N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2018.2518-1726.5**
Volume 5, Number 321 (2018), 33 – 41

UDK 517.951
MRNTI 27.31.15

A.T.Assanova[1], B.Zh.Alikhanova[2] , K.Zh.Nazarova[2]

[1] Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan;
[2] Akhmet Yassawi International Kazakh-Turkish University, Turkestan, Kazakhstan
E-mail: assanova@math.kz; bota_mktu@mail.ru; gjnazarova@mail.ru

WELL-POSEDNESS OF A NONLOCAL PROBLEM WITH INTEGRAL CONDITIONS FOR THIRD ORDER SYSTEM OF THE PARTIAL DIFFERENTIAL EQUATIONS

Abstract. The nonlocal problem with integral conditions for the system of partial differential equations third-order is considered. The existence and uniqueness of classical solution to nonlocal problem with integral conditions for third-order system of partial differential equations are studied and the method for constructing their approximate solutions is proposed. Conditions of an unique solvability to nonlocal problem with integral conditions for third order system of partial differential equations are established. By introduction of new unknown functions, we have reduced the considered problem to an equivalent problem consisting of a nonlocal problem with integral conditions and parameters for a system of hyperbolic equations of second order and a integral relations. We have offered the algorithm for finding approximate solution to investigated problem and have proved its convergence. Sufficient conditions for the existence of unique solution to the equivalent problem with parameters are obtained. Well-posedness of the nonlocal problem with integral conditions for third order system of partial differential equations are established in the terms of well-posedness to nonlocal problem with integral conditions for system of hyperbolic equations second order.

Key Words: third order partial differential equations, nonlocal problem, integral condition, system of hyperbolic equations second order, solvability, algorithm.

1. Introduction. In recent years, there has been a great interest to a nonlocal problems for third order partial differential equations and systems. Such problems are appeared in the mathematical modeling of various natural science processes [1-9]. A lot of many works devoted to the investigate of various problems for third order partial differential equations with two independent variables, bibliography and analysis can be see in [1, 2, 5, 7-9]. The third order system of partial differential equations began to be studied relatively recently [5, 7-9].

In the present paper we consider the nonlocal problem with integral conditions for third order system of partial differential equations at a rectangular domain. We investigate the questions of existence and uniqueness of the classical solution to nonlocal problem with integral conditions for third order system of partial differential equations and its applications. Some types of initial-boundary value problems for third order system of partial differential equations are considered in [7-9].

Methods. For solve to considered problem we use a method of introduction additional functional parameters [10-32]. The original problem is reduced to an equivalent problem consisting from nonlocal problem with integral conditions for system of hyperbolic equations second order, containing functional parameters and integral relations. Sufficient conditions of the unique solvability to investigated problem are established in the terms of unique solvability to nonlocal problem with integral conditions for system of hyperbolic equations. Algorithms for finding of approximate solution to the equivalent problem are

constructed. Well-posedness to nonlocal problem with integral conditions for third order system of partial differential equations are established in the terms of well-posedness to nonlocal problem with integral conditions for system of hyperbolic equations second order.

2. Statement of problem. At the domain $\Omega=[0,T]\times[0,\omega]$ we consider the following initial-boundary value problem for the special system of partial differential equations

$$\frac{\partial^3 u}{\partial t\partial x^2}=A(t,x)\frac{\partial^2 u}{\partial x^2}+B(t,x)\frac{\partial^2 u}{\partial t\partial x}+C(t,x)\frac{\partial u}{\partial x}+D(t,x)\frac{\partial u}{\partial t}+E(t,x)u+f(t,x),\ (t,x)\in\Omega, \quad (1)$$

$$\int_0^T L(\tau,x)\frac{\partial u(\tau,x)}{\partial x}d\tau=\varphi(x),\ x\in[0,\omega], \quad (2)$$

$$u(t,0)=\psi_0(t),\ t\in[0,T], \quad (3)$$

$$\int_0^\omega P(t,\xi)\frac{\partial u(t,\xi)}{\partial \xi}d\xi=\psi_1(t),\ t\in[0,T], \quad (4)$$

where $u(t,x)=col(u_1(t,x),u_2(t,x),...,u_n(t,x))$ is unknown function, the $n\times n$-matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, $D(t,x)$, $E(t,x)$ and n-vector function $f(t,x)$ are continuous on Ω, the $n\times n$-matrix $L(t,x)$ is continuous and continuously differentiable by x on Ω, the n-vector-function $\varphi(x)$ is continuously differentiable on $[0,\omega]$, the $n\times n$-matrix $P(t,x)$ is continuous and continuously differentiable by t on Ω, the n-vector-functions $\psi_0(t)$ and $\psi_1(t)$ are continuously differentiable on $[0,T]$.

