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HIGHER EXCITED STATES OF α+α SYSTEM

Abstract. In this work we investigate the higher excited states of α+α system applying the complex scaling method. The low-lying 0^+, 2^+ and 4^+ states of α+α are measured well but the higher excited states 6^+, 8^+ and 10^+ of α+α are not available by experimentally and these higher excited states have been barely studied by theoretical approaches.

Keywords: Complex scaling method, alpha-alpha system.

INTRODUCTION

The complex scaling method (CSM) [1-5] has been successfully utilized in the description of resonance states in light nuclei. The theory of the complex scaling was proposed mathematically [2] and it has been extensively applied to the atomic and nuclear physics [6-10].

In this work we investigate structure of α+α system. In particular, we focus on the its higher excited 6^+, 8^+ and 10^+ states because there is no experimental evidence for those higher states. But its low-lying 0^+, 2^+ and 4^+ states are experimentally well known. In addition, in this work we apply the CSM and harmonic oscillator wave function in order to calculate both low-lying and higher excited states of α+α.

THEORETICAL FRAMEWORK

Complex Scaling Method

In the CSM the relative coordinate is rotated as like $r \to re^{i\theta}$ in the complex coordinate plane. Therefore, the *Schrödinger* equation

$$\hat{H}\left|\psi\right\rangle = E\left|\psi\right\rangle \tag{1}$$

is rewritten as

$$\hat{H}(\theta)\left|\psi^{\theta}\right\rangle = E^{\theta}\left|\psi^{\theta}\right\rangle, \tag{2}$$

where $\hat{H}(\theta)$ and ψ^{θ} are the complex scaled Hamiltonian and wave function, respectively. The θ is scaling angle being a real number, $U(\theta)$ operate on a function ψ^{θ}, that is

$$\psi^{\theta} = U(\theta)\psi(r) = e^{\frac{3}{2}i\theta}\psi(re^{i\theta}). \tag{3}$$

The eigenvalues and eigenstates are obtained by solving the complex scaled *Schrödinger* equation Eq.(2). The eigenvalues of resonance states are found as $E^{\theta} = E_r - i\Gamma_r/2$, where E_r is resonance energy and Γ_r is the width of resonance. More detailed explanation of the CSM is given in Refs.[1-2].

Two body interaction

For the alpha-alpha system the Hamiltonian is expressed as

$$\hat{H} = \sum_{i=1}^{2} \hat{T}_i - \hat{T}_{c.m.} + V_{\alpha\alpha}^{Nucl}(r) + V_{\alpha\alpha}^{Coul}(r). \tag{4}$$

Harmonic oscillator wave function for radial part is

$$\varphi_{nl}(r) = N_l^n \left(\frac{r}{b_F}\right)^l L_n^{l+\frac{1}{2}}\left(\left(\frac{r}{b_F}\right)^2\right) \exp\left(-\frac{1}{2b_F^2} r^2\right) Y_{lm}(r), \tag{5}$$

here $L_n^{l+\frac{1}{2}}$ are Laguerre polynomials for the angular momentum l and N_l^n denotes the normalization constants as given by $N_l^n = \left\{\frac{2\Gamma(n+1)}{b_F^3 \Gamma\left(l+n+\frac{3}{2}\right)}\right\}^{1/2}$. The size parameter of relative motion of two alpha-cluster b_F is taken as 0.967 fm which corresponds to a single particle size parameter $b_0 = 1.3975$ fm employed to fit the observed r.m.s. radius of ^{4}He [11-12].

Alpha-alpha potential

The α+α potential is constructed by the folding approach for the effective nucleon-nucleon interaction by the Schmid-Wildermuth [13-14] potential. An effective two-nucleon force is written as,

$$v_{ij} = V\{W + BP_\sigma(ij) - HP_\tau(ij) - MP_\sigma(ij)P_\tau(ij)\} \cdot \exp\left(-\mu r^2\right) \tag{6}$$

where $P_\sigma(ij)$ and $P_\tau(ij)$ are the spin and isospin exchange operators. In this work we employ the Schmid-Wildermuth potential as a nucleon-nucleon force, which is given by following parameters:

$$V = -72.98 \text{ MeV}; \ \mu = 0.46 \text{ fm}^{-2};$$
$$W = M = 0.4075; \ B = H = 0.0925. \tag{7}$$

The folding potential of the alpha-alpha system is obtained from such a nucleon-nucleon force and also the Coulomb force.

Its explicit form is

$$V_{\alpha\alpha}^{Nucl} + V_{\alpha\alpha}^{Coul} = 2X_D \left[\frac{2v_\alpha}{2v_\alpha + \frac{3\mu}{2}}\right]^{\frac{3}{2}} V \exp\left(-\frac{v_\alpha \mu}{v_\alpha + \frac{3\mu}{4}} r^2\right) + \frac{4e^2}{r} erf\left(r\sqrt{\frac{4}{3}v_\alpha}\right), \tag{8}$$

where $X_D = 2.445$ and $erf(x)$ is the error function. We use a harmonic oscillator constant $v_\alpha = \frac{M\omega}{2\hbar} = 0.2675$ fm^{-2} which is obtained by using $r_{rms} = 1.63$ fm of the alpha-cluster.

