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**N. Medeubaev¹, S. Menlikozhaeva², A. Seitmuratov²,
M. Ramazanov¹, B. Zharmenova², T. Shamilov³**

¹The E.A. Buketov Karaganda State University, Karaganda;

²The Korkyt Ata Kyzylorda State University, Kyzylorda;

³Azerbaijan University of Architecture and Construction, Azerbaijan, Baku
medeubaev65@mail.ru, angisin_@mail.ru, saulesh_menli@mail.ru,
ramamur@mail.ru, 81_bota@mail.ru, invar59@mail.ru

**AREA OF APPLICABILITY OF APPROXIMATE EQUATIONS
OF VIBRATIONS OF ROD SYSTEMS OF VARIABLE THICKNESS**

Abstract: The approximate equations of torsional vibrations of rod systems of variable thickness given in this work allow us to build approximate theories of oscillations depending on the conditions at the ends of the rod, the order of derivatives of the desired in the approximate equations and initial conditions. Approximate equations of oscillations of rod systems above the second order in derivatives of the desired function in the scientific literature are practically not found. On the other hand, approximate equations containing derivatives above the second order need to study the applicability of a theory based on various approximate equations. For simplicity, we consider the area of applicability for a constant radius rod, when the desired function U_θ or rotational displacement depends on the angular coordinate θ , i.e., we investigate the area of applicability of the approximate equations of oscillation based on the general equation of oscillation

Key words: torsional oscillations, rod, system, variable thickness, cylinder, shell, radius, Bessel function.

In the cylindrical coordinate system (r, θ, z) the spatial problem of torsional vibrations of the cylindrical shell is reduced to the solution of the equation

$$\rho M^{-1} \left(\frac{\partial^2 \psi_1}{\partial t^2} \right) - \Delta \psi_1 = 0 \quad (15)$$

under boundary conditions on the inner and outer surface of the cylindrical shell

$$\sigma_{r\theta} + \frac{(-1)^{i+1}}{\Delta_{oi}} F_i(z) \sigma_{r\theta} = f_{ns_i}^{(i)} \quad i = (1,2) \quad (16)$$

when $r = F_i(z)$, where $r = F_1(z)$ and $r = F_2(z)$ inner and outer shell radii, respectively,

$$\Delta_0 = \sqrt{1 + [F_i]^2} .$$

Presenting ψ_1 in the form of potentials

$$\psi_1 = \int_0^{\infty} \int_{-\cos(kz)}^{\sin(kz)} dk \int_l \psi_{10} \exp(pt) dp \quad (17)$$

and substituting it into equations (15), we again obtain for ψ_{10} expression

$$\frac{d^2\psi_{10}}{dr^2} + \frac{1}{r} \frac{d\psi_{10}}{dr} - \beta^2 \psi_{10} = 0; \quad (18)$$

the general solution of which is

$$\psi_{10} = AI_0(\beta r) + BK_0(\beta r) \quad (19)$$

In this case, the integration constant of B as opposed to the task for the rod is different from zero.

If the shift displacement u_θ search in the offset U_θ

$$u_\theta = \int_0^\infty \int_{-\cos(kz)}^{\sin(kz)} dk \int_{(l)} u_{\theta,0} \exp(pt) dp \quad (20)$$

then, the decomposing of Bessel's function I_0 and K_0 in a row, for $u_{\theta,0}^{(0)}$ we obtain expression

$$u_{\theta,0}^{(0)} = \frac{1}{r} A - \sum_{n=0}^{\infty} \beta_0^{2(n+1)} \left\{ A - B \left[\ln \frac{\beta_0 \xi}{2} - \frac{1}{2} \psi(n+1) - \frac{1}{2} \psi(n+2) \right] \right\} \frac{\left(\frac{r}{2}\right)^{2n+1}}{n!(n+1)!}; \quad (21)$$

where $\psi(n)$ - psy-function.

