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# COEFFICIENT CRITERION OF EXISTENCE OF MULTIPERIODIC SOLUTIONS OF A LINEAR SYSTEM OF FOUR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS ON DIAGONAL

**Abstract.** In the note, we considered a linear system of four differential equations with the differentiation operator $D_e$ in the direction of the main diagonal of the space of time variables $(\tau, t)$. We conducted a study of the $(\theta, \omega, \omega)$-periodicity of the solutions of the linear system of equations with variables but constants coefficients on the diagonal which depend on variables of the eigenvalues of the characteristic equation. The coefficient criterion of the properties of separation and sign-definiteness of distinct real, multiple and complex conjugate eigenvalues are found. Periodicity and continuous differentiability of eigenvalues are investigated. The coefficient sufficient conditions for the existence of periodic solutions are established. The concepts of variable frequency and variable period are introduced.

Investigation of the problems of partial differential equations is closely connected with the theory of ordinary differential equations [1-5]. It is known that the study of the problems of multiperiodic solutions of systems of first-order partial differential $D_e$-equations originates in the works [6-8]. This investigation on the formulation of the question adjoins previous studies [9-14].

**Key words:** linear system, differentiation operator, eigenvalues, characteristic equation, real and imaginary parts, diagonal minors.

Consider a linear system

$$D_e x = A(\sigma)x, \quad (1)$$

where $A(\sigma)$ an $n \times n$-matrix has the properties of periodicity and smoothness of the form

$$A(\sigma + k\omega) = A(\sigma) \in C_{\sigma}^{(e)}(R^m), \quad \forall k \in Z^m, \quad (2)$$

$\sigma = t - e\tau$ is called the characteristic of the operator $D_e = \frac{\partial}{\partial \tau} + \left\langle e, \frac{\partial}{\partial t} \right\rangle$, $\tau \in (-\infty, +\infty) = R$, $t = (t_1, ..., t_m) \in R \times ... \times R = R^m$, $e = (1, ..., 1)$ – $m$-vector, $\langle \, , \rangle$ denotes the scalar product, $k\omega = (k_1\omega_1, ..., k_m\omega_m)$ - a multiple vector-period, $x = (x_1, ..., x_n)$ - the unknown vector.

We form the characteristic equation

$$\det[A(\sigma) - \lambda E] = 0, \quad (3)$$

where $E$ - the identity matrix. The equation (3) for a fixed value $\sigma \in R^m$ is solvable in the field of complex numbers and has roots of the form $\lambda_j(\sigma) = \alpha_j(\sigma) + i\beta_j(\sigma)$, $(j = \overline{1, n})$.

The elucidation of the question of the properties of smoothness, periodicity, sign-definiteness and separation of the eigenvalues of the system (1) in the general case is a serious problem.

But in the well-known particular cases, the roots of equation (3) are determined in the radicals and it is possible to solve this problem on the basis of the conditions imposed on the coefficients of the system.

When investigating various problems of non-linear systems corresponding to the linear system (1), it is important to know that the eigenvalues of the matrix $A(\sigma)$ have the following properties:

$1^0$. Continuous differentiability: $\lambda_j(\sigma) \in C_\sigma^{(e)}(R^m)$, $j = \overline{1,n}$.

$2^0$. Periodicity: $\lambda_j(\sigma + k\omega) = \lambda_j(\sigma)$, $j = \overline{1,n}$, $\sigma \in R^m$, $k \in Z^m$.

$3^0$. Property of having a fixed sign $\lambda_j(\sigma)$ for each $j = \overline{1,n}$:

a) $\lambda_j(\sigma) < 0$, $\forall \sigma \in R^m$ or

b) $\lambda_j(\sigma) = 0$, $\forall \sigma \in R^m$ or

c) $\lambda_j(\sigma) > 0$, $\forall \sigma \in R^m$.

$4^0$. Separation of eigenvalues:

a) for $j \neq l$ $\lambda_j(\sigma) \neq \lambda_l(\sigma)$, $\forall \sigma \in R^m$ or

b) for $j \neq l$ $\lambda_j(\sigma) = \lambda_l(\sigma)$, $\forall \sigma \in R^m$,

i.e. for each value, the eigenvalue $\lambda_j(\sigma)$ has constant multiplicity $k_j = const$ for all $\sigma \in R^m$.

$5^0$. Each of the sets $\operatorname{Re}\{\lambda_j(\sigma)\}$ and $\operatorname{Im}\{\lambda_j(\sigma)\}$ has properties $1^0$-$4^0$.

