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THE CONSTRUCTION OF A SOLUTION OF A RELATED SYSTEM OF THE LAGUERRE TYPE

Abstract. The aim of the work is to study the system of Laguerre type obtained from the degenerate of Horn system by direct selection of parameters, as well as using an exponential transformation. Such a system consisting of two partial differential equations of second order is called related to the basic Laguerre system. The difficulties of studying are that if in the ordinary case there is one degenerate of Kummer's equation and only one degenerate hypergeometric function satisfying it, then in the case of two variables there 20 degenerate systems and 20 degenerate hypergeometric functions of two variables satisfying them appear. It is not known how many systems of Laguerre type exist, and with which of the 20 degenerate systems it links to. There is no general method of a research In this work Frobenius-Latysheva's method which is generalized in this case by Zh.N. Tasmambetov is applied to creation of their normal and regular solution depending on Laguerre's polynomial of two variables. The classification of singular curves using rank and antirank is given, as well as basic information about the features of constructing normal-regular solutions of such systems. The main theorem on the existence of four linearly independent partial solutions is proved. Solutions are determined by the degenerate hypergeometric function of M.P. Humbert in the form of normal-regular ranks of two variables depending on Laguerre's polynomials. The conclusions indicate the relationship of such systems with overridden systems and some representations of Laguerre's polynomial of two variables.

Key words: Related, system, Laguerre-type system, Horn system, normal-regular solution, special curves, rank, antirank, overdetermined.

Introduction

The degenerate hypergeometric function is the root of many well-known functions, and through it all orthogonal polynomials of one variable are expressed [1]-[2]. Indeed, if γ is not an integer, is α a negative integer or zero, then the series

$$G(\alpha,\gamma;x)=1+\frac{\alpha}{\gamma}x+\frac{\alpha(\alpha+1)}{2!\gamma(\gamma+1)}x^2+\ldots \tag{1}$$

representing the degenerate hypergeometric function terminates, and we obtain a polynomial $G(-n,\gamma;x)$ in particular expressing the polynomial of Laguerre. In the theory of orthogonal polynomials, there are several differential equations solutions of which are Laguerre's polynomials and various applications in the problems of mathematical physics, as well as in the theory of the hydrogen atom, etc. [3, c.226]- [4]- [5,115-118]. The generalization of this theory to Laguerre's polynomials of two variables and systems of partial differential equations of the second order, which they satisfy, has not reached this level. The study is complicated by the fact that if in the ordinary case there is only one degenerate hypergeometric equation, then in the case of two variables there are 20 degenerate systems and 20 degenerate hypergeometric functions of two variables satisfying them [6, c.226-230]-[7]. It is not yet known how many systems of the Laguerre type are there and with which of the 20 degenerate systems they are connected. Apparently, this was influenced by the insufficient development of the analytical theory of such systems. Therefore, another direction for studying orthogonal polynomials of two variables as eigenfunctions of linear partial differential operators of the second order was developed [8]-[9]-[10].

In the works of Zh.N.Tasmambetov and A.A.Issenova the system of Horn (Ψ_2) was selected as a binding system and the connection between the degenerate hypergeometric function of Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$ and the Laguerre's function of two variables $L_{n,m}^{(\alpha,\beta)}(x,y)$ was studied.

In [11] it was shown that from the system of Horn

$$\left.\begin{aligned} xZ_{xx}+(\gamma-x)Z_x-yZ_y+nZ=0\\ yZ_{yy}+(\gamma-y)Z_y-xZ_x+nZ=0\end{aligned}\right\} \tag{2}$$

when $\gamma=\alpha+1,\gamma'=\beta+1,\alpha\neq 0,\beta\neq 0,\lambda=-n$ the basic system of Laguerre is obtained

$$\left.\begin{aligned} xZ_{xx}+(\alpha+1-x)Z_x-yZ_y+nZ=0\\ yZ_{yy}+(\beta+1-y)Z_y-xZ_x+nZ=0\end{aligned}\right\} \tag{3}$$

solution of which is a polynomial

$$\Psi_2(-n,\alpha+1,\beta+1;x,y)=\sum_{\mu,\nu=0}^{\infty}\frac{(-n)_{\mu+\nu}}{(\alpha+1)_\mu(\beta+1)_\nu}\cdot\frac{x^\mu}{\mu!}\cdot\frac{y^\nu}{\nu!}. \tag{4}$$