Introducing new values

$$\begin{aligned} \hat{U}_{\theta,0} &= -\frac{1}{2} \beta_0^2 \left\{ A - B \left[\ln \frac{\beta_0 \xi}{2} - \psi(1) - \frac{1}{2} \right] \right\} \\ \hat{U}_{\theta,1} &= -\frac{A}{\xi} \end{aligned} \quad (22)$$

and substituting their expression (21) instead of constants A, B we obtain expression

$$U_{\theta,0}^{(0)} = 2 \sum_{n=0}^{\infty} \frac{\left(\frac{r}{2}\right)^{2n+1}}{n!(n+1)!} \beta_0^{2n} \hat{U}_{\theta,0} + \xi \left\{ \left[\frac{1}{r} + \sum_{n=0}^{\infty} \eta_{l,n}(r) \frac{\left(\frac{r}{2}\right)^{2n+1}}{n!(n+1)!} \beta_0^{2(n+1)} \right] \hat{U}_{\theta,1} \right\} \quad (23)$$

in reference to k and p , for shear displacement, we obtain the final representation

$$U_\theta(r, z, t) = 2 \sum_{n=0}^{\infty} \frac{\left(\frac{r}{2}\right)^{2n+1}}{n!(n+1)!} \lambda_0^{(n)} U_{\theta,0} + \\ + \xi \left\{ \frac{1}{r} + \sum_{n=0}^{\infty} \eta_{1,n}(r) \frac{\left(\frac{r}{2}\right)^{2n+1}}{n!(n+1)!} \lambda_0^{(n+1)} \right\} U_{\theta,1}; \quad (24)$$

where $U_{\theta,0}$; $U_{\theta,1}$ have a obvious mechanical meaning, functions $\eta_{1,n}$ are equal

$$\eta_{1,n}(r) = \ln \frac{r}{\xi} + \frac{n}{2(n+1)} - \sum_{k=1}^n \frac{1}{k} \quad (25)$$

Similarly, for tension $\sigma_{r,\theta}$; $\sigma_{z,\theta}$ we obtain expression

$$M^{-1} \sigma_{r,\theta} = 2 \sum_{n=0}^{\infty} \frac{\left(\frac{r}{2}\right)^{2n+1}}{n!(n+2)!} \lambda_2^{(n+1)} U_{\theta,1} + \frac{\xi}{2} \left\{ \frac{1}{2} \lambda_2^{(1)} - \frac{4}{r^2} + \frac{1}{2} \sum_{n=0}^{\infty} \eta_{2,n}(r) \frac{\left(\frac{r}{2}\right)^{2(n+1)}}{n!(n+2)!} \lambda_2^{(n+2)} \right\} U_{\theta,1} \quad (26)$$

$$M^{-1} \sigma_{z,\theta} = 2 \sum_{n=0}^{\infty} \frac{\left(\frac{r}{2}\right)^{2n+1}}{n!(n+2)!} \lambda_2^{(n)} \frac{\partial U_{\theta,1}}{\partial z} + \xi \left\{ \frac{1}{r} + \sum_{n=0}^{\infty} \eta_{1,n}(r) \frac{\left(\frac{r}{2}\right)^{2n+1}}{n!(n+2)!} \lambda_2^{(n)} \right\} \frac{\partial U_{\theta,1}}{\partial z}$$

where ξ intermediate surface, determined by the formula

$$\xi = \frac{\max[F_1(z)]}{2} \left\{ \chi - \frac{\max[F_1(z)]}{\min[F_2(z)]} \right\} \quad (27)$$

herewith

$$\max[F_1(z)] < \min[F_2(z)]$$

coefficient χ satisfies the inequality

$$2 + \frac{\max[F_1(z)]}{\min[F_2(z)]} \leq \chi \leq 2 \frac{\min[F_2(z)]}{\max[F_1(z)]} + \frac{\max[F_1(z)]}{\min[F_2(z)]}$$

Substituting (25) into boundary conditions (16) to determine $U_{\theta,0}$ и $U_{\theta,1}$ we obtain a system of integro-differential equations, which is also the general equations of torsional vibrations of a circular cylindrical shell.

