \beta) = 0; \qquad L_4(\alpha, \beta) = 0; \qquad (\beta = 0; \pi) \quad (19)$$

The boundary conditions at the edges of the plate depend on the conditions of its fixation or on the free edge from stresses.

Rest of the equation (15)

$$2\lambda \frac{\partial^4 v_3}{\partial \alpha^2 \partial \beta^2} + \lambda D_0 \left(\frac{b}{h}\right)^2 \xi^2 \left(\frac{\partial^2 v_3}{\partial \alpha^2} + \lambda^2 \frac{\partial^2 v_3}{\partial \beta^2}\right) + \lambda_1^4 D_0 \left(\frac{b}{h}\right)^2 \xi^2 \left[D_1 \left(\frac{b}{h}\right)^2 \xi^2 - D_2\right] v_3 + \\ + f^{(1)}(\alpha, \beta) + f^{(2)}(\alpha, \beta) = 0, \quad (20)$$

where $f^{(j)}(\alpha, \beta)$ arbitrary functions the forms of which depend on the boundary-value problems.

Following the decomposition method, we assume that

$$v_3 = \frac{1}{2}[v_1 + v_2] \quad (21)$$

and the condition must be met at given points on the flat element.

The general solutions of the auxiliary problems equations (16) and (18) are

$$v_1 = f_1(\alpha, \beta) + \frac{\alpha^3}{6}\varphi_1(\beta) + \frac{\alpha^2}{2}\varphi_2(\beta) + \alpha\varphi_3(\beta) + \varphi_4(\beta);$$
$$v_1 = f_1(\alpha, \beta) + \frac{\beta^3}{6}\psi_1(\alpha) + \frac{\beta^2}{2}\psi_2(\alpha) + \beta\psi_3(\alpha) + \psi_4(\alpha); \quad (22)$$

where φ_j, ψ_j arbitrary functions of the arguments and are determined from the boundary conditions (17) и (19).

In the following, arbitrary functions in the general form will be represented as

$$f^{(j)}(\alpha, \beta) = \sum_{n=1}^{\infty}\sum_{j=1}^{\infty} a_{n,m}^{(j)} \sin(\alpha n)\sin(\beta m), \quad (23)$$

where $a_{n,m}^{(j)}$ arbitrary constants, and functions $f_j(\alpha, \beta)$ in common solutions (22) are equal

$$f_1(\alpha, \beta) = \sum_{n=1}^{\infty}\sum_{j=1}^{\infty} \frac{a_{n,m}^{(j)}}{n^4} \sin(\alpha n)\sin(\beta m);$$

$$f_2(\alpha,\beta)=\sum_{n=1}^{\infty}\sum_{j=1}^{\infty}\frac{a_{n,m}^{(2)}}{m^4}\sin(\alpha n)\sin(\beta m). \tag{24}$$

Using private solutions of problems under given boundary conditions and using approximate representations (21), to find the unknowns $a_{n,m}^{(j)}$ we obtain a homogeneous linear system of algebraic equations whose nontrivial solution leads to the frequency equation.

We illustrate the decomposition method on a number of particular boundary-value problems of the oscillation of a flat element.

Problem 1. We consider the simplest problem when all edges are hinged. This problem was solved by the direct method (13) and the frequency equation (14) was obtained, where it is necessary to set the relaxation time to infinity.

Boundary conditions have the form

$$\begin{aligned} v_1 &= \frac{\partial^2 v_1}{\partial \alpha^2} = 0 \qquad (\alpha = 0;\pi); \\ v_2 &= \frac{\partial^2 v_2}{\partial \beta^2} = 0 \qquad (\beta = 0;\pi), \end{aligned} \tag{25}$$

satisfying which general solutions (22), we get

$$v_1 = f_1(\alpha,\beta); \qquad \lambda^4 v_2 = f_2(\alpha,\beta) \tag{26}$$

or private solutions are equal

$$\varphi_j(\beta)=\psi_j(\alpha=0) \qquad j=(1,\dots,4)$$

Satisfying solution (26) to conditions (21) and equation (20), for the frequency ξ we again obtain equation (14).

