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DYNAMIC STABILITY OF WAVE PROCESSES OF A ROUND ROD

Abstract: This paper is devoted to the study of the stability dynamics of wave processes of flat and circular elements, and also some axisymmetric problems of oscillation of an elastic layer limited by rigid or deformable boundaries when exposed to normal or rotational shear stresses are considered. Solutions to the problems under consideration were obtained using integral transformations by coordinate or time. The work develops the dynamic stability of a round rod. The loss of stability of a round rod will be investigated on the basis of the mathematical theory and the transverse oscillations of a round rod, described in the work of I.G. Philippov.

Key words: oscillations, stability, wave process, axisymmetric problems, round rod, exponential transformation, shear stress.

The issues of stability in a static formulation have been developed by many authors, and the results of such studies can be found in [1,2].

However, the issues of dynamic stability of elements' construction and structures received much less attention.

In this paper, the dynamic stability of a round rod is developed.

Consider a round elastic rod of length *l*. We will assume that an axial compressive force of intensity is applied to the ends of the rod at any moment of time.

The loss of stability of a round rod will be investigated on the basis of the mathematical theory and the transverse oscillation of a round rod, described in the work of I.G. Philippov [3].

The fourth order equation describing the transverse oscillation of the rod has the form.

$$A_0 \frac{\partial^4 V}{\partial t^4} - A_1 \frac{\partial^4 V}{\partial z^2 \partial t^2} + A_2 \frac{\partial^4 V}{\partial z^4} + \rho \frac{\partial^2 V}{\partial t^2} = F(z,t), \tag{1}$$

where the constants A_j are equal to

$$A_0 = \frac{\mu r_0^2 (4a^2 + b^2)}{8b^4 (a^2 - b^2)}; A_1 = \frac{\mu r_0^2 (13a^2 - 8b^2)}{8b^2 (a^2 - b^2)}; A_2 = \mu r_0^2, \tag{2}$$

r_0 - rod radius, a and b - propagation velocity of longitudinal and transverse waves in the material of the rod, μ - the module of shift модуль сдвига, ρ - rod material density.

Under the action of compressive force for a small period of time Δt when solving bending forces in cross-sectional planes of the rod ΔF , as we know, equals to [2]

$$\Delta F = \Delta P \frac{\partial^2 V}{\partial z^2} \quad (3)$$

Summing (3) over time, we get the power F :

$$F(z,t) = \int_0^t P'(t-\xi) \frac{\partial^2 V}{\partial z^2} d\xi \quad (4)$$

where the lower limit of integration depends on the beginning of the force P and, for example if P power is applied at the $t = 0$ then the lower integration limit is also zero.

Obviously, if the compressive force is constant and is applied at the moment $t = 0$, then it can be written as $P = P_0 H(t)$, где $H(t)$ Heaviside function, then we get the following: (4)

$$F = -P_0 \frac{\partial^2 V}{\partial z^2} \quad (5)$$

Therefore, equation (5) can be written in the following form

$$A_0 \frac{\partial^4 V}{\partial t^4} - A_1 \frac{\partial^4 V}{dz^2 \partial t^4} + A_2 \frac{\partial^4 V}{\partial z^4} + \rho \frac{\partial^2 V}{\partial t^2} - \int_0^t P'(t-\xi) \frac{\partial^2 V}{\partial z^2} d\xi = 0 \quad (6)$$

To study the stability of the rod, it is necessary to formulate the boundary conditions at the ends of the rod. These conditions are: hinged joints $V = \frac{\partial^2 V}{\partial z^2} = 0$, rigid fastening $V = \frac{\partial V}{\partial z} = 0$, free ending $\frac{1}{a^2}\frac{\partial^2 V}{\partial t^2} + 4\frac{\partial^2 V}{\partial z^2} = 0; \frac{\partial^3 V}{\partial z^3} = 0.$ In order to study the stability of the rod, it is necessary to formulate the boundary conditions at the ends of the rod. These conditions are: hinged joints $V = \frac{\partial^2 V}{\partial z^2} = 0$, rigid fastening $V = \frac{\partial V}{\partial z} = 0$, free ending

$$\frac{1}{a^2}\frac{\partial^2 V}{\partial t^2} + 4\frac{\partial^2 V}{\partial z^2} = 0; \frac{\partial^3 V}{\partial z^3} = 0.$$