We present only a system of approximate equations of torsional vibration

$$F_i(z) \left[\frac{F_i(z)}{8} \lambda_2^{(1)} - F_i'(z) \frac{\partial}{\partial z} \right] U_{\theta,0} + \xi \left\{ \frac{1}{2} \lambda_2^{(1)} - \frac{2}{F_i'} - \frac{F_i'(z)}{F_i(z)} \frac{\partial}{\partial z} + \right. \\ \left. + \frac{F_i(z)}{8} \lambda_2^{(1)} \left[\eta_{2,0}(F_i) \frac{F_i}{2} \lambda_2^{(1)} - \eta_{1,0}(F_i) \frac{F_i'(z)}{2} \right] \right\} \frac{\partial U_{\theta,1}}{\partial z} = \Delta_{0i} M^{-1} f_{ns_1}^{(i)} \quad (i=1,2) \quad (28)$$

Where

$$\eta_{1,0} = \ln \frac{r}{\xi}$$

$$\eta_{2,0}(r) = \ln \frac{r}{\xi} - \frac{1}{2}$$

For a shell of constant thickness with radii r_1 and r_2 accordingly, from (28) we obtain the system

$$\frac{r_i^2}{8} \lambda_2^{(1)} U_{\theta,0} + \xi \left\{ \frac{1}{2} \lambda_2^{(1)} - \frac{2}{r_i^2} + \frac{r_i}{2} \lambda_2^{(1)} \left[\frac{\eta_{2,0}(r_i) r_i}{2} \right] \lambda_2^{(1)} \right\} \\ \frac{\partial U_{\theta,1}}{\partial z} = M^{-1} [f_\theta^{(i)}] \quad (29)$$

The boundary conditions for the end of the shell under torsional vibration are derived as for the rod. We present only the boundary conditions at the end of the shell when $z = const$ there is a normal intensity beat $\sigma_{z\theta} = F(r, t)$. The boundary condition will have the form

$$u_{\theta,0} = M^{-1} [F(0, t)] \quad (30)$$

$$\frac{\partial u_{\theta,1}}{\partial z} = 0$$

General equations of longitudinal-radial oscillations of the cylindrical shell are derived as for the rod of variable radius.

For the formulation of various boundary value problems, we first give approximate equations of longitudinal-radial oscillations

$$\left\{ C + \frac{r_i^2}{2} \left[\lambda_2^{(1)} + C \left(\lambda_1^{(1)} + 2 \frac{\partial^2}{\partial z^2} \right) \right] \right\} U_{r,0} - \frac{\partial}{\partial z} \left\{ (1 - C) + \frac{r_i^2}{8} \left[2 \lambda_2^{(1)} - C \left(\lambda_1^{(1)} + 2 \frac{\partial^2}{\partial z^2} \right) \right] \right\} U_{z,0} - \\ - \xi \left\{ - \frac{2}{r_i^2} + \left(\frac{1}{2} + D \ln \frac{r_i}{\xi} \right) \lambda_2^{(1)} + \left(\ln \frac{r_i}{\xi} - \frac{1}{4} \right) (1 - D) \lambda_2^{(1)} + \right.$$

