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THE USE OF A MATHEMATICAL METHOD OF I. G. FILIPPOVA IN THE SOLUTION OF BOUNDARY VALUE PROBLEMS OF VIBRATIONS OF CYLINDRICAL SHELLS

Abstract: In the present work deals with the questions for problems of torsional and longitudinal vibrations of cylindrical shells of variable thickness in a more general setting that allow to formulate a boundary value problem in the solution of private problems of vibrations of a cylindrical shell under various conditions at the end of the shell. A review of the well-known in the scientific literature studies in the field of vibrations of rods and shells is given in many works. However, these works did not reflect and did not formulate the boundary value problems of oscillation: along with the approximate equations of oscillation, there are no strictly justified boundary conditions at the ends of the rods and shells arising from the developed mathematical approach, and the boundary conditions from the static problems were applied. In addition, the necessary number of initial conditions depending on the order of time derivatives of the required functions was not justified and the areas of applicability of the approximate equations of oscillations were not investigated.

Key words: regional tasks, cylindrical shell, oscillation, torsion vibrations, Bessel function, close equalization, three-dimensional task, variable radius.

Cylindrical shells of variable radius are found in many constructions and buildings. In the scientific literature, the theory of oscillations of cylindrical shells is constructed using the hypothesis of flat sections. Below this problem is solved with the use of a mathematical method used in the works of I. G. Filippov and A. Zh. Seitmuratov.

To derive the equations of oscillation of the rod of variable radius we have the following three-dimensional boundary value problem: equations of motion in potentials

$$N(\Delta\Phi) = \rho\frac{\partial^2\Phi}{\partial t^2};$$

$$M(\Delta\psi_2) = \rho\frac{\partial^2\psi_2}{\partial t^2}; \tag{1}$$

$$\vec{U}(u_r, u_z) = grad\ \Phi + rot\left[rot(\vec{e}_z), \psi_2\right]$$

boundary conditions on the rod surface

$$\begin{aligned} &F'^2(z)\sigma_{zz} + \sigma_{rr} - 2F'\sigma_{rz} = f_n\Delta_0^2; \\ &(\sigma_{rr} - \sigma_{zz})F'(z) + \sigma_{rz}\left\{1 - \left[F'^2_z(z)\right]^2\right\} = f_{ns_2}\Delta_0^2 \end{aligned} \tag{2}$$

where $r = F(z)$ - variable radius of the rod; initial conditions are zero. Potentials Φ and ψ_2 let us seek believing

$$\Phi = \int_0^\infty \begin{matrix}\sin(kz)\\ -\cos(kz)\end{matrix} \Big\} dk \int_l \Phi_0 \exp(pt)dp;$$
$$\psi_2 = \int_0^\infty \begin{matrix}\cos(kz)\\ \sin(kz)\end{matrix} \Big\} dk \int_l \psi_{20} \exp(pt)dp \tag{3}$$

For the values Φ_0, ψ_{20} converted by Fourier and Laplace potentials after substitution (3) in the equation (1) we obtain the ordinary Bessel's differential equations of the imaginary argument

$$\frac{d^2\Phi_0}{dr^2} + \frac{1}{r}\frac{d\Phi_0}{dr} - \alpha^2\Phi_0 = 0; \qquad \alpha^2 = k^2 + \rho p^2 N_0^{-1}(p)$$
$$\frac{d^2\psi_{20}}{dr^2} + \frac{1}{r}\frac{d\psi_{20}}{dr} - \beta^2\psi_{20} = 0; \quad \beta^2 = k^2 + \rho p^2 M_0^{-1}(p) \tag{4}$$

whose solution is limited in the case of $r = 0$, equal

$$\Phi_0 = A_0 I_0(\alpha r); \qquad \psi_{20} = B_0 I_0(\beta r) \tag{5}$$

representing the displacements of u_r, u_z in the form

$$u_\theta = \int_0^\infty \begin{matrix}\sin(kz)\\ -\cos(kz)\end{matrix} \Big\} dk \int_{(l)} u_{\theta,0} \exp(pt)dp$$