A function $u(t,x)\in C(\Omega,R^n)$ having partial derivatives $\frac{\partial u(t,x)}{\partial x}\in C(\Omega,R^n)$, $\frac{\partial u(t,x)}{\partial t}\in C(\Omega,R^n)$, $\frac{\partial^2 u(t,x)}{\partial x^2}\in C(\Omega,R^n)$, $\frac{\partial^2 u(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$, $\frac{\partial^3 u(t,x)}{\partial t\partial x^2}\in C(\Omega,R^n)$ is called a classical solution to problem (1)--(4) if it satisfies system (1) for all $(t,x)\in\Omega$, and integral and initial conditions (2), (3) and (4).

We investigate the questions of existence and uniqueness of the classical solutions to nonlocal problem with integral conditions for system of partial differential equations of third order (1)--(4) and the approaches of constructing its approximate solutions. For this goals, we applied the method of introduction additional functional parameters proposed in [10-32] for the solve of nonlocal boundary value problems for systems of hyperbolic equations with mixed derivatives. Considered problem is provided to nonlocal problem with integral conditions for system of hyperbolic equations of second order, including additional functions and integral relations. The algorithm of finding the approximate solution of the investigated problem is proposed and its convergence proved. Sufficient conditions of the existence unique classical solution to problem (1)--(4) are obtained in the terms of unique solvability to nonlocal problem with integral conditions for system of hyperbolic equations second order.

3. Scheme of the method and reduction to equivalent problem.

We introduce a new unknown function $v(t,x)=\frac{\partial u(t,x)}{\partial x}$ and re-write nonlocal problem with integral conditions (1)--(4) in the following form

$$\frac{\partial^2 v}{\partial t\partial x}=A(t,x)\frac{\partial v}{\partial x}+B(t,x)\frac{\partial v}{\partial t}+C(t,x)v+D(t,x)\frac{\partial u}{\partial t}+E(t,x)u+f(t,x),\ (t,x)\in\Omega, \quad (5)$$

$$\int_0^T L(\tau,x)v(\tau,x)d\tau=\varphi(x),\ x\in[0,\omega], \quad (6)$$

$$\int_0^{\omega} P(t,\xi)v(t,\xi)d\xi = \psi_1(t),\ t\in[0,T], \tag{7}$$

$$u(t,x)=\psi_0(t)+\int_0^x v(t,\xi)d\xi\,,\ \frac{\partial u(t,x)}{\partial t}=\dot{\psi}_0(t)+\int_0^x \frac{\partial v(t,\xi)}{\partial t}d\xi\,,\ (t,x)\in\Omega\,. \tag{8}$$

Here the condition (3) is taken account in (8).

A pair functions $(v(t,x),u(t,x))$, where the function $v(t,x)\in C(\Omega,R^n)$ has partial derivatives $\frac{\partial v(t,x)}{\partial x}\in C(\Omega,R^n)$, $\frac{\partial v(t,x)}{\partial t}\in C(\Omega,R^n)$, $\frac{\partial^2 v(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$, the function $u(t,x)\in C(\Omega,R^n)$ has partial derivatives $\frac{\partial u(t,x)}{\partial x}\in C(\Omega,R^n)$, $\frac{\partial u(t,x)}{\partial t}\in C(\Omega,R^n)$, $\frac{\partial^2 u(t,x)}{\partial x^2}\in C(\Omega,R^n)$, $\frac{\partial^2 u(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$, $\frac{\partial^3 u(t,x)}{\partial t\partial x^2}\in C(\Omega,R^n)$, is called a solution to problem (5)--(8) if it satisfies of the system of hyperbolic equations (5) for all $(t,x)\in\Omega$, the boundary conditions (6), (7), and the integral relation (8).

At fixed $u(t,x)$ the problem (5)--(7) is the nonlocal problem with integral conditions for the system of hyperbolic equations with respect to $v(t,x)$ on Ω. The integral relations (8) allow us to determine the unknown functions $u(t,x)$ and its partial derivative $\frac{\partial u(t,x)}{\partial t}$ for all $(t,x)\in\Omega$.

4. Algorithm for finding of the approximate solution to problem (5—(8).

The unknown function $v(t,x)$ will be determined from nonlocal problem with integral conditions for system of hyperbolic equations (5)--(7). The unknown function $u(t,x)$ and its partial derivative $\frac{\partial u(t,x)}{\partial t}$ will be found from integral relations (8). If we known the function $u(t,x)$ and its partial derivative $\frac{\partial u(t,x)}{\partial t}$, then from nonlocal problem with integral conditions (5)--(7) we find the function $v(t,x)$. Conversely, if we known the function $v(t,x)$ and its partial derivative $\frac{\partial v(t,x)}{\partial t}$, then from integral relations (8) we find the function $u(t,x)$ and its partial derivative $\frac{\partial u(t,x)}{\partial t}$. Since the functions $u(t,x)$ and $v(t,x)$ are unknowns together for finding of the solution to problem (5)--(8) we use an iterative method.