The solution of equation (6) will be sought believing that

$$V(z,t) = V_0(z)\exp(i\omega t) \quad (7)$$

where ω is oscillation frequency. Putting (7) into the equation (6), for для V_0 we get the equation

$$A_2 \frac{d^4 V}{dz^4} + [A_1 \omega^4 - \int_0^\infty P'(\xi)\exp(i\omega\xi)d\xi] \frac{d^2 V_0}{dz^2} + A_0 \omega^4 V_0 - \rho \omega^2 V_0 = 0 \quad (8)$$

Denoting by $Q(\omega)$ Fourier's exponential transform of rate of change of compressive force $P'(t)$, equation (8) we write in the form

$$\frac{d^4V_0}{dz^4}+B_0\frac{d^2V_0}{dz^2}+B_1V_0=0 \tag{9}$$

where the coefficients are B_0, B_1 are equal to

$$B_0=\frac{A_1\omega^2-Q(\omega)}{A_2}; \qquad B_1=\frac{A_0\omega^4-\rho\omega^2}{A_2} \tag{10}$$

For simplicity, we consider the case when the ends of the rod are hinged, i.e. at $z=0$ and $z=l$ conditions are met

$$V=\frac{\partial^2V}{\partial z^2}=0 \tag{11}$$

The solution of equation (9) is sought in the form of

$$V_0=\sum_{n=1}^{\infty}V_{0n}\sin\left(\frac{\pi n}{l}z\right) \tag{12}$$

at the same time, boundary conditions (11) are satisfied automatically.
Putting (12) into the equation (9) for the length $Q(\omega)$ we get

$$Q(\omega)=A_1\omega^2-A_2\left(\frac{l}{\pi n}\right)^2\left[B_1+\left(\frac{\pi n}{l}\right)^4\right] \tag{13}$$

Введем безразмерные величины
Let's introduce dimensionless quantities

$$Q_0(\xi)=\frac{Q(\omega)}{\mu\pi^4};\xi=\frac{\omega l}{\pi b};c=\pi\left(\frac{r_0}{l}\right);\frac{b^2}{a^2}=\frac{1-2\upsilon}{2(1-\upsilon)} \tag{14}$$

And then for $Q_0(\xi)$ we get

$$Q_0(\xi)=\frac{2}{c^2}\left[\frac{(9-5\upsilon)c^2+4}{9-10\upsilon}\right]\xi^2-\xi^4-\frac{8}{9-10\upsilon} \tag{15}$$

where υ - Poisson's ratio.

The physical meaning of the influence of the rate of change of the compressive force is that, it is important that the impulse of this force or the rate of increase of the force over time as in the impact theory.

You can also consider other types of binding. For example, you can search for solutions of equation (9) in the form

$$V_0 = C_1\left[\frac{\cos(\alpha_0 z)}{\alpha_0^n}+\frac{\cos(\alpha_1 z)}{\alpha_1^n}\right]+C_2\left[\frac{\cos(\alpha_0 z)}{\alpha_0^n}-\frac{\cos(\alpha_1 z)}{\alpha_1^n}\right]+ \\ +C_3\left[\frac{\sin(\alpha_0 z)}{\alpha_0^m}+\frac{\sin(\alpha_1 z)}{\alpha_1^m}\right]+C_4\left[\frac{\sin(\alpha_0 z)}{\alpha_0^m}-\frac{\sin(\alpha_1 z)}{\alpha_1^m}\right] \tag{16}$$

where C_j - arbitrary constants, integers (n,m) are selected from conditions at $Z=0$ и $Z=l$

$\dot{a}_j$ - the roots of the characteristic equation and are equal

$$a_j=\sqrt{\frac{B_0}{2}\pm\sqrt{\frac{B_0^2}{4}-B_1}}$$

In the case of rigid fastening of both ends of the rod integers $(n=0, m=1)$ and to find $Q(\omega)$ we get the transcendental equation

$$2-\frac{\alpha_0^2+\alpha_1^2}{\alpha_0\alpha_1}\sin(\alpha_0 l)\sin(\alpha_1 l)-2\cos(\alpha_0 l)\cos(\alpha_1 l)=0, \tag{17}$$

Similarly, you can consider other combinations of conditions for binding the ends of the rod.