$$\begin{aligned}
& + D \left(\lambda_1^{(1)} + 2 \frac{\partial^2}{\partial z^2} \right) \ln \frac{r_i}{\xi} - D \left(\frac{3}{4} \lambda_1^{(1)} + \frac{\partial^2}{\partial z^2} \right) \frac{r_i^2}{16} \Big\} \lambda_2^{(1)} U_{r,1} + \\
& + \xi \left\{ \frac{1}{2} + (1 - D) \ln \frac{r_i}{\xi} + \frac{r_i^2}{8} \left[- D \left(\ln \frac{r_i}{\xi} - \frac{3}{4} \right) \lambda_1^{(1)} + D \left(\ln \frac{r_i}{\xi} - \frac{1}{4} \right) \lambda_2^{(1)} + \right. \right. \\
& \left. \left. + \left(2 \ln \frac{r_i}{\xi} - 1 \right) \left(\lambda_2^{(1)} - D \frac{\partial^2}{\partial z^2} \right) \right] \right\} \frac{\partial U_{z,1}}{\partial z} = M^{-1} \left(f_r^{(i)} \right) \quad (i = 1, 2) \\
& \left\{ (1 - C) + \frac{r_i^2}{8} \left[(1 - C) \lambda_2^{(1)} - 2C \lambda_1^{(1)} \right] \right\} \frac{\partial U_{r,0}}{\partial z} + \\
& + \left\{ (1 + C) \lambda_1^{(1)} + \frac{r_i^2}{8} \lambda_1^{(1)} \left[(1 + C) \lambda_2^{(1)} - 2C \frac{\partial^2}{\partial z^2} \right] \right\} U_{z,0} - \\
& - \xi \left\{ \frac{2}{r_i^2} + (1 - D) \ln \frac{r_i}{\xi} \lambda_2^{(1)} + \frac{r_i^2}{8} \left(\ln \frac{r_i}{\xi} - \frac{3}{4} \right) \left[(1 - D) \lambda_2^{(1)} - 2D \lambda_1^{(1)} \right] \lambda_2^{(1)} \right\} \times \\
& \times \frac{\partial U_{r,1}}{\partial z} - \xi \left\{ \frac{2}{r_i^2} + \left(\lambda_2^{(1)} - 2D \frac{\partial^2}{\partial z^2} \right) \ln \frac{r_i}{\xi} + \frac{r_i^2}{\xi} \left(\ln \frac{r_i}{\xi} - \frac{1}{2} \right) \times \right. \\
& \left. \times \left[\left(\lambda_2^{(1)} - D \frac{\partial^2}{\partial z^2} \right) \lambda_2^{(1)} - 2D \lambda_1^{(1)} \frac{\partial^2}{\partial z^2} \right] \right\} U_{z,1} = \frac{2}{r_i^2} M^{-1} \left(f_{rz}^{(i)} \right) \quad (i = 1, 2)
\end{aligned} \tag{31}$$

where $f_r^{(i)}, f_{rz}^{(i)}$ the loads on the outer and inner surfaces of the shells u_r, u_z are approximately equal

$$\begin{aligned}
u_z &= U_{z,0} - \xi \left(\ln \frac{r}{\xi} + \frac{1}{2} \right) U_{z,1} \\
u_r &= \frac{r}{2} U_{r,0} - \frac{r\xi}{2} \left[\frac{2}{r^2} + (1 - D) \lambda_2^{(1)} \ln \frac{r}{\xi} \right] U_{r,1} + \frac{r\xi}{2} D \ln \frac{r}{\xi} \frac{\partial U_{z,1}}{\partial z}
\end{aligned} \tag{32}$$

tension

$$M^{-1} \sigma_{zz} = (1 + C) \frac{\partial U_{z,0}}{\partial z} - (1 - C) U_{r,0} - \xi \left(\frac{1}{2} + \ln \frac{r}{\xi} \right) \left[(1 + 2D) \frac{\partial U_{z,1}}{\partial z} - (1 - 2D) \lambda_2^{(1)} U_{r,1} \right] \tag{33}$$

$$\begin{aligned}
M^{-1} \sigma_{r,z} &= \frac{r}{2} \left\{ (1 - C) \frac{\partial U_{r,0}}{\partial z} + (1 + C) \lambda_1^{(1)} U_{z,0} - \xi \left[\frac{2}{r^2} + (1 - 2D) \lambda_2^{(1)} \ln \frac{r}{\xi} \right] \frac{\partial U_{r,1}}{\partial z} - \right. \\
& \left. - \xi \left[\frac{2}{r^2} + \left(\lambda_2^{(1)} - 2D \frac{\partial^2}{\partial z^2} \right) \ln \frac{r}{\xi} \right] U_{z,1} \right\};
\end{aligned}$$