Thus, an approximate decomposition method gives the same result as the exact direct method. Consequently, the decomposition method can be applied with a sufficient degree of reliability in the solution of other boundary-value problems.

Problem 2. A rigidly fixed plate on the edges. Boundary conditions have the form

$$\begin{aligned} v_1 &= \frac{\partial v_1}{\partial \alpha} = 0 \qquad (\alpha = 0;\pi); \\ v_1 &= \frac{\partial v_2}{\partial \beta} = 0 \qquad (\beta = 0;\pi); \end{aligned} \tag{27}$$

Using general solutions (22) and boundary-value solutions (27), for the unknown quantities v_1, v_2 get expressions

$$v_1 = f_1(\alpha,\beta)-\frac{\alpha^3}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{a_{n,m}^{(1)}}{n^3}\left[1+(-1)^n\right]\sin(\beta m)+$$

$$+\frac{\alpha^2}{\pi}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{a_{n,m}^{(1)}}{n^3}\left[2+(-1)^n\right]\sin(\beta,m)-\alpha\frac{\alpha^3}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{a_{n,m}^{(1)}}{n^3}\sin(\beta m);$$

$$v_2 = f_2(\alpha,\beta) - \frac{\beta^3}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{a_{n,m}^{(1)}}{m^3}\left[1+(-1)^m\right]\sin(\alpha m) +$$
$$+\frac{\beta^2}{\pi}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{a_{n,m}^{(1)}}{m^3}\left[2+(-1)^m\right]\sin(\alpha m) - \beta\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{a_{n,m}^{(1)}}{m^3}\sin(\alpha n). \tag{28}$$

We confine ourselves to the first coefficients in the series of arbitrary functions (23) and the condition $v_1, = v_2; (\alpha,\beta) = \frac{\pi}{2}$, we get a system of algebraic equations

$$\left[a_{1,1}^{(1)} + \lambda^{-4}a_{1,1}^{(2)}\right]\left\{\lambda^2\left(1-\frac{2}{\pi}\right) + \frac{(2-\gamma)}{2}\lambda_1^2\xi^2\left[\frac{2}{\pi} - 1 + \lambda^2\left(\frac{\pi}{4}-1\right)\right] +\right.$$
$$\left.+\frac{1}{2}\lambda_1^4\xi^2\left[\frac{(7-8v)}{8}\xi^2 - \frac{3(1-v)}{2}\right]\left(1+\frac{\pi}{4}\right)+\frac{1}{2}\right\} = 0;$$
$$a_{1,1}^{(1)} = \lambda^{-4}a_{1,1}^{(2)} \tag{29}$$

Nontrivial solution of system (29) to the frequency equation

$$\lambda_1^4\frac{(7-8v)}{8}\xi^4 - \frac{\lambda_1^2}{2}\left[3-(1-v)\lambda_1^2 + (2-v)\left(2-\frac{1}{\pi}\right)\left(1+\lambda^6\right)\right]\xi^2 +$$
$$+\left[2\lambda^2\left(1-\frac{1}{\pi}\right)+\left(1+\lambda^4\right)\right] = 0 \tag{30}$$

Problem 3. The edges of the plate $\beta = 0; \beta = \pi$ are rigidly fixed and the edges $\alpha = 0; \alpha = \pi$ are free from stresses i.e. we have boundary conditions

$$\frac{\partial^2 v_1}{\partial\alpha^2} + Q_0 v_1 = 0; \frac{\partial^3 v_1}{\partial\alpha^3} = 0, (\alpha = 0;\pi)$$
$$Q_0 = \left(\frac{3-2v}{7-4v}\right)\left[2\lambda^2\frac{\partial^2}{\partial\beta^2} + \lambda_1^2\xi^2\right]; \tag{31}$$
$$v_2 = \frac{\partial v^2}{\partial\beta} = 0 \ (\beta = 0;\pi)$$

The solution of the problem to determine V_2 has the form (2 8).