The solution to problem (5)--(8) is the pair functions $(v^*(t,x),u^*(t,x))$ we defined as a limit of sequence of pairs $(v^{(k)}(t,x),u^{(k)}(t,x))$, $k=0,1,2,\ldots$, according to the following algorithm:

Step 0. 1) Suppose in the right-hand part of system (5) $u(t,x)=\psi_0(t)$, $\frac{\partial u(t,x)}{\partial t}=\dot{\psi}_0(t)$, from nonlocal problem with integral conditions (5)--(7) we find the initial approximation $v^{(0)}(t,x)$ and its partial derivatives for all $(t,x)\in\Omega$;

2) From integral relations (8) under $v(t,x)=v^{(0)}(t,x)$, $\frac{\partial v(t,x)}{\partial t}=\frac{\partial v^{(0)}(t,x)}{\partial t}$, we find the functions $u^{(0)}(t,x)$ and $\frac{\partial u^{(0)}(t,x)}{\partial t}$ for all $(t,x)\in\Omega$.

Step 1. 1) Suppose in the right-hand part of system (5) $u(t,x)=u^{(0)}(t,x)$, $\dfrac{\partial u(t,x)}{\partial t}=\dfrac{\partial u^{(0)}(t,x)}{\partial t}$, from nonlocal problem with integral conditions (5)--(7) we find the first approximation $v^{(1)}(t,x)$ and its partial derivatives for all $(t,x)\in\Omega$.

2) From integral relations (8) under $v(t,x)=v^{(1)}(t,x)$, $\dfrac{\partial v(t,x)}{\partial t}=\dfrac{\partial v^{(1)}(t,x)}{\partial t}$, we find the functions $u^{(1)}(t,x)$ and $\dfrac{\partial u^{(1)}(t,x)}{\partial t}$ for all $(t,x)\in\Omega$.

And so on.

Step k. 1) Suppose in the right-hand part of system (5) $u(t,x)=u^{(k-1)}(t,x)$, $\dfrac{\partial u(t,x)}{\partial t}=\dfrac{\partial u^{(k-1)}(t,x)}{\partial t}$, from nonlocal problem with integral conditions (5)--(7) we find the k-th approximation $v^{(k)}(t,x)$ and its partial derivatives for all $(t,x)\in\Omega$:

$$\frac{\partial^2 v^{(k)}}{\partial t\partial x}=A(t,x)\frac{\partial v^{(k)}}{\partial x}+B(t,x)\frac{\partial v^{(k)}}{\partial t}+C(t,x)v^{(k)}+D(t,x)\frac{\partial u^{(k-1)}}{\partial t}+E(t,x)u^{(k-1)}+f(t,x),\ (t,x)\in\Omega, \quad (9)$$

$$\int_0^T L(\tau,x)v^{(k)}(\tau,x)d\tau=\varphi(x),\ x\in[0,\omega], \quad (10)$$

$$\int_0^\omega P(t,\xi)v^{(k)}(t,\xi)d\xi=\psi_1(t),\ t\in[0,T]. \quad (11)$$

2) From integral relations (8) under $v(t,x)=v^{(k)}(t,x)$, $\dfrac{\partial v(t,x)}{\partial t}=\dfrac{\partial v^{(k)}(t,x)}{\partial t}$, we find the function $u^{(k)}(t,x)$ and $\dfrac{\partial u^{(k)}(t,x)}{\partial t}$ for all $(t,x)\in\Omega$:

$$u^{(k)}(t,x)=\psi_0(t)+\int_0^x v^{(k)}(t,\xi)d\xi,\ \frac{\partial u^{(k)}(t,x)}{\partial t}=\dot{\psi}_0(t)+\int_0^x\frac{\partial v^{(k)}(t,\xi)}{\partial t}d\xi,\ (t,x)\in\Omega. \quad (12)$$

$k=1,2,3,\ldots$.

5. The main result.

Consider nonlocal problem with integral conditions (5)-(7) at fixed $u(t,x)$. Then nonlocal problem with integral conditions for system of hyperbolic equations can be have the following form

$$\frac{\partial^2 v}{\partial t\partial x}=A(t,x)\frac{\partial v}{\partial x}+B(t,x)\frac{\partial v}{\partial t}+C(t,x)v+F(t,x),\ (t,x)\in\Omega, \quad (13)$$

$$\int_0^T L(\tau,x)v(\tau,x)d\tau=\varphi(x),\ x\in[0,\omega], \quad (14)$$

$$\int_0^\omega P(t,\xi)v(t,\xi)d\xi=\psi_1(t),\ t\in[0,T], \quad (15)$$

where n-vector function $F(t,x)$ is continuous on Ω.