In Eq. (8) the simplified notations can be applied:

$$V_0 = 2X_D V \left[\frac{2v_\alpha}{2v_\alpha + \frac{3\mu}{2}}\right]^{\frac{3}{2}},$$

$$\beta = \frac{\nu_\alpha \mu}{\nu_\alpha + \frac{3\mu}{4}},$$

$$\alpha = \sqrt{\frac{4}{3}\nu_\alpha}.$$

RESULTS

Complex Scaling Method

Figure 1 displays the complex energy eigenvalues of 4^+ state which is obtained by diagonalization of Eq.(2) with $N_{\max} = 50$ for $\theta = 13^0$. We can see all energies are on lines of $\arg(E_0) = 2\theta$ which correspond to the branch cut of the complex energy plane. When we take larger values of θ, we observe isolated energy points, being resonance states, whose positions are almost unchanged by varying $\theta \geq \frac{1}{2}\left|\arg\left(E_\theta^R\right)\right|$.

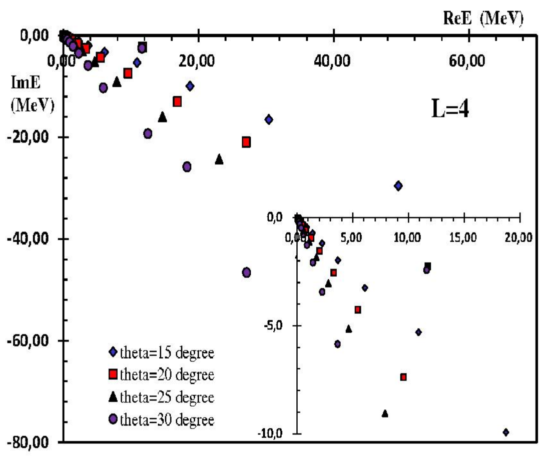

Figure 1. The resonance eigenvalues at $J^\pi=4^+$ for the different θ

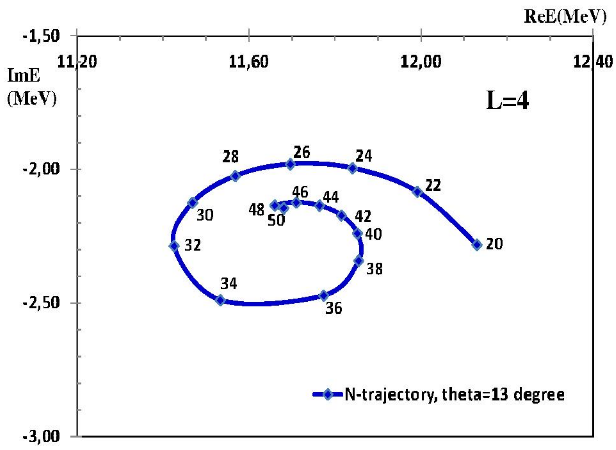

Figure 2. N-trajectory at $J^\pi=4^+$. The folding potential and harmonic oscillator wave function are used

In Fig. 2 presents the eigenvalues of 4^+ state which is calculated with $N_{\max} = 20 \sim 50$. This is called the $N_{\max}$-trajectory. As the number of basis states is increased, the $N_{\max}$-trajectory shows the spiral convergence. The radius of curvature of the spiral depends on the values of θ and b_0. In calculation of the $N_{\max}$-trajectory, we fixed to $\theta = 13^0$ and $b_0 = 1.3975$ fm for 4^+. The computed decay widths for the experimentally unknown 6^+, 8^+ and 10^+ higher states are rather large, however, they can be recognized as resonances on the complex energy plane.

Table I - Experimental and calculated resonance energies with corresponding decay widths of $\alpha+\alpha$ system

States	Experimental data [12]		Present work	
	E_r(MeV)	Γ_r(MeV)	E_r(MeV)	Γ_r(MeV)
4^+	11.35	~3.5	11.7	4.4
6^+	-	-	30.5	36.8
8^+	-	-	51.6	120
10^+	-	-	70.0	180

For the case of 6^+, 8^+ and 10^+ states, there are no measured data, however, we calculated a resonance energy with a broad decay width applying harmonic oscillator wave function. The calculated results are given in table 1.

SUMMARY

Positions and widths of low-lying and higher excited states of ^{8}Be are calculated by using the CSM and the two-body model. The result of recent calculation indicates that reasonably good agreement with measured data for 4^+ state. It is remarkable that the energies with decay widths of the higher excited 6^+, 8^+ and 10^+ states are calculated.

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α+α ЖҮЙЕСІНІҢ ЖОҒАРҒЫ ҚОЗҒАН КҮЙЛЕРІ

Аннотация. Бұл жұмыста кешенді масштабтау әдісін қолдана отырып α+α жүйесінің жоғары қозған күйлері зерттелді. α+α-ның төмен 0^+, 2^+ және 4^+ күйлері өлшенген, бірақ α+α жүйесінің 6^+, 8^+ және 10^+ жоғары қозған күйлерінің экспериментaлды мәндері белгісіз болғандықтан, бұл жоғары қозған күйлер теориялық тәсілдермен есептелінді.

Түйін сөздер: кешенді масштабтау әдісі, альфа-альфа жүйесі.

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ВЫСОКИЕ ВОЗБУЖДЕННЫЕ СОСТОЯНИЯ α+α СИСТЕМЫ

Аннотация. В данной работе мы исследуем высокие возбужденные состояния α+α системы, применяя метод комплексного масштабирования. Низколежащие 0^+, 2^+ и 4^+ состояния α+α системы хорошо известны, но высокие возбужденные состояния 6^+, 8^+ и 10^+ α+α системы не доступны экспериментально, поэтому эти высокие возбужденные состояния были изучены теоретическими подходами.

Ключевые слова: метод комплексного масштабирования, альфа-альфа-система.

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