By analogy, this polynomial is called the generalized Laguerre's polynomials of two variables and is denoted by

$$L_{n,m}^{(\alpha,\beta)}=\Psi_2(-n,\alpha+1,\beta+1;x,y). \tag{5}$$

Basic information

According to the general theory of systems of the form (2), when the condition of compatibility and integrality is performed [12], it has up to four linearly independent solutions $Z_i\left(i=\overline{1,4}\right)$, and the general solution depends on arbitrary constants and is represented as a sum

$$Z(x,y)=C_1Z_1(x,y)+C_2Z_2(x,y)+C_3Z_3(x,y)+C_4Z_4(x,y) \tag{6}$$

where $C_i\left(i=\overline{1,4}\right)$ are arbitrary constants, $Z=Z(x,y)$ is a general unknown.

The system has a regular $(0,0)$ singularity and an irregular (∞,∞) singularity. To classify regular and irregular singularities K.Ya. Latysheva used the notion of rank

$$p=1+k,k=\max_{(1\leq s\leq n)}\frac{\beta_s-\beta_0}{s} \tag{7}$$

introduced by H. Poincare and antirank

$$m=-1-\chi,\chi=\min_{(1\leq s\leq n)}\frac{\pi_s-\pi_0}{s} \tag{8}$$

introduced by L. Tome.

These concepts were generalized to the case of the studied system of (2) Zh.N.Tasmambetov[13].

If the rank is $p\leq 0$, then the special curve $(x=\infty,y=\infty)$ is regular, when $p>0$ the special curve is irregular. When $m\leq 0$ a special curve $(0,0)$ is regular, and if $m>0$ special is irregular [13].

Definition 1. If the rank $p>0$ and antirank $m\leq 0$, then system (2) has a solution

$$Z(x,y)=\exp Q(x,y)\cdot x^{\rho_i}y^{\sigma_i}\sum_{\mu,\nu=0}^{\infty}A_{\mu,\nu}^{(i)}x^{\mu}y^{\nu},\ A_{0,0}\neq 0, \tag{9}$$

where $\rho_i,\sigma_i\ (i=\overline{1,4}),\ A_{\mu,\nu}^{(i)}\ (\mu,\nu=0,1,2,...)$ - unknown constants; $Q(x,y)$ - polynomial of two variables

$$Q(x,y)=\frac{\alpha_{p0}}{p}x^p+\frac{\alpha_{0p}}{p}y^p+...+\alpha_{11}xy+\alpha_{10}x+\alpha_{01}y, \tag{10}$$

with unknown coefficients $\alpha_{p0},\alpha_{0p},...,\alpha_{11},\alpha_{10},\alpha_{01}$. The solution of the form (9) is called normal-regular.

If the special curve $(0,0)$ is regular, then the polynomial $Q(x,y)\equiv 0$ and the solution of the system exists in the form of a generalized power series of two variables

$$Z(x,y)=x^{\rho_i}y^{\sigma_i}\sum_{\mu,\nu=0}^{\infty}A_{\mu,\nu}^{(i)}x^{\mu}y^{\nu},\ A_{0,0}\neq 0, \tag{11}$$

where $\rho_i,\sigma_i\ (i=\overline{1,4}),\ A_{\mu,\nu}^{(i)}\ (\mu,\nu=0,1,2,...)$ - the unknown constants.

The highest degree of the polynomial $Q(x,y)$ is determined by the rank p.

