WELL-POSEDNESS OF A NONLOCAL PROBLEM WITH INTEGRAL CONDITIONS FOR THIRD ORDER SYSTEM OF THE PARTIAL DIFFERENTIAL EQUATIONS

Abstract. The nonlocal problem with integral conditions for the system of partial differential equations third-order is considered. The existence and uniqueness of classical solution to nonlocal problem with integral conditions for third-order system of partial differential equations are studied and the method for constructing their approximate solutions is proposed. Conditions of an unique solvability to nonlocal problem with integral conditions for third order system of partial differential equations are established. By introduction of new unknown functions, we have reduced the considered problem to an equivalent problem consisting of a nonlocal problem with integral conditions and parameters for a system of hyperbolic equations of second order and a integral relations. We have offered the algorithm for finding approximate solution to investigated problem and have proved its convergence. Sufficient conditions for the existence of unique solution to the equivalent problem with parameters are obtained. Well-posedness of the nonlocal problem with integral conditions for third order system of partial differential equations are established in the terms of well-posedness to nonlocal problem with integral conditions for system of hyperbolic equations second order.

Key Words: third order partial differential equations, nonlocal problem, integral condition, system of hyperbolic equations second order, solvability, algorithm.

1. Introduction. In recent years, there has been a great interest to a nonlocal problems for third order partial differential equations and systems. Such problems are appeared in the mathematical modeling of various natural science processes [1-9]. A lot of many works devoted to the investigate of various problems for third order partial differential equations with two independent variables, bibliography and analysis can be see in [1, 2, 5, 7-9]. The third order system of partial differential equations began to be studied relatively recently [5, 7-9].

In the present paper we consider the nonlocal problem with integral conditions for third order system of partial differential equations at a rectangular domain. We investigate the questions of existence and uniqueness of the classical solution to nonlocal problem with integral conditions for third order system of partial differential equations and its applications. Some types of initial-boundary value problems for third order system of partial differential equations are considered in [7-9].

Methods. For solve to considered problem we use a method of introduction additional functional parameters [10-32]. The original problem is reduced to an equivalent problem consisting from nonlocal problem with integral conditions for system of hyperbolic equations second order, containing functional parameters and integral relations. Sufficient conditions of the unique solvability to investigated problem are established in the terms of unique solvability to nonlocal problem with integral conditions for system of hyperbolic equations. Algorithms for finding of approximate solution to the equivalent problem are
constructed. Well-posedness to nonlocal problem with integral conditions for third order system of partial differential equations are established in the terms of well-posedness to nonlocal problem with integral conditions for system of hyperbolic equations second order.

2. Statement of problem. At the domain \( \Omega = [0, T] \times [0, \omega] \) we consider the following initial-boundary value problem for the special system of partial differential equations

\[
\frac{\partial^3 u}{\partial t \partial x^2} = A(t, x) \frac{\partial^2 u}{\partial x^2} + B(t, x) \frac{\partial^2 u}{\partial t \partial x} + C(t, x) \frac{\partial u}{\partial x} + D(t, x) \frac{\partial u}{\partial t} + E(t, x) u + f(t, x), \quad (t, x) \in \Omega, \tag{1}
\]

\[
\int_0^T L(t, x) \frac{\partial u(t, x)}{\partial x} dt = \phi(x), \quad x \in [0, \omega], \tag{2}
\]

\[
u(t, 0) = \psi_0(t), \quad t \in [0, T], \tag{3}
\]

\[
\int_0^\omega P(t, \xi) \frac{\partial u(t, \xi)}{\partial \xi} d\xi = \psi_1(t), \quad t \in [0, T], \tag{4}
\]

where \( u(t, x) = (u_1(t, x), u_2(t, x), \ldots, u_n(t, x)) \) is unknown function, the \( n \times n \)-matrices \( A(t, x), B(t, x), C(t, x), D(t, x), E(t, x) \) and \( n \)-vector function \( f(t, x) \) are continuous on \( \Omega \), the \( n \times n \)-matrix \( L(t, x) \) is continuous and continuously differentiable by \( x \) on \( \Omega \), the \( n \)-vector-function \( \phi(x) \) is continuously differentiable on \( [0, \omega] \), the \( n \times n \)-matrix \( P(t, x) \) is continuous and continuously differentiable by \( t \) on \( \Omega \), the \( n \)-vector-functions \( \psi_0(t) \) and \( \psi_1(t) \) are continuously differentiable on \( [0, T] \).