$$M^{-1}\sigma_{r,r} = \left\{ CU_{r,0} - (1-C)\frac{\partial U_{z,0}}{\partial z} + \xi \left[\left(D \ln \frac{r}{\xi} - \frac{1}{2} \right) \lambda_2^{(1)} + \frac{2}{r^2} \right] U_{r,1} + \xi \left[(1-D) \ln \frac{r}{\xi} + \frac{1}{2} \right] \frac{\partial U_{z,1}}{\partial z} \right\};$$

$$M^{-1}\sigma_{\theta\theta} = \left\{ CU_{r,0} - (1-C)\frac{\partial U_{z,0}}{\partial z} + \frac{\xi}{2} \left[(1-D) \lambda_2^{(1)} + \frac{2}{r^2} \right] U_{r,1} + \xi \left[1 - D + 2 \ln \frac{r}{\xi} \right] \frac{\partial U_{z,1}}{\partial z} \right\}$$

$$C = 1 - NM^{-1}; \quad D = 1 - MN^{-1}$$

Here $U_{z,0}$ and $U_{r,1}$ moving intermediate surface points, $U_{z,1}$ and $U_{r,0}$ deformations of these points along the radial coordinate.

At the beginning we formulate three-dimensional boundary conditions for the shell end $z = const$. Free or loaded by normal tension of the end

$$\sigma_{zz} = -F(t); \quad \sigma_{rz} = 0 \quad (34)$$

Rigidly fixed end

$$u_r = u_z = 0 \quad (35)$$

Perfectly fixed end

$$u_z = 0; \quad \sigma_{rz} = 0 \quad (36)$$

On the basis of the approximate expression (33) for tension conditions (34) for unknown U_{rj}, U_{zj} lead to boundary conditions

$$(1+C)\frac{\partial U_{z,0}}{\partial z} - (1-C)U_{r,0} - \frac{\xi}{2}(1+2C)\frac{\partial U_{z,1}}{\partial z} + \frac{\xi}{2}(1-2C)\lambda_2^{(1)}U_{r,1} = -M^{-1}(F)(1-C)\frac{\partial U_{z,0}}{\partial z} + (1+C)\lambda_1^{(1)}U_{z,0} = 0$$

$$\frac{\partial U_{r,1}}{\partial z} + U_{z,1} = 0 \quad (37)$$

Similarly, conditions (35) and (36) lead to boundary conditions

$$U_{r,0} = U_{r,1} = 0; \quad U_{r,0} = \frac{\xi}{2}U_{z,1} \quad (38)$$

And

$$U_{z,0} = \frac{\xi}{2}U_{z,1}; \quad \frac{\partial U_{r,1}}{\partial z} + U_{z,1} = 0 \quad (39)$$

$$(1-C)\frac{\partial U_{r,0}}{\partial z} + (1-C)\lambda_1^{(1)}U_{z,0} = 0$$

Boundary conditions (37) - (39) differ from those available in the scientific literature, obtained on the basis of certain hypotheses and suggestions of mechanical and geometric nature, the presence of operators $\lambda_j^{(1)}$, reflecting the principle of D'Alembert's mechanics.

Consider a more complex sealing of the end of the cylindrical shell.

Let the end of the cylindrical shell is in interaction with another deformable body. In particular, such a deformable body should be a plate-plate perpendicular to the axis of the shell.

We also consider two types of contact in a three-dimensional setting.

Rigid contact.

$$\begin{aligned} W_z^{(1)} &= W_z^{(2)}; & \bar{u}_r^{(1)} &= u_r^{(2)}; & \bar{u}_r^{(1)} &= \frac{1}{\xi} \int_{r_1}^{r_2} u_r^{(1)} dr, \\ \sigma_{zz}^{(1)} &= \sigma_{zz}^{(2)}; & \bar{\sigma}_{rz}^{(1)} &= \sigma_{rz}^{(2)}; & \bar{\sigma}_{rz}^{(1)} &= \frac{1}{\xi} \int_{r_1}^{r_2} \sigma_r^{(1)} dr \end{aligned} \quad (40)$$

Perfect contact

$$W_z^{(1)} = W_z^{(2)}; \quad \sigma_{zz}^{(1)} = \sigma_{zz}^{(2)}; \quad \bar{\sigma}_{rz}^{(1)} = 0; \quad \sigma_{rz}^{(2)} = 0 \quad (41)$$