We note that if eigenvalues of the matrix $A(\sigma)$ possess these indicated properties, then the system (1) has a solution of the same structure as in the case of constant coefficients.

For the purpose of illustration, we consider the case when equation (1) in scalar form has the form

$$D_e x_i = \sum_{j=1}^{4} a_{ij}(\sigma) x_j, \quad j = \overline{1,4}, \tag{4}$$

the equation (3) is always solvable in radicals, and the characteristic equation of the matrix $A(\sigma) = [a_{ij}(\sigma)]_1^4$ can be represented in the form

$$\lambda^4 - a_1(\sigma)\lambda^3 + a_2(\sigma)\lambda^2 - a_3(\sigma)\lambda + a_4(\sigma) = 0, \tag{5}$$

where coefficients $a_j(\sigma)$ are determined by the sum of all diagonal minors $M_j(\sigma)$ of order $j$, in particular, $a_1(\sigma) = SpA(\sigma)$, $a_4(\sigma) = \det A(\sigma)$ and the coefficients $a_2(\sigma)$ and $a_3(\sigma)$ are determined by expressions

$$a_2(\sigma) = \sum_{\substack{j=1 \\ \alpha_j < \beta_j}}^{6} \begin{vmatrix} a_{\alpha_j\alpha_j}(\sigma) & a_{\alpha_j\beta_j}(\sigma) \\ a_{\beta_j\alpha_j}(\sigma) & a_{\beta_j\beta_j}(\sigma) \end{vmatrix},$$

$$a_3(\sigma) = \sum_{\substack{j=1 \\ \alpha_j < \beta_j < \gamma_j}}^{4} \begin{vmatrix} a_{\alpha_j\alpha_j}(\sigma) & a_{\alpha_j\beta_j}(\sigma) & a_{\alpha_j\gamma_j}(\sigma) \\ a_{\beta_j\alpha_j}(\sigma) & a_{\beta_j\beta_j}(\sigma) & a_{\beta_j\gamma_j}(\sigma) \\ a_{\gamma_j\alpha_j}(\sigma) & a_{\gamma_j\beta_j}(\sigma) & a_{\gamma_j\gamma_j}(\sigma) \end{vmatrix}.$$

Substituting

$$\lambda = \eta - \frac{a(\sigma)}{4} \tag{6}$$

we reduce equation (5) to the form

$$\eta^4 + p(\sigma)\eta^2 + q(\sigma)\eta + r(\sigma) = 0, \quad (7)$$

where $p(\sigma) = a_2(\sigma) - \frac{3}{8}a_1^2(\sigma)$, $q(\sigma) = a_3(\sigma) - \frac{1}{2}a_1(\sigma)a_2(\sigma) + \frac{1}{8}a_1^3(\sigma)$,

$r(\sigma) = \frac{1}{16}a_1^2(\sigma)a_2(\sigma) - \frac{1}{4}a_1(\sigma)a_3(\sigma) - \frac{3}{256}a_1^4(\sigma) + a_4(\sigma)$.

Then, left-hand side of this equation identically rearranges by means of an accessory parameter $\rho$:

$$\left(\eta^2 + \frac{p(\sigma)}{2} + \rho\right)^2 - \left[2\rho\eta^2 - q(\sigma)\eta + \left(\rho^2 + p(\sigma)\rho - r(\sigma) + \frac{p^2(\sigma)}{4}\right)\right] = 0. \quad (8)$$

We now choose $\rho$ so that the polynomial in square brackets becomes a complete square. For this, it must have one double root, i.e. equality must hold

$$q^2(\sigma) - 4\cdot 2\rho\left(\rho^2 + p(\sigma)\rho - r(\sigma) + \frac{p^2(\sigma)}{4}\right) = 0,$$

which is a cubic equation in unknown $\rho$:

$$8\rho^3 + 8\rho^2 p(\sigma) - 2\rho[4r(\sigma) - p^2(\sigma)] - q^2(\sigma) = 0. \quad (9)$$

Substituting

$$\rho = \mu - \frac{p(\sigma)}{3}$$

we reduce equation (9) to the form

$$\mu^3 + p_1(\sigma)\mu + q_1(\sigma) = 0 \quad (10)$$

with coefficients

$$p_1(\sigma) = \frac{1}{12}p^2(\sigma) - r(\sigma), \quad q_1(\sigma) = -\frac{1}{108}p^3(\sigma) + \frac{1}{8}q^2(\sigma) + \frac{1}{3}p(\sigma)r(\sigma).$$

We calculate the discriminant $\Delta(\sigma)$ of equation (10)

$$\Delta(\sigma) = -108\left(\frac{q_1^2(\sigma)}{4} + \frac{p_1^3(\sigma)}{27}\right)$$

and require that it be sign-definite:

a) $\Delta(\sigma) < 0$, $\sigma \in R^m$ or

b) $\Delta(\sigma) = 0$, $\sigma \in R^m$ or (11)

c) $\Delta(\sigma) > 0$, $\sigma \in R^m$.