The following theorem gives conditions of feasibility and convergence of the constructed algorithm and the conditions of the existence unique solution to problem (5)--(8).

Theorem 1. *Suppose that*

i) the $n\times n$-matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, $D(t,x)$, $E(t,x)$, and n-vector function $f(t,x)$

are continuous on Ω;

ii) the $n\times n$*-matrix* $L(t,x)$ *is continuous and continuously differentiable by* x *on* Ω; *and the* n*-vector-function* $\varphi(x)$ *continuously differentiable on* $[0,\omega]$;

iii) the $n\times n$*-matrix* $P(t,x)$ *is continuous and continuously differentiable by* t *on* Ω; *the* n*-vector-functions* $\psi_0(t)$ *and* $\psi_1(t)$ *are continuously differentiable on* $[0,T]$;

iv) Nonlocal problem with integral conditions for system of hyperbolic equations (13)--(15) has a unique classical solution.

Then equivalent nonlocal problem for system of hyperbolic equations with integral conditions and parameters (5)—(8) has a unique solution.

Proof. Let the conditions i) - iv) of Theorem 1 are fulfilled. By the algorithm, proposing above, on 0th step we have

$$\frac{\partial^2 v^{(0)}}{\partial t\partial x}=A(t,x)\frac{\partial v^{(0)}}{\partial x}+B(t,x)\frac{\partial v^{(0)}}{\partial t}+C(t,x)v^{(0)}+D(t,x)\dot{\psi}_0(t)+E(t,x)\psi_0(t)+f(t,x),\ (t,x)\in\Omega, \quad (16)$$

$$\int_0^T L(\tau,x)v^{(0)}(\tau,x)d\tau=\varphi(x),\ x\in[0,\omega], \quad (17)$$

$$\int_0^\omega P(t,\xi)v^{(0)}(t,\xi)d\xi=\psi_1(t),\ t\in[0,T]. \quad (18)$$

Since condition iv) is valid, problem (16)—(18) has a unique classical solution $v^{(0)}(t,x)$ and the following inequality holds

$$\max\left(\max_{(t,x)\in\Omega}\left\|v^{(0)}(t,x)\right\|,\max_{(t,x)\in\Omega}\left\|\frac{\partial v^{(0)}(t,x)}{\partial x}\right\|,\max_{(t,x)\in\Omega}\left\|\frac{\partial v^{(0)}(t,x)}{\partial t}\right\|\right)\le$$

$$\le K(x)\max\left(\max_{(t,x)\in\Omega}\left\|f(t,x)\right\|,\max_{t\in[0,T]}\left\|\psi_1(t)\right\|,\max_{x\in[0,\omega]}\left\|\varphi(x)\right\|,\max_{(t,x)\in\Omega}\left\|F^{(0)}(t,x)\right\|\right),$$

where the function $K(x)$ is continuous on $[0,\omega]$, positive and independent on functions $f(t,x)$, $\psi_1(t)$, $\varphi(x)$, $F^{(0)}(t,x)=D(t,x)\dot{\psi}_0(t)+E(t,x)\psi_0(t)$.

Initial approximations $u^{(0)}(t,x)$ and $\dfrac{\partial u^{(0)}(t,x)}{\partial t}$ are determined by the following form

$$u^{(0)}(t,x)=\psi_0(t)+\int_0^x v^{(0)}(t,\xi)d\xi,\ \frac{\partial u^{(0)}(t,x)}{\partial t}=\dot{\psi}_0(t)+\int_0^x\frac{\partial v^{(0)}(t,\xi)}{\partial t}d\xi,\ (t,x)\in\Omega. \quad (19)$$

Then functions $u^{(0)}(t,x)$ and $\dfrac{\partial u^{(0)}(t,x)}{\partial t}$ satisfy the estimate

$$\max\left(\max_{t\in[0,T]}\left\|u^{(0)}(t,x)\right\|,\max_{t\in[0,T]}\left\|\frac{\partial u^{(0)}(t,x)}{\partial t}\right\|\right)\le\max\left(\max_{t\in[0,T]}\left\|\psi_0(t)\right\|,\max_{t\in[0,T]}\left\|\dot{\psi}_0(t)\right\|\right)+$$

$$+\int_0^x\max\left(\max_{t\in[0,T]}\left\|v^{(0)}(t,\xi)\right\|,\max_{t\in[0,T]}\left\|\frac{\partial v^{(0)}(t,\xi)}{\partial t}\right\|\right)d\xi.$$