Some axisymmetric problems of oscillation of an elastic layer bounded by rigid or deformable boundaries when exposed to a normal or rotational shear stress are considered below. Solutions to the problems under consideration were obtained using integral transformations by coordinate or time.

First we consider the problem for half-space under the assumption that the half-space $z>0$ is an anisotropic medium with the axis of symmetry of the mechanical properties (axis) (axis z), and the surface of which is subjected to an impulse voltage at the time $\xi_{z0}=-f(r,t)$.

Because of the symmetry of the mechanical properties of the medium relative to the axis z of the unique nonzero component of the displacement vector, $U_0(r,z,t)$ only the stresses and ξ_{r0} and ξ_{z0} the ones determined by formulas

$$\xi_{r0}=C_{nn}\left(\frac{\partial U_0}{\partial r}-\frac{U_0}{r}\right), \\ \xi_{z0}=C_{mm}\frac{\partial U_0}{\partial z} \tag{18}$$

The equation of motion reduces to one

$$\frac{\partial \xi_{r0}}{\partial r}+\frac{\partial \xi_{r0}}{\partial z}+\frac{2\xi_{r0}}{r}=\rho\frac{\partial^2 U_0}{\partial t^2} \tag{19}$$

Substituting the expressions for ξ_{r0} and ξ_{z0} from (18) into equation (19), we bring it to the form:

$$\frac{\partial^2 U_0}{\partial r^2}+\frac{1}{r}\frac{\partial U_0}{\partial r}-\frac{U_0}{r^2}+\gamma^2\frac{\partial^2 U_0}{\partial z^2}=\frac{1}{b^2}\frac{\partial^2 U_0}{\partial t^2} \tag{20}$$

where

$$b^2 = \frac{C_{nn}}{\rho}; \ \gamma^2 = \frac{C_{mm}}{C_{nn}}$$

If the half-space is isotropic, then $\gamma = 1$ and $b = \sqrt{\frac{\mu}{\rho}}$.

The boundary conditions for U_0 has:

$$\xi_{z0} = -f(r,t) \text{ under } z = 0, \ t \geq 0 \tag{21}$$

$$U_0 \to 0 \text{ under } z \to \infty \tag{22}$$

The initial conditions of the problem are zero

$$U_0 = \frac{\partial U_0}{\partial t} = 0 \qquad \text{under } t = 0 \tag{23}$$

The solution of equation (20) for the boundary (21) - (22) and the initial conditions (6) will be sought, by applying the Laplace transform. Suppose,

$$U(r,z,p) = \int_0^\infty U_0(r,z,t)e^{-pt}dt, \ \operatorname{Re} p > 0 \tag{24}$$

Definitely, for the function $U(r,z,p)$ we obtain the equation

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} - \left(\frac{1}{r^2} + \frac{p^2}{b^2}\right)U + \gamma^2 \frac{\partial^2 U}{\partial z^2} = 0 \tag{25}$$

And U must satisfy the boundary conditions:

$$\frac{\partial U}{\partial z} = -\frac{f_0(r,p)}{C_{66}} \qquad \text{under } z = 0, \ t > 0 \tag{26}$$

$$U_0 \to 0 \text{ under } z \to \infty \tag{27}$$

where

$$f_0(r,p) = \int_0^\infty (r,t)e^{-pt}dt.$$

The general solution of equation (25) is sought by the method of separation of variables (the Fourier method) and has the form:

$$U(r,z,p) = \int_0^\infty \alpha \left[A(\alpha,p)e^{-\frac{z}{\gamma}\sqrt{\alpha^2 + \frac{p^2}{b^2}}} + B(\alpha,p)e^{\frac{z}{\gamma}\sqrt{\alpha^2 + \frac{p^2}{b^2}}} \right] J_1(\alpha r)d\alpha. \tag{28}$$

where $A(\alpha,p)$ and $B(\alpha,p)$ are defined from boundary conditions (26)-(27) from the condition (29) follows that

$$B(\alpha,p) = 0 \tag{29}$$

Using boundary condition (26), for defining $A(\alpha, p)$ we'll take integral equation:

$$\int_0^\infty \alpha A(\alpha, p)\sqrt{\alpha^2 + \frac{p^2}{b^2}} J_1(\alpha r) d\alpha = \frac{\gamma}{C_{66}} f_0(r, p) \tag{30}$$