for $u_{r,0}, u_{z,0}$ obtaining the expression

$$u_{r,0} = \sum_{n=0}^{\infty}\left(A_0\alpha^{2n+2} - kB_0\beta^{2(n+1)}\right)\frac{\left(\frac{r}{2}\right)^{2n+1}}{n!(n+1)!};$$
$$u_{z,0} = \sum_{n=0}^{\infty}\left(kA_0\alpha^{2n} - B_0\beta^{2(n+1)}\right)\frac{\left(\frac{r}{2}\right)^{2n}}{n!} \tag{6}$$

after decomposition of the Bessel's function in power series of the argument.

Instead of continuous integration A_0, B_0 we introduce a new

$$U_0 = \alpha^2 A_0 - k\beta^2 B_0; \qquad W_0 = kA_0 - \beta^2 B_0 \tag{7}$$

herewith W_0 - the transformed offset of the axis points z of the rod U_0 - s derived from the displacement $u_{r,0}$ along the radial coodinate r also at $r = 0$. In the new variables U_0, W_0 as before $u_{r,0}, u_{z,0}$ will be written in the form

$$u_{r,0}=\sum_{n=0}^{\infty}\left[\left(\alpha^2C_0Q_n^{(0)}+\beta^{2n}\right)U_0-k\alpha^2C_0Q_n^{(0)}W_0\right]\frac{\left(\frac{r}{2}\right)^{2n+1}}{n!(n+1)!};$$

$$u_{z,0}=\sum_{n=0}^{\infty}\left[k^2C_0Q_n^{(0)}U_0-\left(k^2C_0Q_n^{(0)}-\beta^{2n}\right)W_0\right]\frac{\left(\frac{r}{2}\right)^{2n}}{(n!)^2}; \tag{8}$$

By reference (8) to k , p , for displacement u_r, u_z we obtain expressions

$$u_r=\sum_{n=0}^{\infty}\left[\left(\lambda_1^{(1)}CQ_n+\lambda_2^{(n)}\right)U+\frac{\partial}{\partial z}\lambda_1^{(1)}CQ_nW\right]\frac{\left(\frac{r}{2}\right)^{2n+1}}{n!(n+1)!};$$

$$u_z=\sum_{n=0}^{\infty}\left[\frac{\partial}{\partial z}CQ_nU+\left(\frac{\partial^2}{\partial z^2}CQ_n+\lambda_2^{(n)}\right)W\right]\frac{\left(\frac{r}{2}\right)^{2n}}{(n!)^2} \tag{9}$$

где

$$\lambda_1^{(1)}=\rho N^{-1}\left(\frac{\partial^2}{\partial t^2}\right)-\frac{\partial^2}{\partial z^2};\qquad \lambda_2^{(1)}=\rho M^{-1}\left(\frac{\partial^2}{\partial t^2}\right)-\frac{\partial^2}{\partial z^2};$$

$$Q_n=\sum_{q=1}^{n-1}\lambda_1^{(q)}\lambda_2^{(q-n+1)};\qquad Q_0=0;\qquad Q_1=1$$

$\lambda_1^{(1)}$, $\lambda_2^{(1)}$ - describes the propagation of longitudinal and transverse waves along the Z .

Substituting the general solutions of the three-dimensional problem (9) in the boundary conditions (2), to determine U,W we obtain a system of equation

$$\begin{aligned}L_{1n}(U)+M_{1n}(W)&=f_n\Delta_0^2;\\ L_{2n}(U)+M_{2n}(W)&=f_{ns_2}\Delta_0^2\end{aligned} \tag{10}$$