For kth approximations determined by relations (9)—(12), we have

$$\max\left(\max_{(t,x)\in\Omega}\left\|v^{(k)}(t,x)\right\|,\max_{(t,x)\in\Omega}\left\|\frac{\partial v^{(k)}(t,x)}{\partial x}\right\|,\max_{(t,x)\in\Omega}\left\|\frac{\partial v^{(k)}(t,x)}{\partial t}\right\|\right)\le$$

$$\leq K(x)\max\left(\max_{(t,x)\in\Omega}\|f(t,x)\|,\max_{t\in[0,T]}\|\psi_1(t)\|,\max_{x\in[0,\omega]}\|\varphi(x)\|,\max_{(t,x)\in\Omega}\|F^{(k-1)}(t,x)\|\right), \tag{20}$$

$$\max\left(\max_{t\in[0,T]}\|u^{(k)}(t,x)\|,\max_{t\in[0,T]}\left\|\frac{\partial u^{(k)}(t,x)}{\partial t}\right\|\right)\leq\max\left(\max_{t\in[0,T]}\|\psi_0(t)\|,\max_{t\in[0,T]}\|\dot{\psi}_0(t)\|\right)+$$

$$+\int_0^x\max\left(\max_{t\in[0,T]}\|v^{(k)}(t,\xi)\|,\max_{t\in[0,T]}\left\|\frac{\partial v^{(k)}(t,\xi)}{\partial t}\right\|\right)d\xi\,, \tag{21}$$

where $F^{(k-1)}(t,x)=D(t,x)\dfrac{\partial u^{(k-1)}(t,x)}{\partial t}+E(t,x)u^{(k-1)}(t,x)$.

Let $\Delta v^{(k)}(t,x)=v^{(k+1)}(t,x)-v^{(k)}(t,x)$, $\Delta_x v^{(k)}(t,x)=\dfrac{\partial v^{(k+1)}(t,x)}{\partial x}-\dfrac{\partial v^{(k)}(t,x)}{\partial x}$,

$$\Delta_t v^{(k)}(t,x)=\frac{\partial v^{(k+1)}(t,x)}{\partial t}-\frac{\partial v^{(k)}(t,x)}{\partial t},\ \Delta u^{(k)}(t,x)=u^{(k+1)}(t,x)-u^{(k)}(t,x),$$

$$\Delta_t u^{(k)}(t,x)=\frac{\partial u^{(k+1)}(t,x)}{\partial t}-\frac{\partial u^{(k)}(t,x)}{\partial t},\ k=1,2,3,\ldots.$$

Then, for differences $\Delta v^{(k)}(t,x)$, $\Delta_x v^{(k)}(t,x)$, $\Delta_t v^{(k)}(t,x)$, $\Delta u^{(k)}(t,x)$, $\Delta_t u^{(k)}(t,x)$ are valid the following inequalities

$$\max\left(\max_{(t,x)\in\Omega}\|\Delta v^{(k)}(t,x)\|,\max_{(t,x)\in\Omega}\|\Delta_x v^{(k)}(t,x)\|,\max_{(t,x)\in\Omega]}\|\Delta_t v^{(k)}(t,x)\|\right)\leq$$

$$\leq K(x)\max\left(\max_{(t,x)\in\Omega}\|\Delta u^{(k-1)}(t,x)\|,\max_{(t,x)\in\Omega]}\|\Delta_t u^{(k-1)}(t,x)\|\right), \tag{22}$$

$$\max\left(\max_{t\in[0,T]}\|\Delta u^{(k)}(t,x)\|,\max_{t\in[0,T]}\|\Delta_t u^{(k)}(t,x)\|\right)\leq\int_0^x\max\left(\max_{t\in[0,T]}\|\Delta v^{(k)}(t,\xi)\|,\max_{t\in[0,T]}\|\Delta_t v^{(k)}(t,\xi)\|\right)d\xi\,. \tag{23}$$

From estimates (20)—(23) is follows

$$\max\left(\max_{t\in[0,T]}\|\Delta u^{(k)}(t,x)\|,\max_{t\in[0,T]}\|\Delta_t u^{(k)}(t,x)\|\right)\leq\frac{[\hat{K}x]^k}{k!}\cdot\max\left(\max_{(t,x)\in\Omega}\|\Delta v^{(0)}(t,x)\|,\max_{(t,x)\in\Omega}\|\Delta_t v^{(0)}(t,x)\|\right),$$

where $\hat{K}=\max\limits_{x\in[0,\omega]}K(x)$.