Definition 2. The values of number p determined by the equality (7) are called series order (9) and can be an integer or a fractional number (positive or negative).

CONCLUSION OF THE RELATED SYSTEM OF THE LAGUERRE TYPE AND THE CONSTRUCTION OF ITS SOLUTION

Formulation of the problem

From the system of Horn (Ψ_2) by means of converting

$$Z=\exp\left(\frac{x}{2}+\frac{y}{2}\right)\cdot x^{-\frac{\alpha+1}{2}}y^{-\frac{\beta+1}{2}}\cdot U(x,y) \tag{12}$$

a system of Laguerre type is installed

$$\left.\begin{aligned} x^2U_{xx}-xyU_y+\left(-\frac{x^2}{4}-\frac{xy}{2}+kx+\frac{1}{4}-\alpha^2\right)\cdot U=0\\ y^2U_{yy}-xyU_x+\left(-\frac{y^2}{4}-\frac{xy}{2}+ky+\frac{1}{4}-\beta^2\right)\cdot U=0 \end{aligned}\right\} \tag{13}$$

where $k=(\alpha+\beta+2-2\lambda)/2$ is related with the basic Laguerre system (3).

Such systems belong to the Whitaker-type system [7]. By applying Frobenius-Latysheva method [13] we want to establish distinctive features of the system (12) and construct its normal-regular solution dependent on Laguerre's polynomials of two variables.

MAIN RESULTS

Theorem 1. The system of second order partial differential equations (13) has four linearly independent partial solutions, which are expressed through the degenerate hypergeometric function of M.R. Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$ in the form of normal-regular series

$$\begin{aligned} U(x,y)&=\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\cdot x^{\frac{\alpha+1}{2}}y^{\frac{\beta+1}{2}}\cdot\Psi_2(-n,\alpha+1,\beta+1;x,y)=\\ &=\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\cdot x^{\frac{\alpha+1}{2}}y^{\frac{\beta+1}{2}}\cdot L_{n,n}^{(\alpha,\beta)}(x,y) \end{aligned} \tag{14}$$

dependent on the Laguerre's polynomial of two variables

$$L_{n,n}^{(\alpha,\beta)}(x,y)=1-\frac{n}{1!(\alpha+1)}x-\frac{n}{1!(\beta+1)}y+\frac{n(n-1)}{1!(\alpha+1)(\beta+1)}xy+\frac{n(n-1)}{1!(\alpha+1)(\alpha+2)}x^2+$$
$$+\frac{n(n-1)}{1!(\beta+1)(\beta+2)}y^2+...+(-1)^n\frac{n(n-1)...1}{n!(\alpha+1)...(\alpha+n)}x^n+ \qquad (15)$$
$$+(-1)^n\frac{n(n-1)...1}{n!(\alpha+1)...(\alpha+n-1)(\beta+1)}x^{n-1}y+...+(-1)^n\frac{n(n-1)...1}{n!(\beta+1)...(\beta+n)}y^n$$

Evidence. For the proof Frobenius-Latysheva method is used. Like the degenerate system (2) the system (12) has a regular $(0,0)$ singularity and an irregular (∞,∞) singularity. By highest degrees of independent variables x and y certain subranks: $k_1'=0, k_1''=0$ and rank $p=1+k=1$. Then according to the method of Frobenius-Latysheva for the construction of normal-regular solution of (9), in the system (13) the transformation is correct:

$$U=\exp(\alpha_{10}x+\alpha_{01}y)\Phi(x,y) \qquad (16)$$

where α_{10} and α_{01} are uncertain coefficients, which need to be determined from the newly obtained support system.