Having approximate expressions for displacements and stresses at the end points of the cylindrical shell (32) and (33), the material parameters will be denoted by the index "1", and the plates-by the index "2", while the approximate expressions for the necessary displacements and stresses at the points of the plate have the form

$$\begin{aligned} W_z^{(2)} &= W_2 + [(1-C_2)\lambda_{222}^{(1)} + C_2\Delta]W_2 \frac{h^2}{2}, \\ u_r^{(2)} &= -\frac{\partial W_2}{\partial z}h + \frac{\partial}{\partial r}[C_2\Delta - (1-C_2)\lambda_{22}^{(1)}]W_2 \frac{h_3}{6}; \\ M_2^{-1}\sigma_{zz}^{(2)} &= [(\lambda_{22}^{(1)} + \Delta)W_2]h + [4C_2\Delta\lambda_{22}^{(1)} + \lambda_{12}^{(1)}(\lambda_{12}^{(1)} + \Delta)]W_2 \frac{h^3}{6}; \\ M_2^{-1}\sigma_{rz}^{(2)} &= [2\lambda_{12}^{(1)} - C(2\lambda_{222}^{(1)} - \Delta)]\frac{\partial W_2}{\partial z} \frac{h^2}{2}. \end{aligned} \quad (42)$$

Using the approximate expressions (32), (33) and (42), from the conditions (40) we obtain the system

$$\begin{aligned} U_{z,0}^{(1)} - \frac{\xi}{2}U_{r,1}^{(1)} &= W_2; \\ \frac{\partial U_{r,1}^{(1)}}{\partial z} + U_{z,1}^{(1)} &= 0; \\ (1+C_1)\frac{\partial U_{z,0}^{(1)}}{\partial z} - (1-C_1)U_{r,0}^{(1)} - \frac{\xi}{2} \times \\ \times \left[(1+2C_1)\frac{\partial U_{z,1}^{(1)}}{\partial z} - (1-2C_1)\lambda_{21}^{(1)}U_{r,1}^{(1)} \right] &= M_1^{-1}M_2[(\lambda_{22}^{(1)} + \Delta)W_2h]. \end{aligned} \quad (43)$$

$$\begin{aligned} & \left[(1-C_1) \frac{\partial U_{r,0}^{(1)}}{\partial z} + (1+C_1) \lambda_{11}^{(1)} U_{r,0}^{(1)} \right] \frac{r_2^2 - r_1^2}{2\xi} = \\ & M_1^{-1} M_2 \left\{ \left[2\lambda_{12}^{(1)} - C_1 (2\lambda_{22}^{(1)} - \Delta) \right] \frac{\partial W_2}{\partial z} \right\} \frac{h^2}{2}; \\ & \frac{r_2^2 - r_1^2}{4\xi} U_{r,0}^{(1)} - \ln \frac{r_2}{r_1} U_{r,1}^{(1)} = - \frac{\partial W_2}{\partial z} h \end{aligned}$$

or, after elimination, we obtain the boundary conditions of the elastic end fitting shells taking into account the influence of plate-plate deformability

$$\begin{aligned} & (1+C_1) \frac{\partial U_{z,0}^{(1)}}{\partial z} - (1-C_1) U_{r,0}^{(1)} - \frac{\xi}{2} \left[(1+2C_1) \frac{\partial U_{z,1}^{(1)}}{\partial z} - (1-2C_1) \lambda_{21}^{(1)} U_{r,1}^{(1)} \right] = M_1^{-1} M_2 \left\{ \lambda_{22}^{(1)} + \Delta \right\} \times \\ & \times \left(U_{z,0}^{(1)} - \frac{\xi}{2} U_{r,1}^{(1)} \right) h; \end{aligned}$$