If any of the conditions (11) is satisfied, equation (10), and, consequently, equation (9), in accordance with the conditions for the existence of the real roots of the cubic equations, has at least one real root $\mu = \mu_0(\sigma)$. Then we assume that $\rho = \rho_0(\sigma) = \mu_0(\sigma) - \frac{p(\sigma)}{3}$ is sign-definite, namely, we set

$$\rho_0(\sigma) = \mu_0(\sigma) - \frac{p(\sigma)}{3} > 0. \quad (12)$$

For this value $\rho$, the polynomial in square brackets in (8) has a double root $\frac{q(\sigma)}{4\rho_0(\sigma)}$. Therefore, equation (8) simplifies to

$$\left(\eta^2+\frac{p(\sigma)}{2}+\rho_0(\sigma)\right)^2-2\rho_0(\sigma)\left(\eta-\frac{q(\sigma)}{4\rho_0(\sigma)}\right)^2=0,$$

which splits into two quadratic equations, the roots are determined by the formulas

$$\eta_i(\sigma)=\frac{\sqrt{\rho_0(\sigma)}\pm\sqrt{\delta(\sigma)}}{\sqrt{2}},\quad i=1,2, \tag{13}$$

$$\eta_j(\sigma)=\frac{-\sqrt{\rho_0(\sigma)}\pm\sqrt{\delta(\sigma)}}{\sqrt{2}},\quad j=3,4,$$

where the function $\delta(\sigma)$ has the form

$$\delta(\sigma)=-\rho_0(\sigma)-p(\sigma)-\frac{q(\sigma)}{\sqrt{2\rho_0(\sigma)}}. \tag{14}$$

We impose the conditions of sign-definiteness on the function $\delta(\sigma)$:

a) $\delta(\sigma)<0$, $\sigma\in R^m$ or

b) $\delta(\sigma)=0$, $\sigma\in R^m$ or (15)

c) $\delta(\sigma)>0$, $\sigma\in R^m$.

Obviously, in the case (15a), equation (9) has four distinct real roots, in case (15b) it has two distinct double roots, and in case (15c) there are four mutually conjugate complex roots, and

$$\operatorname{Re}\eta_j(\sigma)\neq 0,\ j=\overline{1,4},\ \sigma\in R^m. \tag{16}$$

In the case of sign-negative $\rho_0(\sigma)$ and sign-positive $\delta(\sigma)$, i.e. under conditions

$$\rho_0(\sigma)<0,\ \delta(\sigma)>0,\ \sigma\in R^m \tag{17}$$

roots $\eta_j(\sigma)$, $j=\overline{1,4}$ are complex-valued with nonzero real parts, therefore, we have property (16).

Under condition

$$\rho_0(\sigma)<0,\ \delta(\sigma)<0,\ \sigma\in R^m \tag{18}$$

these roots are either pure imaginary or zero.

As can be seen from the above eigenvalues $\lambda_j(\sigma)$, $j=\overline{1,4}$ of matrix $A(\sigma)$ of the system (4) are determined by the relations (6), (12), (13), (14), which by conditions (2), (11), (12), (15) , (17) and (18) have the properties $1^0$-$5^0$.

In other words, eigenvalues $\lambda_j(\sigma)$, $j=\overline{1,4}$ of the matrix $A(\sigma)=\left[a_{ij}(\sigma)\right]_1^4$ are complex-valued functions of real arguments $\sigma\in R^m$ and its are continuously differentiable, $\omega$-periodic, either coincide or do not intersect, their real and imaginary parts have definite signs or are identically equal to zero.

Thus, the results obtained can be formulated as the following lemma.

**Lemma 1.** If one of the conditions (15), (17) or (18) is fulfilled along with the conditions (2), (11), (12), then eigenvalues $\lambda_j(\sigma)$, $j=\overline{1,4}$ of matrix $A(\sigma)=\left[a_{ij}(\sigma)\right]_1^4$ of the system (4) have the properties $1^0$-$5^0$.