Hence, we obtain the uniform convergence of the sequences $\{u^{(k)}(t,x)\}$ and $\left\{\dfrac{\partial u^{(k)}(t,x)}{\partial t}\right\}$ to functions $u^*(t,x)$ and $\dfrac{\partial u^*(t,x)}{\partial t}$ on Ω, respectively, as $k\to\infty$.

Then the sequences $\{v^{(k)}(t,x)\}$, $\left\{\dfrac{\partial v^{(k)}(t,x)}{\partial x}\right\}$, and $\left\{\dfrac{\partial v^{(k)}(t,x)}{\partial t}\right\}$ also will be convergent to functions $v^*(t,x)$, $\dfrac{\partial v^*(t,x)}{\partial x}$, and $\dfrac{\partial v^*(t,x)}{\partial t}$ on Ω, respectively, as $k\to\infty$.

The uniqueness of solution to problem (5)—(8) is proved by contradiction.

Theorem 1 is proved.

From equivalence of problem (1)—(4) and (5)—(8) follows the following assertion.

Theorem 2. *Suppose that the conditions i) - iv) of Theorem 1 are fulfilled.*

Then nonlocal problem with integral conditions for third order system of partial differential

equations (1)--(4) has a unique classical solution.

Conditions of Theorem 1 are sufficient of the well-posedness to nonlocal problem with integral solutions for third order system of partial differential equations (1)—(4).

Funding. This results are partially supported by grant of the Ministry education and science of Republic Kazakhstan No. AP 05131220 for 2018-2020 years.

REFERENCES

[1] Ptashnyck B.I. Ill-posed boundary value problems for partial differential equations, Naukova Dumka, Kiev (1984) (in Russian).

[2] Nakhushev A.M. Problems with shift for a partial differential equations, Nauka, Moskow (2006). ISBN: 5-02-034076-6 (in Russian).

[3] Kiguradze T., Lakshmikantham V. On initial-boundary value problems in bounded and unbounded domains for a class of nonlinear hyperbolic equations of the third order, Journal of Mathematical Analysis and Applications, **324**:2 (2006), 1242--1261.

DOI: 10.1016/j.jmaa.2005.12.067

[4] Dzhokhadze O.M. The Riemann function for higher-order hyperbolic equations and systems with dominated lower terms, Differential Equations, **39**:10 (2003), 1440--1453.

DOI: 0012-2661/03/3910-1440

[5] Andreev A.A., Yakovleva J.O. The Goursat problem for one hyperbolic system of the third order differential equations with two independent variables, Journal of Samara State Technical University, Ser. Physical and Mathematical Sciences, **15**:3(24) (2011), 35--41. DOI: https://doi.org/10.14498/vsgtu996

[6] Assanova A.T., Imanchiev A.E. Solvability of multipoint-integral boundary value problem for a third-order differential equation and parametrization method, in: Functional Analysis in Interdisciplinary Applications, Springer Proceedings in Mathematics & Statistics, T.Sh. Kalmenov et al. (eds.), Springer International Publishing, Cham, **216** (2017), 113--122. https://doi.org/10.1007/978-3-319-67053-9_11

[7] Assanova A.T., Sabalakhova A.P., Toleukhanova Z.M. On the solving of initial-boundary value problem for system of partial differential equations of the third order, News of the National Academy of Sciences of the RK. Physico-Mathematical Series. **3**:319 (2018), 67–73.

[8] Assanova A.T., Imanchiyev A.E., Kadirbayeva Z.M. On the solvability of multipoint problem for the loaded third order partial differential equation, Mathematical Journal. **18**:1(67) (2018), 27-35.

[9] Assanova A.T., Imanchiyev A.E. On the unique solvability of a family of multipoint-integral boundary value problems for a third order differential equation, Bulletin of the Karaganda University-Mathematics. 2(90) (2018), 25-33.

[10] Asanova A.T., Dzhumabaev D.S. Unique solvability of the boundary value problem for systems of hyperbolic equations with data on the characteristics, Computational Mathematics and Mathematical Physics. **42**:11 (2002), 1609--1621.

[11] Asanova A.T., Dzhumabaev D.S. Correct solvability of a nonlocal boundary value problem for systems of hyperbolic equations, Doklady Mathematics. **68**:1 (2003), 46--49.

[12] Asanova A.T., Dzhumabaev D.S. Unique solvability of nonlocal boundary value problems for systems of hyperbolic equations, Differential Equations. **39**:10 (2003), 1414--1427. 0012-2661/03/3910-1414

[13] Asanova A.T., Dzhumabaev D.S. Periodic solutions of systems of hyperbolic equations bounded on a plane, Ukrainian Mathematical Journal. **56**:4 (2004), 682--694.