To find the unknown function V_1 from boundary conditions

$$\frac{\partial^3 v_1}{\partial\alpha^3} = 0 \text{ at } \alpha = 0;\pi$$

we obtain

$$\varphi_1 = -\left.\frac{\partial^3 f_1}{\partial a^3}\right|_{a=0;}\varphi_1 = -\left.\frac{\partial^3 f_1}{\partial a^3}\right|_{a=\pi}; \tag{32}$$

which can be fulfilled at $n = 2q$ that is odd values of unknowns $a_{n,m}^{(1)}$ must be set to zero.

Conditions (31) at $\alpha = 0; \pi$ lead to the system

$$[\pi\varphi_1 + \varphi_2] + \left(\frac{3-2\nu}{7-4\nu}\right)\left[\left(\frac{\pi^3}{6}\frac{\partial^2\varphi_1}{\partial\beta^2} + \frac{\pi^2}{2}\frac{\partial^2\varphi_2}{\partial\beta^2} + \pi\frac{\partial^2\varphi_3}{\partial\beta^2} + \frac{\partial^2\varphi_4}{\partial\beta^2}\right) + \right.$$

$$\left. + \lambda_1^2\xi^2\left(\frac{\pi^3}{6}\varphi_1 + \frac{\pi^2}{2}\varphi_2\pi\varphi_3 + \varphi_4\right)\right] = 0$$

$$\varphi_2 = -\left(\frac{3-2\nu}{7-4\nu}\right)\left(\frac{\partial^2\varphi_4}{\partial\beta^2} + \lambda_1^2\xi^2\varphi_4\right) \tag{33}$$

Two equations (33) connect three unknown functions. Since we are looking for private solutions of problems without limiting the generality, the unknown function φ_3 can be put equal to $\varphi_3 = 0$.

уравнения (33) связывают три неизвестные функции. Так как ищем частные решения задач, то не ограничивая общности, неизвестную функцию φ_3 можно положить равной $\varphi_3 = 0$.

From the system (33) we get the equation for φ_4 :

$$\frac{\partial^4\varphi_4}{\partial\beta^4} + 2\lambda_1^2\xi^2\frac{\partial^2\varphi_4}{\partial\beta^2} + \lambda_1^4\xi^4\varphi_4 = -\frac{2}{\pi}\left(\frac{7-4\nu}{3-2\nu}\right)^2\left\{\pi\left[1 + \left(\frac{3-2\nu}{7-4\nu}\right)\frac{\pi^2}{6}\lambda_1^2\xi^2\right]\frac{\partial^3 f_1}{\partial\alpha^2}\Bigg|_{\alpha=0} + \right.$$

$$\left. + \frac{\pi^3}{6}\left(\frac{3-2\nu}{7-4\nu}\right)\frac{\partial^5 f_1}{\partial\alpha^3\partial\beta^2}\Bigg|_{\alpha=\pi}\right\}, \tag{34}$$

whose particular solution is equal to

$$\varphi_4 = \sum_{q=1}^{\infty}\sum_{m=1}^{\infty} a_{2q,m}^{(1)} A_{q,m}^{(1)} \sin(\beta m), \tag{35}$$

где

$$A_{q,m}^{(1)} = \frac{(m^2-1)}{2q}\left(m^4 - 2m^2\lambda_1^2\xi^4\right)^{-1} \tag{36}$$