$$\left.\begin{aligned}&x^2\Phi_{xx}+2\alpha_{10}^2x^2\Phi_x-xy\Phi_y+\left(\left(\alpha_{10}^2-\frac{1}{4}\right)x^2-\left(\alpha_{01}^2-\frac{1}{2}\right)xy+kx+\frac{1}{4}-\frac{\alpha^2}{4}\right)\cdot\Phi=0\\&y^2\Phi_{yy}+2\alpha_{01}^2y^2\Phi_y-xy\Phi_x+\left(\left(\alpha_{01}^2-\frac{1}{4}\right)y^2-\left(\alpha_{10}^2-\frac{1}{2}\right)xy+ky+\frac{1}{4}-\frac{\beta^2}{4}\right)\cdot\Phi=0\end{aligned}\right\} \qquad (17)$$

By equating coefficients to zero at the highest degrees of independent variables x^2 and y^2 at unknown $\Phi(x,y)$, we define a system of characteristic equations

$$\begin{aligned}&b_{10}^{(1)}(\alpha_{10},\alpha_{01})=\alpha_{10}^2-\frac{1}{4}=0,\\&b_{01}^{(2)}(\alpha_{10},\alpha_{01})=\alpha_{01}^2-\frac{1}{4}=0.\end{aligned} \qquad (18)$$

This shows the fulfillment of the first necessary condition for the existence of a normal-regular solution (9) connected with the definition of the unknown coefficients of the $Q(x,y)$ polynomial [13].

Theorem 2. Equality (18) is required for a supporting system to have at least one solution of the form (9).

The system (17) has four root pairs:

$$\begin{aligned}&\left(\alpha_{10}^{(1)},\alpha_{01}^{(1)}\right)=\left(\frac{1}{2},\frac{1}{2}\right),\left(\alpha_{10}^{(1)},\alpha_{01}^{(2)}\right)=\left(\frac{1}{2},-\frac{1}{2}\right),\\&\left(\alpha_{10}^{(2)},\alpha_{01}^{(1)}\right)=\left(-\frac{1}{2},\frac{1}{2}\right),\left(\alpha_{10}^{(2)},\alpha_{01}^{(2)}\right)=\left(-\frac{1}{2},-\frac{1}{2}\right),\end{aligned} \qquad (19)$$

defining four polynomials of the first degree of the form (10), since the rank of the system is equal to one:

$$Q_i(x,y)=\alpha_{10}^{(i)}x+\alpha_{01}^{(i)}y,\ i=\overline{1,4}.$$

Four $\left(\alpha_{10}^{(l)},\alpha_{01}^{(l)}\right), l=1,2$ pairs in (18) define four systems from the auxiliary system (18). Each of them can have up to four linearly independent particular solutions. Thus, the initial system should have up to 16 private solutions. However, a detailed study shows that out of the four systems, only the system

$$\left.\begin{aligned} x^2\Phi_{xx} + x^2\Phi_x - xy\Phi_y + \left(kx + \frac{1}{4} - \frac{\alpha^2}{4}\right)\cdot\Phi = 0 \\ y^2\Phi_{yy} + y^2\Phi_y - xy\Phi_x + \left(ky + \frac{1}{4} - \frac{\beta^2}{4}\right)\cdot\Phi = 0 \end{aligned}\right\} \tag{20}$$

has four linearly independent particular solutions. All of them are expressed through degenerate hypergeometric function of Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$.

Indeed, by making up the system of characteristic functions of the system (20) we make sure that the system of defining equations with respect to the singularity $(0,0)$

$$\left.\begin{aligned} f_{00}^{(1)}(\rho,\sigma) = \rho(\rho-1) + \frac{1}{4} - \frac{\alpha^2}{4} = 0, \\ f_{00}^{(2)}(\rho,\sigma) = \sigma(\sigma-1) + \frac{1}{4} - \frac{\beta^2}{4} = 0, \end{aligned}\right\} \tag{21}$$

has four pairs of roots:

$$\begin{aligned} (\rho_1,\sigma_1) = \left(\frac{1}{2}+\frac{\alpha}{2}, \frac{1}{2}+\frac{\beta}{2}\right), (\rho_1,\sigma_2) = \left(\frac{1}{2}+\frac{\alpha}{2}, \frac{1}{2}-\frac{\beta}{2}\right), \\ (\rho_2,\sigma_1) = \left(\frac{1}{2}-\frac{\alpha}{2}, \frac{1}{2}+\frac{\beta}{2}\right), (\rho_2,\sigma_2) = \left(\frac{1}{2}-\frac{\alpha}{2}, \frac{1}{2}-\frac{\beta}{2}\right). \end{aligned} \tag{22}$$

This shows the fulfillment of the second necessary condition.