0041–5995/04/5604–0682

[14] Asanova A.T., Dzhumabaev D.S. Well-posed solvability of nonlocal boundary value problems for systems of hyperbolic equations, Differential Equations. **41**:3 (2005), 352--363.

0012-2661/05/4103-0352

[15] Asanova A.T. A nonlocal boundary value problem for systems of quasilinear hyperbolic equations, Doklady Mathematics. **74**:3 (2006), 787--791.

DOI: 10.1134/S1064562406060019

[16] Asanova A.T. On the unique solvability of a nonlocal boundary value problem with data on intersecting lines for systems of hyperbolic equations, Differential Equations. **45**:3 (2009), 385--394. DOI: 10.1134/S0012266109030082

[17] Asanova A.T. On a boundary-value problem with data on noncharacteristic intersecting lines for systems of hyperbolic equations with mixed derivative, Journal of Mathematical Sciences (United States). **187**:4 (2012), 375--386. 1072-3374/12/1874–0375

[18] Asanova A.T. On a nonlocal boundary-value problem for systems of impulsive hyperbolic equations, Ukrainian Mathematical Journal. **65**:3 (2013), 349--365.

0041-5995/13/6503–0349

[19] Asanova A.T., Dzhumabaev D.S. Well-posedness of nonlocal boundary value problems with integral condition for the system of hyperbolic equations, Journal of Mathematical Analysis and Applications, **402**:1 (2013), 167--178. doi:10.1016/j.jmaa.2013.01.012

[20] Asanova A.T. Well-posed solvability of a nonlocal boundary-value problem for systems of hyperbolic equations with

impulse effects, Ukrainian Mathematical Journal. **67**:3 (2015), 333--346. DOI: 10.1007/s11253-015-1083-3

[21] Asanova A.T. On solvability of nonlinear boundary value problems with integral condition for the system of hyperbolic equations, Electronic Journal of Qualitative Theory of Differential Equations. 63 (2015), 1--13. doi: 10.14232/ejqtde.2015.1.63

[22] Asanova A.T., Imanchiev A.E. On conditions of the solvability of nonlocal multi-point boundary value problems for quasi-linear systems of hyperbolic equations, Eurasian Mathematical Journal. **6**:4 (2015), 19--28.

[23] Asanova A.T. Multipoint problem for a system of hyperbolic equations with mixed derivative, Journal of Mathematical Sciences (United States), **212**:3 (2016), 213--233.

DOI: 10.1007/s10958-015-2660-6

[24] Asanova A.T. Criteria of solvability of nonlocal boundary-value problem for systems of hyperbolic equations with mixed derivatives, Russian Mathematics. 60:1 (2016), 1-17.

DOI: 10.3103/S1066369X16050017

[25] Assanova A.T. On the solvability of nonlocal boundary value problem for the systems of impulsive hyperbolic equations with mixed derivatives, Journal of Discontinuity, Nonlinearity and Complexity. **5**:2 (2016), 153--165. DOI: 10.5890/DNC.2016.06.005

[26] Assanova A.T. Periodic solutions in the plane of systems of second-order hyperbolic equations, Mathematical Notes. **101**:1 (2017), 39—47. DOI: 10.1134/S0001434617010047

[27] Assanova A.T. Nonlocal problem with integral conditions for a system of hyperbolic equations in characteristic rectangle, Russian Mathematics. **61**:5 (2017), 7—20.

DOI: 10.3103/S1066369X17050024

[28] Assanova A.T. Solvability of a nonlocal problem for a hyperbolic equation with integral conditions, Electronic Journal of Differential Equations. **2017**:170 (2017), 1-12.

[29] Assanova A.T. An approach to solution of a nonlocal problem for a system of hyperbolic equations with integral conditions, Nonlinear Oscillations, 20:4 (2017), 435-450. (in Russian)

[30] Asanova A.T., Kadirbaeva Zh. M., and Bakirova E. A. On the unique solvability of a nonlocal boundary-value problem for systems of loaded hyperbolic equations with impulsive actions, Ukrainian Mathematical Journal. **69**:8 (2018), 1175—1195.

DOI: 10.1007/s11253-017-1424-5

[31] Assanova A.T. On a nonlocal problem with integral conditions for the system of hyperbolic equations, Differential Equations. **54**:2 (2018), 201–214.

DOI: 10.1134/S0012266118020076

[32] Assanova A.T., Kadirbayeva Z.M. Periodic problem for an impulsive system of the loaded hyperbolic equations, Electronic Journal of Differential Equations. **2018**:72 (2018), 1-8.