Let

$$f_0(r, p) = \int_0^\infty \alpha f_1(\alpha, p) J_1(\alpha r) d\alpha \tag{31}$$

Then

$$A(\alpha, p) = \frac{\gamma f_1(\alpha, p)}{C_{mm}\sqrt{\alpha^2 + \frac{p^2}{b^2}}} \tag{32}$$

Substituting expression (29) and (32) into formula (28), we obtain the following expression:

$$U(r, z, p) = \frac{\gamma}{C_{mm}} \int_0^\infty \frac{\alpha f_1(\alpha, p)}{\sqrt{\alpha^2 + \frac{p^2}{b^2}}} J_1(\alpha r) e^{-\frac{z}{\gamma}\sqrt{\alpha^2 + \frac{p^2}{b^2}}} d\alpha \tag{33}$$

We consider the special case when

$$f_0(r, p) = \frac{\varphi_0(p)}{r} \tag{34}$$

In case (34) function

$$f_1(\alpha, p) = \frac{\varphi_0(p)}{\alpha}$$

and (33) has

$$U(r,z,p) = \frac{\gamma}{C_{mm}} \int_0^\infty \frac{\varphi_0(p)}{\sqrt{\alpha^2 + \frac{p^2}{b^2}}} e^{-\frac{z}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}} J_1(\alpha r) d\alpha = \frac{1}{C_{mm}} \varphi_0(p) I_{\frac{1}{2}}\left[\frac{p}{2b}\left(\sqrt{\frac{z^2}{\gamma^2} + r^2} - \frac{z}{\gamma}\right)\right] K_{\frac{1}{2}}\left[\frac{p}{2b}\left(\sqrt{\frac{z^2}{\gamma^2} + r^2} + \frac{z}{\gamma}\right)\right]$$

where $K_{\frac{1}{2}}, I_{\frac{1}{2}}$ Bessel functions of imaginary argument. Using the representations of the functions $I_{\frac{1}{2}}(\zeta)$ and $K_{\frac{1}{2}}(\zeta)$, for $U(r, z, p)$ we find that

$$U(r, z, p) = \frac{b\varphi_0(p)}{C_{mm} r p}\left[e^{-\frac{pz}{\gamma a}} - e^{-\frac{p}{a}\sqrt{\frac{z^2}{\gamma^2} + r^2}}\right] \tag{35}$$

Paying attention to the expression (35) in p, for the required value $U_0(r,z,t)$, we'll take the expression

$$U_0(r,z,t)=\frac{b}{\gamma C_{mm} r}\int_0^t f_1(t-\xi)\left[H\left(\xi-\frac{z}{\gamma b}\right)-H\left(\xi-\frac{1}{b}\sqrt{\frac{z^2}{\gamma^2}+r^2}\right)\right]d\xi \tag{36}$$

where

$$f(r,t)=\frac{f_1(t)}{r}.$$

The resulting expression $U_0(r,z,t)$ consists of two terms, the first term corresponding to plane wave propagating in half-space with velocity γb and parallel to the plane $z = 0$, and the second term to the diffracted wave, which has the form of half-ellipsoid of revolution (hemisphere at $\gamma = 1$) and in contact with plane wave on the rotation axis at $z = b\gamma t$.

Besides, from (36) follows, that $U_0(r,z,t)$ fades out from r as $1/r$.