Obviously, under the conditions (2), (11) and (18), the eigenvalues $\lambda_j(\sigma)$, $j=\overline{1,4}$ of the systems (4), according to Lemma 1, are either pure imaginary or identically zero. Consequently, the homogeneous system (4), in accordance with the general theory [15-16], admits multiperiodic solutions $x$ in $\tau\in R$ with frequencies $\nu_j(\sigma)=\operatorname{Im}\lambda_j(\sigma)$ in the case $\operatorname{Im}\lambda_j(\sigma)\neq 0$ and constant solutions $x=const$ in the case of zero eigenvalues.

Thus, we give the following lemma.

**Lemma 2.** Under conditions (2), (11), and (18) the system (4) has an infinitely many of multiperiodic in $\tau$ solutions $x = x(\tau, \sigma)$ nonzero, and their periods are equal to $\theta_j(\sigma) = 2\pi\nu_j^{-1}(\sigma)$, $\sigma \in R^m$, $j = \overline{1,4}$.

If conditions (2), (11), (12) and one of the conditions (15) or (17) are satisfied, then, according to Lemma 1, inequality (16) is satisfied.

Then homogeneous linear system (4) does not have bounded solutions with bounded initial conditions except for the zero solution.

Hence the lemma holds.

**Lemma 3.** Assume that conditions (2), (11), (12) and one of the conditions (15) or (17) are satisfied. Then, system (4) does not have multiperiodic solutions $x(\tau, \sigma)$ except for trivial.

If the conditions of Lemma 3 are satisfied, the eigenvalues $\lambda_j(\sigma)$ can be represented in the form

$$\lambda_j(\sigma) = \alpha_j(\sigma) \pm i\beta_j(\sigma),\ j = \overline{1,4},\ \sigma \in R^m$$

with $\omega$-periodic smooth in $\sigma$ real and imaginary parts, and, according to (16), we have $\operatorname{Re}\lambda_j(\sigma) = \alpha_j(\sigma) \neq 0$. For definiteness, we shall suppose that

$$\operatorname{Re}\lambda_j(\sigma) = \alpha_j(\sigma) < 0,\ j = \overline{1,4},\ \sigma \in R^m. \tag{19}$$

We also note that $\omega$-periodicity of the solutions $x$ of system (1) under conditions of Lemma 1 is determined by $\omega$-periodicity of the initial data in $t \in R^m$ if $\tau = 0$.

Then it is easy to show that matriciant $X(\tau, \sigma)$ of the system (4) satisfies the estimate

$$|X(\tau, \sigma)| \leq \gamma e^{-\delta\tau},\ \tau \in R \tag{20}$$

where $\gamma \geq 1$, $\delta > 0$ are constants and

$$X(\tau, \sigma + k\omega) = X(\tau, \sigma), \quad k \in Z^m, \quad \sigma \in R^m. \tag{21}$$

We introduce an inhomogeneous system of equations

$$D_e x_i = \sum_{j=1}^{4} a_{ij}(\sigma)x_j + f_j(\tau, t, \sigma), \quad j = \overline{1,4}, \tag{22}$$

where $f_j(\tau, t, \sigma)$ satisfy the condition

$$f_j(\tau + \theta, t + k\omega, \sigma + q\omega) = f_j(\tau, t, \sigma) \in C_{\tau,t,\sigma}^{(0.e.e)}(R \times R^m \times R^m), \quad k, q \in Z^m, \tag{23}$$

the period $\theta = \theta(\sigma) \neq 0$, $\sigma \in R^m$ and has properties

$$\theta(\sigma + k\omega) = \theta(\sigma) \in C_\sigma^{(e)}(R^m), \quad k \in Z^m. \tag{24}$$

Putting $x = (x_1, ..., x_4)$ and $f(\tau, t, \sigma) = (f_1(\tau, t, \sigma), ..., f_4(\tau, t, \sigma))$ system (22) can be represented in the vector-matrix form

$$D_e x = A(\sigma)x + f(\tau, t, \sigma) \tag{22'}$$

which is convenient for the formulation of the following theorem.