Restricting to the first components $a_{2,1}^{(1)}; a_{1,1}^{(2)}$, as in the previous problem, we obtain the frequency equation

$$\frac{\pi^2}{192}\lambda_1^4(7-8\nu)\xi^4 - \left\{\left(\frac{2-\nu}{2}\right)\lambda_1^2\left[\left(\frac{\pi^2}{24} - 1\right) + \frac{\lambda^2\pi^2\left(2 - \frac{\pi}{4} - \frac{2}{\pi}\right)}{24\left(1 - \frac{\pi}{4}\right)}\right] - \frac{3(1-\nu)}{48}\lambda_1^4\pi^2\right\}\xi^2 +$$

$$+ \left\{\lambda^2\left[\frac{\pi^2\left(1 - \frac{2}{\pi}\right)}{24\left(1 - \frac{\pi}{4}\right)} - 1\right] + \left[1 - \lambda^4\frac{\pi^3}{48\left(1 - \frac{\pi}{4}\right)}\right]\right\} = 0 \tag{37}$$

Problem 4. The edges of a plate $(\beta = 0; \pi)$; $\alpha = 0$ rigidly clamped and the edge $\alpha = \pi$ is free from stress.

In this problem the desired function v_2 is determined in the previous problems and v_1 is equal to

$$v_1 = f_1(\alpha, \beta) + \frac{a^3}{6}\varphi_1(\beta) + \frac{a^2}{2}\varphi_2(\beta) + a\varphi_3\beta;$$

$$\varphi_4 = 0;\ \varphi_1 = -\left.\frac{\partial^3 f_1}{\partial a^3}\right|_{a=\pi};\qquad \varphi_3 = -\left.\frac{\partial f_1}{\partial a}\right|_{a=0}; \tag{38}$$

where

$$\frac{\pi^2}{2}\left(\frac{3-2v}{7-4v}\right)\frac{\partial^2\varphi_2}{\partial\beta^2} + \left[1 + \left(\frac{3-2v}{7-4v}\right)\frac{\pi^2}{2}\lambda_1^2\xi^2\right]\varphi_2 = \left[\pi\left.\frac{\partial^3 f_1}{\partial a^3}\right|_{a=\pi} + \frac{\pi^3}{6}\left(\frac{3-2v}{7-4v}\right)\left.\frac{\partial^5 f_1}{\partial a^3\partial\beta^2}\right|_{a=\pi} + \right.$$
$$+\left(\frac{3-2v}{7-4v}\right)\pi\left.\frac{\partial^3 f_1}{\partial a^3}\right|_{a=\pi} + \left(\frac{3-2v}{7-4v}\right)\pi\lambda_1^2\xi^2\left.\frac{\partial f_1}{\partial \alpha}\right|_{a=0} \tag{39}$$

As in the previous problems, we obtain the frequency equation

$$\lambda^2\left[\left(1+\frac{\pi}{2} - B_1 + C_1\left(1-\frac{2}{\pi}\right)\right] + \frac{(2-v)}{2}\lambda_1^2\xi^2\left\{\left[\left(B_1 - \frac{\pi}{2} - 1\right) - \right.\right.$$
$$\left.-C_1\left(1+\frac{\pi}{2}\right)\right] + \lambda^2\left[-\left(1 - \frac{\pi}{2} - \frac{\pi^3}{48} + \frac{\pi^2}{8}B_1\right) + C_1\left(\frac{2}{\pi} - 1\right)\right]\right\} +$$
$$+\lambda_1^4\xi^2\left[\left(\frac{7-8v}{8}\right)\xi^2 - \frac{3}{2}(1-v)\right]\left[\left(1 - \frac{\pi^3}{48} - \frac{\pi}{2} + \frac{\pi^2}{48}B_1\right) + \right.$$
$$\left. + C_1\left(1 - \frac{\pi}{4}\right)\right] + \left(1 + C_1\lambda^4\right) = 0; \tag{40}$$