Theorem 3. In order for the system (13) to have a normal-regular solution of the form (9), it is necessary for the pair (ρ,σ) to be the root of the defining equations with respect to the $(0,0)$singularity of the form (20) obtained from the auxiliary system (17) by substituting instead of the unknown $Z(x,y) = x^\rho \cdot y^\sigma$.

The existence of four pairs of roots (22) ensures the existence of four linearly independent particular solutions of the system (20) in the form of generalized power series (12). Since, the values of ρ and σ are defined, it remains to find the unknown constants $A_{\mu,\nu}$ $(\mu,\nu = 0,1,2,...)$ with the help of a system of recurrent sequences

$$\sum A_{m-\mu,n-\nu}^{(i)} \cdot f_{\mu,\nu}^{(i)}(\rho+m-\mu, \sigma+n-\nu), (m,n = 0,1,2,...; i = 1,2; \mu,\nu = 0,1,2,...).$$

Thus, the constructed particular solutions of (19) have the form of the

$$\Phi_1(x,y) = x^{\frac{\alpha+1}{2}} y^{\frac{\beta+1}{2}} \cdot \Psi_2\left(\frac{\alpha+\beta}{2} + 1 - k, \alpha+1, \beta+1; x, y\right)$$

$$\Phi_2(x,y) = x^{\frac{\alpha+1}{2}} y^{\frac{1-\beta}{2}} \cdot \Psi_2\left(\frac{\alpha-\beta}{2} + 1 - k, \alpha+1, 1-\beta; x, y\right)$$

$$\Phi_3(x,y) = x^{\frac{1-\alpha}{2}} y^{\frac{\beta+1}{2}} \cdot \Psi_2\left(\frac{\beta-\alpha}{2} + 1 - k, 1-\alpha, \beta+1; x, y\right)$$

$$\Phi_4(x,y) = x^{\frac{1-\alpha}{2}} y^{\frac{1-\beta}{2}} \cdot \Psi_2\left(\frac{-\alpha-\beta}{2} + 1 - k, 1-\alpha, 1-\beta; x, y\right)$$

Considering $k = (\alpha+\beta+2-2\lambda)/2$, these solutions are represented in the form

$$\Phi_1(x,y) = x^{\frac{\alpha+1}{2}} y^{\frac{\beta+1}{2}} \cdot \Psi_2(-n, \alpha+1, \beta+1; x, y)$$

$$\Phi_2(x,y)=x^{\frac{\alpha+1}{2}}y^{\frac{1-\beta}{2}}\cdot\Psi_2(-n-\beta,\alpha+1,1-\beta;x,y)$$
$$\Phi_3(x,y)=x^{\frac{1-\alpha}{2}}y^{\frac{\beta+1}{2}}\cdot\Psi_2(-n-\alpha,1-\alpha,\beta+1;x,y) \qquad (23)$$
$$\Phi_4(x,y)=x^{\frac{1-\alpha}{2}}y^{\frac{1-\beta}{2}}\cdot\Psi_2(-n-\alpha-\beta,1-\alpha,1-\beta;x,y)$$

Hence, it is not difficult to notice that the system solution we are interested in (20):

$$\Phi_1(x,y)=x^{\frac{\alpha+1}{2}}y^{\frac{\beta+1}{2}}\cdot\Psi_2(-n,\alpha+1,\beta+1;x,y)=x^{\frac{\alpha+1}{2}}y^{\frac{\beta+1}{2}}\cdot L_{n,n}^{(\alpha,\beta)}(x,y) \qquad (24)$$

and the remaining solutions will not be considered in the future.