УДК 517.951
МРНТИ 27.31.15

А.Т. Асанова[1], Б.Ж. Алиханова[2] , К.Ж.Назарова[2]

[1]Институт математики и математического моделирования, Алматы, Казахстан
[2]Международный Казахско-Турецкий университет им. Х.А.Яссави, Туркестан, Казахстан

КОРРЕКТНАЯ РАЗРЕШИМОСТЬ НЕЛОКАЛЬНОЙ ЗАДАЧИ С ИНТЕГРАЛЬНЫМИ ДЛЯ СИСТЕМЫ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ В ЧАСТНЫХ ПРОИЗВОДНЫХ ТРЕТЬЕГО ПОРЯДКА

Аннотация. Рассматривается нелокальная задача с интегральными условиями для системы дифференциальных уравнений в частных производных третьего порядка. Исследуются вопросы существования и единственности классического решения нелокальной задачи для системы дифференциальных уравнений в частных производных третьего порядка и предлагаются методы построения их приближенных решений. Установлены условия однозначной разрешимости нелокальной задачи для системы дифференциальных уравнений в частных производных третьего порядка. Путем введения новой неизвестной функции исследуемая задача сведена к эквивалентной задаче, состоящей из нелокальной задачи для системы гиперболических уравнений второго порядка с интегральными условиями и функциональными параметрами и интегрального соотношения. Предложены алгоритмы нахождения приближенного решения исследуемой задачи и доказана их сходимость. Получены достаточные условия существования единственного решения эквивалентной задачи с параметрами. Корректная разрешимость нелокальной задачи с интегральными условиями для системы дифференциальных уравнений в частных производных третьего порядка получены в терминах корректной разрешимости нелокальной задачи с интегральными условиями для системы гиперболических уравнений второго порядка.

Ключевые слова: дифференциальное уравнение в частных производных третьего порядка, нелокальная задача, интегральное условие, система гиперболических уравнений второго порядка, разрешимость,

алгоритм.

УДК 517.951
МРНТИ 27.31.15

А.Т. Асанова[1], Б.Ж. Алиханова[2] , К.Ж. Назарова[2]

[1]Математика және математикалық моделдеу институты, Алматы, Қазақстан
[2]Қ.А.Ясауи атындағы Халықаралық қазақ-түрік университеті, Түркістан, Қазақстан

ҮШІНШІ РЕТТІ ДЕРБЕС ТУЫНДЫЛЫ ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕР ЖҮЙЕСІ ҮШІН ИНТЕГРАЛДЫҚ ШАРТТАРЫ БАР БЕЙЛОКАЛ ЕСЕПТІҢ КОРРЕКТІЛІ ШЕШІЛІМДІЛІГІ

Аннотация. Үшінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі үшін интегралдық шарттары бар бейлокал есеп қарастырылады. Үшінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі үшін бейлокал есептің классикалық шешімінің бар болуы мен жалғыздығы мәселелері зерттеледі және олардың жуық шешімдерін тұрғызу әдістері ұсынылады. Үшінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі үшін интегралдық шарттары бар бейлокал есептің бірмәнді шешілімдігінің шарттары тағайындалған. Жаңа белгісіз функция енгізу арқылы зерттеліп отырған есеп гиперболалық теңдеулер жүйесі үшін интегралдық шарттары және параметрлері бар бейлокал есептен және интегралдық қатынастан тұратын пара-пар есепке келтірілген. Зерттеліп отырған есептің жуық шешімін табу алгоритмдері ұсынылған және олардың жинақтылығы дәлелденген. Параметрлері бар пара-пар есептің жалғыз шешімінің бар болуының жеткілікті шарттары алынған. Үшінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі үшін интегралдық шарттары бар бейлокал есептің корректілі шешілімділігінің шарттары екінші ретті гиперболалық теңдеулер жүйесі үшін интегралдық шарттары бар бейлокал есептің корректілі шешілімділігі терминінде алынған.

Түйін сөздер: үшінші ретті дербес туындылы дифференциалдық теңдеу, бейлокал есеп, интегралдық шарт, екінші ретті гиперболалық теңдеулер жүйесі, шешілімділік, алгоритм.

Information about authors:

Assanova Anar Turmaganbetkyzy - the member of Editorial Board of journal “News of the NAS RK. Physico-Mathematical Series”, Institute of Mathematics and Mathematical Modeling, chief scientific researcher, Doctor of Physical and Mathematical Sciences, professor, anarasanova@list.ru, assanova@math.kz, https://orcid.org/0000-0001-8697-8920;

Alikhanova Botakoz Zhorakhanovna - Akhmet Yassawi International Kazakh-Turkish University, teacher, bota_mktu@mail.ru, .: https://orcid.org/0000-0003-2912-1300;

Nazarova Kulzina Zharkimbekovna - Akhmet Yassawi International Kazakh-Turkish University, head teacher, gjnazarova@mail.ru, https://orcid.org/0000-0002-2093-1879;