**Theorem.** Under conditions of Lemma 3 with the additional condition (19), (23), and (24) system (22) has a unique $(\theta, \omega, \omega)$-periodic solution

$$x^*(\tau, t, \sigma) = \int_{-\infty}^{\tau} X(\tau - s, \sigma) f(s, t - e\tau + es, \sigma) ds. \tag{25}$$

Proof. The existence of the function $x^*(\tau, t, \sigma)$ is ensured by the convergence of improper integral (25) by theorem and estimate (20). Given that

$$D_e X(\tau, \sigma) = A(\sigma)x + X(\tau, \sigma)$$

it is not difficult to verify that by (25) it is possible to represent the solution of system (22′), hence, of the system (22). On the basis of (21), (23) and (24), it is easy to verify that this solution is of $(\theta, \omega, \omega)$-periodicity. The uniqueness follows from the estimate (20).

In conclusion, we note that idea of a method of obtaining the results of this study can be generalized to the more general case with nonlinearities on the basis of the methods of [17-20].

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## ТҰРАҚТЫ КОЭФФИЦИЕНТТІ ТӨРТ ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕРДІҢ СЫЗЫҚТЫ ЖҮЙЕСІНІҢ КӨППЕРИОДТЫ ШЕШІМІНІҢ БАР БОЛУЫНЫҢ КОЭФФИЦИЕНТТІК БЕЛГІЛЕРІ

**Аннотация.** Заметкада $(\tau, t)$ уақыттық айнымалыларының кеністігінің негізгі диагоналының бағыты бойынша $D_e$ дифференциалдау операторлы төрт дифференциалдық теңдеулердің сызықты жүйесі қарастырылған. Характеристикалық теңдеудің меңшікті мәндерінің айнымалыларынан тәуелділікте айнымалы, бірақ диагоналда тұрақты коэффициентті қарастырылатын сызықты теңдеулер жүйесінің шешімінің $(\theta, \omega, \omega)$ -периодтылығын зерттеу жүргізілген. Әртүрлі нақты, еселі және комплекс түйіндес меншікті мәндердің ажыратылу және таңба анықталған қасиеттерінің коэффициентілік белгілері анықталған. Меншікті мәндердің үзілісіз диффе-ренциалданатындығы және периодтылығы зерттелген. Периодты шешімдердің бар болуының коэффициентті жеткіліктілік шарттары орнатылған. Айнымалы период және айнымалы жиілік ұғымдары енгізілген.

Дербес туындылы теңдеулер мәселесің зерттеу қарапайым дифференциалдық теңдеулер [1-5] сұрақтарымен тығыз байланысты. Бірінші ретті дербес туындылы $D_e$-теңдеулер жүйесінің көппериодты шешімдері сұрақтарының зерттеуі өз бастауын [6-8] еңбектерінен алатындығы белгілі. Сұрақтың қойылуы бойынша берілген зерттеу бұрын жүргізілген зерттеулерге [9-14] қосылады.

**Түйін сөздер:** сызықты жүйе, дифференциалдық оператор, меншікті мәндер, характеристикалық теңдеу, нақты және жорамал бөліктер, диагоналдық минорлар.

УДК 35В10

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## КОЭФФИЦИЕНТНЫЕ ПРИЗНАКИ СУЩЕСТВОВАНИЯ МНОГОПЕРИОДИЧЕСКИХ РЕШЕНИЙ ЛИНЕЙНОЙ СИСТЕМЫ ЧЕТЫРЕХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С ПОСТОЯННЫМИ НА ДИАГОНАЛИ КОЭФФИЦИЕНТАМИ

**Аннотация.** В заметке рассмотрена линейная система четырех дифференциальных уравнений с оператором дифференцирования $D_e$ по направлению главной диагонали пространства временных переменных $(\tau, t)$. Проведено исследование $(\theta, \omega, \omega)$-периодичности решений рассматриваемой линейной системы уравнений с переменными, но постоянными на диагонали коэффициентами в зависимости от переменных собственных значений характеристического уравнения. Выяснены коэффициентные признаки свойств разделенности и знакоопределенности различных вещественных, кратных и комплексно сопряженных собственных значений. Исследованы периодичность и непрерывная дифференцируемость собственных значений. Установлены коэффициентные достаточные условия существования периодических решений. Введены понятия переменной частоты и переменного периода.

Исследование проблем уравнений в частных производных тесно связано с вопросами теории обыкновенных дифференциальных уравнений [1-5]. Известно, что исследование вопросов многопериодических решений систем $D_e$ -уравнений в частных производных первого порядка берет свое начало в трудах [6-8]. Данное исследование по постановке вопроса примыкает к ранее проведенным исследованиям [9-14].

**Ключевые слова:** линейная система, дифференциальный оператор, собственные значения, характеристическое уравнение, действительные и мнимые части, диагональные миноры.

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