The fulfillment of two necessary conditions ensures the existence of a normal-regular solution (14), dependent on the Laguerre's polynomial of two variables (15). The theorem is proved.

The General solution of the system (12) on the basis of (6), taking into account formulas (23), is presented as

$$U(x,y)=C_1U_1(x,y)+C_2U_2(x,y)+U_3Z_3(x,y)+C_4U_4(x,y)=$$
$$=C_1\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\Phi_1(x,y)+C_2\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\Phi_2(x,y)+$$
$$+C_3\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\Phi_3(x,y)+C_4\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\Phi_4(x,y),$$

where $C_i\left(i=\overline{1,4}\right)$ - are arbitrary constants.

On the basis of the above reasoning, some statements can be made.

Theorem 4. The system (16) with respect to $\Phi(x,y)$, obtained from (13) by conversion

$$U(x,y)=\exp Q(x,y)\cdot\Phi(x,y) \qquad (25)$$

has the same rank as the system (13).

Indeed, since the rank of the system (12) is equal to one, we present transformations (25) in the form of (15) and obtain a system with respect to $\Phi(x,y)$, where the rank is $p=1$. The proof of theorem for the General case is given in the monograph [12].

Theorem 5. The system (13) for which $p>0$, $m\le 0$ has normally regular solution (14), which is expressed through the generalized Laguerre's polynomial of two variables and the right-hand side (14) converges near the singularity $(x=0,y=0)$.

The system (13) is said to be related the system of the Laguerre type. As we have seen, its solutions are also expressed through the degenerate hypergeometric function of M.R. Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$ in the form of normal-regular series (14) dependent on the Laguerre's polynomial of two variables (15).

Conclusion: Thus, using the transformation (12), we have established the form of a system of Laguerre's type (3) related to the main system. The application of the Frobenius-Latysheva's method allowed us to construct normal-regular solutions of the derived related system (13) near the singularity $(0,0)$. Generalized Laguerre polynomials also have representations through other hypergeometric functions of two variables [14, p.358].

The limit transition formula is fair [15]

$$\lim_{\alpha\to\infty}L_{m,n}^{(\alpha,\beta,\gamma)}\left(\frac{x}{\alpha},\frac{y}{\alpha}\right)=L_m^{(\beta-1)}(x)\cdot L_n^{(\gamma-1)}(y).$$

Formula (24) can be similarly represented using the same limit transition as a product of Laguerre polynomials in variables x and y.

In the work [16, p. 6-17] the connection of considered systems with the overdetermined systems, studied in the works of Tajik Mathematical School [17] - [18] - [19] was indicated.

The research in this work can be extended to the case of three variables. The connection of the generalized Laguerre's polynomials of one and two variables with generalized hypergeometric functions [20], [21] of many variables was considered in [14] - [15], [22]. However, for this case the main theorem 1 and theorems 2–5 presented here haven't been proved yet. Also, the question of the computational application of special functions of several variables, as in the monograph [23] hasn't been touched upon. Following [24], it is necessary to develop a numerical method for calculating the values of the degenerate hypergeometric Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$ functions through the products of Laguerre polynomials in variables and using our formula (24). The problem of the asymptotic expansion, given in [25], is also important when studying the properties of special functions of several variables. We have obtained an asymptotic expansion near the origin $(0,0)$.