where B_1, C_1 are equal to

$$B_1 = \left[\frac{\pi}{4} - \frac{\pi^3}{6}\left(\frac{3-2v}{7-4v}\right) - \pi\left(\frac{3-2v}{7-4v}\right) + \pi\lambda_1^2\xi^2\left(\frac{3-2v}{7-4v}\right)\right]\times$$
$$\times\left[1 + \frac{\pi^2}{2}\left(\frac{3-2v}{7-4v}\right)\lambda_1^2\xi^2 - \frac{\pi^2}{2}\left(\frac{3-2v}{7-4v}\right)\right]^{-1}; \tag{41}$$
$$C_1 = \left(1 - \frac{\pi}{2} - \frac{\pi^3}{48} + \frac{\pi^2}{8}B_1\right)\left(1 - \frac{\pi}{4}\right)^{-1}$$

Frequency equation (40) defines three frequencies unlike the previous ones which is apparently, connected with the fact that the edge $a = \pi$ is free from stresses and the waves are reflected from the rigidly fixed edge $a = 0$.

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ДЕКОМПОЗИЦИЯ ӘДІСІМЕН ШЫҒАРЫЛҒАН КЕЙБІР ТЕРБЕЛІС ЕСЕБІНІҢ ШЕШІМДЕРІН САНДЫҚ ТАЛДАУ

Аннотация: Тік бұрышты пішіндегі жазық пластинкалар құрылыс конструкцияларының және ғимараттарының негізгі элементтерінің бірі болып табылады. Тік бұрышты жазық элементтердің қолданбалы тербеліс есептерін шешу кезінде шеттік есептер үшін, жазық элементтердің бастапқы шарттары мен шетіндегі шегаралық шарттарына байланысты әр-түрлі жоғары санатты есептер пайда болады. Тербелістер теориясында меншікті тербелістің жиілігін анықтау, жазық элементтердің еріксіз тербеліс есебің шешу және ондағы гармоникалық толқындардың таралуын зерттеу негізгі кезең болып табылады. Бұл жұмыста жазық элементтердің өзіндік және еріксіз тербелісін, материал элементтерінің қатпарлылығын, тұтқыр реологиялық қасиетін, қоршаған ортаның әсерін, негізінің деформацияға ұшырауы, анизотропиясын және тағы басқа да жасалған зеріттеу шешімдері келтіріледі, себебі көрсетілген факторлардың әсері жазық элементтердің өзіндік және еріксіз тербелісі есебіндегі гармоникалық толқындардың таралу процесін зертеуді айтарлықтай қиындатады.

Түйін сөздер: +өзіндік тербеліс, еріксіз тербеліс, жиіліктік теңдеулері, трансценденттік теңдеулер, декомпозици әдісі, таралу уақыты, кернеу, пластинка

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ЧИСЛЕННЫЙ АНАЛИЗ РЕШЕНИЯ НЕКОТОРЫХ ЗАДАЧ КОЛЕБАНИЯ МЕТОДОМ ДЕКОМПОЗИЦИИ

Аннотация: Плоские пластинки прямоугольной формы являются одними из основных элементов строительных конструкций и сооружений. При решении прикладных задач колебания прямоугольных плоских элементов возникает широкий класс задач колебаний, связанных с различными краевыми задачами: приближёнными уравнениями колебания, различными граничными условиями на краях плоского элемента и начальными условиями. В теории колебания важным моментом является определение частот собственных колебаний, решение задач о вынужденных колебаниях плоского элемента и исследование распространения гармонических волн в них. В данной работе приводятся результаты по исследованию собственных и вынужденных колебаний плоских элементов с учётом слоистости материала элемента, реологических вязких свойств, влияния окружающей среды, деформируемого основания, анизотропии и т.д. Влияние указанных факторов значительно затрудняет исследование задач о собственных и вынужденных колебаниях плоского элемента, о распространении в них гармонических волн.

Ключевые слова: собственная колебания, вынужденная колебания, частотные уравнения, трансцендентные уравнения, метод декомпозиции, время релаксации, напряжения, пластинка

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