$$\begin{aligned} & \frac{\partial U_{r,1}^{(1)}}{\partial z} + U_{z,1}^{(1)} = 0; \\ & \left[(1-C_1) \frac{\partial U_{r,0}^{(1)}}{\partial z} + (1+C_1) \lambda_{11}^{(1)} U_{r,0}^{(1)} \right] \frac{r_2^2 - r_1^2}{2\xi} = M_1^{-1} M_2 \left\{ 2\lambda_{12}^{(1)} - C_1 (2\lambda_{22}^{(1)} - \Delta) \right\} \left[\frac{r_2^2 - r_1^2}{4\xi} U_{r,0}^{(1)} - \ln \frac{r_2}{r_1} U_{r,1}^{(1)} \right] \frac{h}{2} \end{aligned}$$

Similarly, for an ideal contact (41) we obtain a system

$$\begin{aligned} & U_{z,0}^{(1)} - \frac{\xi}{2} U_{r,1}^{(1)} = W_2; \quad \frac{\partial U_{r,1}^{(1)}}{\partial z} + U_{z,1}^{(1)} = 0; \quad \frac{\partial W_2}{\partial z} = 0; \quad (44) \\ & (1+C_1) \frac{\partial U_{z,0}^{(1)}}{\partial z} - (1-C_1) U_{r,0}^{(1)} - \frac{\xi}{2} \left[(1+2C_1) \frac{\partial U_{z,1}^{(1)}}{\partial z} - (1-2C_1) \times \right. \\ & \times \left. \lambda_{21}^{(1)} U_{r,1}^{(1)} \right] = M_1^{-1} M_2 \left\{ (\lambda_{22}^{(1)} + \Delta) W_2 h \right\} \\ & \left[(1-C_1) \frac{\partial U_{r,0}^{(1)}}{\partial z} + (1+C_1) \lambda_{11}^{(1)} U_{r,0}^{(1)} \right] = 0 \end{aligned}$$

and for the end of the shell

$$\begin{aligned} & (1+C_1) \frac{\partial U_{z,0}^{(1)}}{\partial z} - (1-C_1) U_{r,0}^{(1)} - \frac{\xi}{2} \left[(1+2C_1) \frac{\partial U_{z,1}^{(1)}}{\partial z} - (1-2C_1) \times \right. \\ & \times \left. \lambda_{21}^{(1)} U_{r,1}^{(1)} \right] = M_1^{-1} M_2 \left\{ (\lambda_{22}^{(1)} + \Delta) \left(U_{z,0}^{(1)} - \frac{\xi}{2} U_{r,1}^{(1)} \right) h \right\} \\ & \left[(1-C_1) \frac{\partial U_{r,0}^{(1)}}{\partial z} + (1+C_1) \lambda_{11}^{(1)} U_{r,0}^{(1)} \right] = 0; \quad \frac{\partial U_{r,1}^{(1)}}{\partial z} + U_{z,1}^{(1)} = 0; \end{aligned} \quad (45)$$

The initial conditions for the variables $U_{z,j}^{(1)}, U_{r,j}^{(1)}$ it is easy to obtain three-dimensional initial conditions for the displacements

$$\begin{aligned} u_r &= f_1(r, z); & u_z &= f_2(r, z); \\ \frac{\partial u_r}{\partial t} &= f_3(r, z); & \frac{\partial u_z}{\partial t} &= f_4(r, z) \quad (t=0) \end{aligned} \tag{46}$$

The results obtained allow us to formulate boundary value problems in solving specific problems of the cylindrical shell oscillation under different conditions at the end of the shell.

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**Н.Қ.Медеубаев¹, С.Менліхожаева², А.Ж.Сейтмұратов²,
М.И.Рамазанов¹, Б.К.Жарменова², Т.Шамилов³**

¹Е. Бекетов атындағы Қарағанды мемлекеттік университеті,
Қазақстан Республикасы. Қарағанды қ.;

²Қорқыт Ата атындағы Қызылорда мемлекеттік университеті, Қазақстан Республикасы. Қызылорда қ.;

³Азербайджан Архитектура және Құрылым университеті, Азербайджан Республикасы. Бақу қ.