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ЛАГЕРРА ТЕКТЕС ТУЫСТАС ЖҮЙЕНІҢ ШЕШІМДЕРІН ТҰРҒЫЗУ

Аннотация. Жұмыстың мақсаты – Горнның туындалған жүйесінен параметрлерді тікелей таңдау және экспоненциал түрлендіру көмегімен алынған Лагерра тектi жүйені зерттеу. Мұндай екінші ретті дербес туындылы екі теңдеулерден тұратын дифференциалдық теңдьеулер жүйесін біз, Лагерра текті негізгі жүйемен туыстас деп атадық. Аталған жүйелерді зерттеудің қиындығы мынада: егер жай дифференциалдық теңдеулер жағдайында Куммердің бір туындалған теңдеуі бар және оны қанағаттандыратын бір ғана гипергеометриялық туындалған функциясы бар болса, онда екі айнымалы жағдайында 20 туындалған жүйелер пайда болады және оларды қанағаттандыратын 20 туындалған гипергеометриялық функциялар белгілі. Әзірге, Лагерра тектес қанша жүйелер бар екендігі және олардың туындалған жүйелердің қайсысымен байланыста екендігі белгісіз. Жалпыға ортақ зерттеу әдісі жоқ. Ұсынылған жұмыста екі айнымалының Лагерра көпмүшелігіне тәуелді қалыпты-регуляр шешімдер тұрғызу үшін, екі айнымалы жағдайына Ж.Н.Тасмамбетов жалпылаған Фробениус-Латышева әдісі пайдаланылады. Ранг және антиранг түсініктерін пайдаланып, ерекше қисықтардың классификациясы жасалған және мұндай жүйелердің қалыпты-регуляр шешімдерін тұрғызуға қатысты негізгі түсініктер келтірілген. Төрт сызықты-тәуелсіз дербес шешімдердің бар болатындығы туралы негізгі теорема дәлелденген. Ол дербес шешімдер Лагерраның екі айнымалының көпмүшелігіне тәуелді М.П.Гумберттің туындалған гипергеометриялық функциясы арқылы өрнектелген қалыпты-регуляр қатар арқылы анықталады. Қорытындысында, зерттелген жүйенің артығымен анықталған жүйелермен байланысы және екі айнымалының Лагерра көпмүшелігінің кейбір басқаша өрнектелуі келтірілген.

Түйін сөздер: туыстас, жүйе, Лагерра текті жүйе, Горн жүйесі, қалыпты-регуляр шешім, ерекше қисықтар, ранг, антиранг, артығымен анықталған.

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ПОСТРОЕНИЯ РЕШЕНИЯ РОДСТВЕННОЙ СИСТЕМЫ ТИПА ЛАГЕРРА

Аннотация. Целью работы является изучение системы типа Лагерра, полученной из вырожденной системы Горна непосредственным подбором параметров, а также с помощью экспоненциального преобразования. Такая система, состоящая из двух дифференциальных уравнений в частных производных второго порядка, нами названа родственной с основной системой типа Лагерра. Трудности изучения состоят в том, что если в обыкновенном случае имеет место одно вырожденное уравнение Куммера и только одна вырожденная гипергеометрическая функция, удовлетворяющая ему, то в случае двух переменных появляются 20 вырожденных систем и 20 вырожденных гипергеометрических функций двух переменных удовлетворяющих им. Пока не известно, сколько существуют систем типа Лагерра, и с какими из 20-ти вырожденных систем они связаны. Отсутствует общий метод исследования. В данной работе для построения их нормально-регулярного решения, зависящего от многочлена Лагерра двух переменных, применен обобщенный на этот случай Ж.Н.Тасмамбетовым метод Фробениуса-Латышевой. Приведена классификация особых кривых с помощью ранга и антиранга, а также основные сведения об особенностях построения нормально-регулярных решений таких систем. Доказана основная теорема о существовании четырех линейно-независимых частных решений, которые определяются через вырожденную гипергеометрическую функцию М.П.Гумберта в виде нормально-регулярных рядов зависящих от многочленов Лагерра двух переменных. В выводах указана связь таких систем с переопределенными системами и некоторыми представлениями многочлена Лагерра двух переменных.

Ключевые слова: Родственная, система, система типа Лагерра, система Горна, нормально-регулярное решение, особые кривые, ранг, антиранг, переопределенный

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