If the acting function $f(r,t)$ is arbitrary, then we represent it in the form of Schlemmich series:

$$f(r,t)=\frac{1}{r}\sum_{j=0}^{\infty}a_j(t)J_0(jr)=\frac{f_r(t)}{r} \tag{37}$$

where

$$a_0(t)=\frac{1}{\pi}\int_0^{\pi}\left\{f_r(0,t)+U\int_0^1\frac{\frac{\partial f_r(\xi U,t)}{\partial r}}{\sqrt{1-\xi^2}}d\xi\right\}dU;$$

$$a_j(t)=\frac{2}{\pi}\int H\cos(jU)\left\{\int_0^1\frac{\frac{\partial f_r(\xi U,t)}{\partial r}}{\sqrt{1-\xi^2}}d\xi\right\}dU;\ j=1,2,... \tag{38}$$

For $f(r,t)$ type (37) function $f_1(\alpha,p)$, entering to the formula (33), is

$$f_1(\alpha,p)=\frac{1}{\alpha}\sum_{j=0}^{\infty}a_{j0}(p)H(\alpha-j) \tag{39}$$

where

$$a_{j0}(p)=\int_0^{\infty}a_j(p)e^{-pt}dt. \tag{40}$$

Following,

$$U(r,z,p)=\frac{\gamma}{C_{66}}\sum_{j=0}^{\infty}a_{j0}(p)\int_0^{\infty}\frac{e^{-\frac{z}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}}{\sqrt{\alpha^2+\frac{p^2}{b^2}}}J_1(\alpha r)d\alpha \tag{41}$$

Paying attention (41) on p and using theory about convolution, we'll take

$$U_0(r,z,p)=\frac{\gamma}{2C_{66}}\sum_{j=0}^{\infty}\int_g^t a_j(t-\xi)T_j(r,z,\xi)d\xi \tag{42}$$

where

$$T_j(r,z,\xi)=b\int_j^{\infty}J_1(\alpha r)J_0\left[\alpha\sqrt{b^2t^2-\frac{z^2}{\gamma^2}}\right]d\alpha \tag{43}$$

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ДӨҢГЕЛЕК ӨЗЕКТІҢ ТОЛҚЫНДЫҚ ПРОЦЕСТЕРІНІҢ ДИНАМИКАЛЫҚ ТҰРАҚТЫЛЫҒЫ

Аннотация. Бұл жұмыс жазық және айналмалы элементтердің толқындық процестерінің орнықтылық динамикасын зерттеуге арналған, сондай-ақ мақалада қалыпты немесе айналмалы жанасу кернеулерінің әсер етуі кезінде қатты немесе деформацияланатын шекаралармен шектелген серпімді қабат тербелісінің кейбір осесимметриялық есептері қарастырылады. Қарастырылатын есептердің шешімдері координат бойынша немесе уақыт бойынша интегралды түрлендірулерді пайдалана отырып алынған. Жұмыста дөңгелек өзектің динамикалық тұрақтылығы жайлы есепбін терең зеріттеледі. Дөңгелек өзектің орнықтылығының ауытқуы И. Г. Филипповтың жұмысында баяндалғандай, дөңгелек өзектің көлденең тербелісі математикалық теория негізінде зерттелетін болады.

Түйін сөз: тербелістер, орнықтылық, толқындық процесс, осесимметриялық есептер, дөңгелек өзек, экспоненциалды түрлендіру, жанама кернеу.

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ДИНАМИЧЕСКАЯ УСТОЙЧИВОСТЬ ВОЛНОВЫХ ПРОЦЕССОВ КРУГЛОГО СТЕРЖНЯ

Аннотация: Данная работа посвящена изучению динамики устойчивости волновых процессов плоских и круговых элементов, а также рассматривается некоторые осесимметричные задачи колебания упругого слоя ограниченные жесткими или деформируемыми границами при воздействии на него нормального или вращательного касательного напряжений. Решения рассматриваемых задач получены с использованием интегральных преобразований по координате или по времени. В работе развивается динамическая устойчивость круглого стержня. Потеря устойчивости круглого стержня будет исследоваться на основе математической теории и поперечного колебания круглого стержня, изложенной в работе И.Г.Филиппова.

Ключевые слова: колебания, устойчивость, волновой процесс, осесимметричные задачи, круглый стержнь, экспоненциальное преобразование, касательная напряжения.

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