ҚАЛЫНДЫҒЫ АЙНЫМАЛЫ БОЛАТЫН СЫРЫҚТЫҚ ЖҮЙЕНІН АЙНАЛМА ТЕРБЕЛІСІНІҢ ЖУЫҚ ТЕНДЕУІНІҢ ҚОЛДАНУ АУМАҒЫ

Аннотация: Макалада қалындығы айнымалы болатын сырыйтық жүйенін айналмалы тербелісінің жуық тендеуінің туындысының реті, сырыйтың сыртқы шартына тәуелді жағдайындағы тербелісінің жуық теориясын құруға мүмкіндік беретінін қарастырады. Берілген функцияның туындысы екіден жоғары болатын сырыйтық жүйенін тербелісінің жуық тендеулері жайлы ғылыми әдебиеттерде іс-жүзінде мүлдем аз қарастырылған. Дегенмен туындысы екінші реттен жоғары болатын жуық тендеулер түрі, әр түрлі негізде болатын тендеулер теориясының жуық есебін зерттеу қажеттілікті тудырады. Сол себептен есепте радиусы тұрақты болатын сырыйтар үшін берілген U_θ функциясы немесе θ бұрыштық координатасынан тәуелді болатын ығысу айналымының зерттеу аумағын қарастырады.

Түйін сөз: айналмалы тербеліс, сырый, жүйе, қалындығы айнымалы, цилиндр, қабықша, радиус, Бессел функциясы.

**Н.К.Медеубаев¹, С.Менлихожаева², А.Ж.Сейтмуратов²,
М.И.Рамазанов¹, Б.К.Жарменова², Т.Шамилов³**

¹ Карагандинский государственный университет им.Букетова, Республика Казахстан. г.Караганда;

²Кызылординский государственный университет им.Коркыт Ата, Республика Казахстан. г.Кызылорда;

³ Азербайджанский университет Архитектуры и Строительства, Республика Азербайджан. г.Баку

ОБЛАСТЬ ПРИМЕНИМОСТИ ПРИБЛИЖЁННЫХ УРАВНЕНИЙ СТЕРЖНЕВЫХ СИСТЕМ ПЕРЕМЕННОЙ ТОЛЩИНЫ

Аннотация: Приведённые в данной работе приближённые уравнения крутильного колебаний стержневых систем переменной толщины позволяют строить приближённые теории колебания в зависимости от условий на торцах стержня, порядка производных от искомых в приближённых уравнениях и начальных условий.

Приближённые уравнения колебаний стержневых систем выше второго порядка по производным от искомой функции в научной литературе практически не встречаются.

С другой стороны, приближённые уравнения, содержащие производные выше второго порядка, нуждаются в исследовании области применимости той или иной теории на основе различных приближённых уравнений. Для простоты рассмотрим область применимости для стержня постоянного радиуса, когда искомая функция U_θ или вращательное смещение зависит от угловой координаты θ , т.е. исследуем область применимости приближённых уравнений колебания на основе общего уравнения колебания

Ключевые слова: крутильная колебания, стержень, система, переменная толщина, цилиндр, оболочка, радиус, функция Бесселя

Information about authors:

Medeubaev Nurbolat -a senior teacher of department is "Algebra, mate. logic and geometry", Buketov. Karaganda State University.Karaganda.

Seitmuratov Angisin – Doktor of Physical and Mathematical Sciences, Professoz, The Korkyt Ata Kyzylorda State University. Kyzylorda.

Menlikozhaeva Sauleh Koilibaevna - kandidate of Pedagogical sciences,associate professor, The Korkyt Ata Kyzylorda State University, Kyzylorda.

Ramazanov Murat- Doktor of Physical and Mathematical Sciences, Professoz, Buketov. Karaganda State University.Karaganda.

Zharmenova Botagoz Kuanyshhevna- Master degree of mathematical sciences, The Korkyt Ata Kyzylorda State University, Kyzylorda.

Shamilov Tefriz- Candidate of Technical Sciences, Professoz, Departament of «Physics»,Azerbaijan University of Architecture and Construction, Azerbaijan, Baku.