where operators L_{jn},M_{jn} are equal

$$L_{1n}=\sum_{n=0}^{\infty}\frac{\left(\frac{F}{2}\right)^{2n}}{(n!)^2}\left\{\left[-2CQ_n\frac{\partial^2}{\partial z^2}+(1-C)\lambda_1^{(n)}-\frac{1}{n+1}\left(\lambda_1^{(n)}-CQ_n\frac{\partial^2}{\partial z^2}\right)\right]+\left(F_z'\right)^2\left[2CQ_n\frac{\partial^2}{\partial z^2}-(1+C)\lambda_1^{(n)}\right]+\right.$$
$$\left.+\frac{FF_z'}{n+1}\left[CQ_n\left(\lambda_2^{(1)}-\frac{\partial^2}{\partial z^2}\right)+(1+C)\lambda_1^{(n)}\right]\frac{\partial}{\partial z}\right\};$$

$$L_{2n}=\sum_{n=0}^{\infty}\frac{\left(\frac{F}{2}\right)^{2n}}{(n!)^2}\left\{F_z^{'}\left[2\lambda_1^{(n)}-4CQ_n\frac{\partial^2}{\partial z^2}-\frac{1}{n+1}\left(\lambda_1^{(n)}-CQ_n\frac{\partial^2}{\partial z^2}\right)\right]-\frac{F\left(F_z^{'}\right)^2}{2(n+1)}\left[CQ_n\left(\lambda_2^{(1)}-\frac{\partial^2}{\partial z^2}\right)+(1+C)\lambda_1^{(n)}\right]\frac{\partial}{\partial z}\right\}; \quad (11)$$

$$M_{1n}=\sum_{n=0}^{\infty}\frac{\left(\frac{F}{2}\right)^{2n}}{(n!)^2}\left\{\left[-2CQ_n\lambda_1^{(1)}+(1-C)\lambda_1^{(n)}+\frac{1}{n+1}CQ_n\lambda_1^{(n)}\right]\frac{\partial}{\partial z}+\left(F_z^{'}\right)^2\right.$$

$$\left.\times\left[2CQ_n\lambda_1^{(1)}-(1-C)\lambda_1^{(n)}\right]\frac{\partial}{\partial z}+\frac{FF_z^{'}}{n+1}\left[CQ_n\left(\lambda_2^{(1)}-\frac{\partial^2}{\partial z^2}\right)\lambda_1^{(n)}-(1-C)\lambda_1^{(n+1)}\right]\right\}$$

$$f_n^{(1)}=M^{-1}f_n\Delta_0^2; \qquad f_{ns_2}^{(1)}=f_{ns_2}\Delta_0^2;$$

$$\Delta_0^2=1+\left(F_z^{'}\right)^2$$

The equations (10) are the general equations of longitudinal oscillation of a rod of variable radius.

Due to the complexity of the equations (10), we obtain from them the approximate equations of the longitudinal vibration of the rod. Limited to the first summands in the series of operators (11), we obtain a known equation for the constant radius rod

$$\frac{1}{\tilde{n}_{cm}^2}\frac{\partial^2 W}{\partial t^2}-\frac{\partial^2 W}{\partial z^2}=0; \qquad \left(f_n=f_{ns_2}=0\right) \quad (12)$$

in this case, the speed of the longitudinal wave of compression in the rod is equal to

$$c_{cm}^2=b\left(3a^2-4b^2\right)*\left(a^2-b^2\right)^{-1}$$

and is given in various textbooks and books.

If we limit ourselves to the first two terms in the series of operators (11), we obtain a refined equation that generalizes the equation (12).

If the rod of variable radius, then from the general equations of oscillation (10) we obtain the simplest equation of the second order

$$\rho_0(z)W+\rho_1(z)\frac{\partial W}{\partial z}+\rho_2(z)\frac{\partial^2 W}{\partial z^2}=\rho N^{-1}\rho_3(z)\frac{\partial^{2W}}{\partial t^2} \quad (13)$$

where the variable coefficients of the $\rho_j(z)$ in the derivative of W are equal

$$\rho_0(z)=2\left[(1-C)F(z)F^{'''}(z)F^{'}(z)-(1+C)F^{''}(z)\right]$$

$$\rho_1(z)=\left[\frac{1}{2}(1-C)^2F_z^{'}()-(5+C)F_z^{'2}(z)+\frac{1}{2}(1-C^2)F_z^{'5}(z)--(1+C)(3+C)F(z)F_z^{'}(z)F_z^{''}(z)5-\right.$$

$$-\frac{1}{2}(1-C^2)F_z^{''}(z)+(1-C^2)\,F_2^{'2}(z)F_z^{''}(z)\left(FF_z^{'}(z)-\frac{5}{2}F\right)+(1-C^2)\left(F_z^{''2}(z)+F_z^{'}(z)F_z^{'''}(z)\right)$$

$$\rho_2(z)=\left\{-\frac{1}{2}\left[(1+C)^2-C(1-C)\right]F(z)+\frac{1}{2}(3C^2-5C-5)F(z)F_z^{'2}(z)+\frac{5}{2}(1-C)^2F(z)F_z^{'4}(z)+\right.$$

$$\left.+3(1-C^2)F^2(z)F_z^{'2}(z)F_z^{''}(z)\right\}$$

$$\rho_3(z)=\left[\frac{C}{2}(1-C)F(z)-\frac{1}{2}(C^2-3C+1)F(z)F_z'^2(z)-(1-C^2)F(z)F_z'^4(z)-(1-C^2)F^2(z)F_z'^2(z)F_z''(z)\right] \quad (14)$$

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ЦИЛИНДІРЛІК ҚАБЫҚШАЛАРДЫҢ ШЕТТІК ТЕРБЕЛІС ЕСЕБІ ҮШІН И.Г.ФИЛИППОВТЫҢ МАТЕМАТИКАЛЫҚ ШЕШУ ӘДІСІН ҚОЛДАНУ

Аннотация: Берілген мақалада айналмалы және көлбеу цилиндірлік қабықшалар есебінен туындайтын сұрақтар жайлы мәселе қозғалады. Негізінен қабықшалар айналасында болатын әр-түрлі шарттарға байланысты шеттік есептерді қалыптастыру мақсатында цилиндірлік қабықшасының айнымалы қалыңдығына байланысты жалпы есебі қарастырылады. Осыған дейінгі жасалған сырықтар мен қабықшалар тербелісі жұмыстарына көз жүгірте отырып, қарастырылған тербеліс шеттік есептерінде нақты қатаң түрде математикалық тұжырымы жеткілікзі көрсетілген. Сондықтан бұл жұмыста белгілі ғалым И.Г.Филипповтың тербеліс есептерін шешудегі математикалық әдісін қолдандып тербеліс теңдеулерінің жуық шамасының қолдану шарттары анықталды.

Түйін сөз: шеттік есеп, цилиндірлік қабықша, тербеліс, айналмалы тербеліс, Бессел функциясы, жуық теңдеу, үш өлшемді есеп, айнымалы радиус.

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ПРИМЕНЕНИЕ МАТЕМАТИЧЕСКОГО МЕТОДА И.Г.ФИЛИППОВА ПРИ РЕШЕНИИ КРАЕВЫХ ЗАДАЧ КОЛЕБАНИЯ ЦИЛИНДРИЧЕСКИХ ОБОЛОЧЕК

Аннотация: В настоящей работе рассматриваются вопросы для задач крутильного и продольного колебаний цилиндрических оболочек переменной толщины в более общей постановке, которые позволяют формулировать краевые задачи при решении частных задач колебания цилиндрической оболочки при различных условиях на торце оболочки. Обзор известных в научной литературе исследований в области колебаний стержней и оболочек приведён во многих работах. Однако в этих работах не нашли отражения и не были сформулированы краевые задачи колебания: наряду с приближёнными уравнениями колебания отсутствуют строго обоснованные граничные условия на торцах стержней и оболочек, вытекающие из развиваемого математического подхода, а применялись граничные условия из задач статики. Кроме того, не обосновывалось необходимое число начальных условий в зависимости от порядка производных по времени от искомых функций и не исследовались области применимости приближённых уравнений колебаний.

Ключевые слова: краевые задачи, цилиндрическая оболочка, колебание, крутильная колебания, функция Бесселя, приближенные уравнение, трехмерная задача, переменный радиус.

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