A function \( u(t, x) \in C(\Omega, R^n) \) having partial derivatives \( \frac{\partial u(t, x)}{\partial x} \in C(\Omega, R^n) \), \( \frac{\partial^2 u(t, x)}{\partial t \partial x} \in C(\Omega, R^n) \), \( \frac{\partial^2 u(t, x)}{\partial x^2} \in C(\Omega, R^n) \), \( \frac{\partial^3 u(t, x)}{\partial t \partial x^2} \in C(\Omega, R^n) \), \( \frac{\partial^3 u(t, x)}{\partial t \partial x} \in C(\Omega, R^n) \) is called a classical solution to problem (1)–(4) if it satisfies system (1) for all \( (t, x) \in \Omega \), and integral and initial conditions (2), (3) and (4).

We investigate the questions of existence and uniqueness of the classical solutions to nonlocal problem with integral conditions for system of partial differential equations of third order (1)–(4) and the approaches of constructing its approximate solutions. For this goals, we applied the method of introduction additional functional parameters proposed in [10–32] for the solve of nonlocal boundary value problems for systems of hyperbolic equations with mixed derivatives. Considered problem is provided to nonlocal problem with integral conditions for system of hyperbolic equations of second order, including additional functions and integral relations. The algorithm of finding the approximate solution of the investigated problem is proposed and its convergence proved. Sufficient conditions of the existence unique classical solution to problem (1)–(4) are obtained in the terms of unique solvability to nonlocal problem with integral conditions for system of hyperbolic equations second order.

3. Scheme of the method and reduction to equivalent problem.

We introduce a new unknown function \( v(t, x) = \frac{\partial u(t, x)}{\partial x} \) and re-write nonlocal problem with integral conditions (1)–(4) in the following form

\[
\frac{\partial^3 v}{\partial t \partial x} = A(t, x) \frac{\partial v}{\partial x} + B(t, x) \frac{\partial v}{\partial t} + C(t, x)v + D(t, x) \frac{\partial u}{\partial t} + E(t, x) u + f(t, x), \quad (t, x) \in \Omega, \tag{5}
\]

\[
\int_0^T L(t, x) v(t, x) dt = \phi(x), \quad x \in [0, \omega], \tag{6}
\]
\[ \int_0^T P(t, \xi) v(t, \xi) d\xi = \psi_1(t), \; t \in [0, T], \]  
(7)

\[ u(t, x) = \psi_0(t) + \int_0^x v(t, \xi) d\xi, \quad \frac{\partial u(t, x)}{\partial t} = \psi_0(t) + \int_0^x \frac{\partial v(t, \xi)}{\partial t} d\xi, \; (t, x) \in \Omega. \]  
(8)

Here the condition (3) is taken account in (8).

A pair functions \( (v(t, x), u(t, x)) \), where the function \( v(t, x) \in C(\Omega, R^n) \) has partial derivatives

\[ \frac{\partial v(t, x)}{\partial x} \in C(\Omega, R^n), \quad \frac{\partial v(t, x)}{\partial t} \in C(\Omega, R^n), \quad \frac{\partial^2 v(t, x)}{\partial t \partial x} \in C(\Omega, R^n), \]  
the function \( u(t, x) \in C(\Omega, R^n) \) has partial derivatives

\[ \frac{\partial u(t, x)}{\partial x} \in C(\Omega, R^n), \quad \frac{\partial u(t, x)}{\partial t} \in C(\Omega, R^n), \quad \frac{\partial^2 u(t, x)}{\partial t \partial x} \in C(\Omega, R^n), \]

\[ \frac{\partial^2 u(t, x)}{\partial t \partial x^2} \in C(\Omega, R^n), \]  
is called a solution to problem (5)–(8) if it satisfies of the system of hyperbolic equations (5) for all \( (t, x) \in \Omega \), the boundary conditions (6), (7), and the integral relation (8).

At fixed \( u(t, x) \) the problem (5)–(7) is the nonlocal problem with integral conditions for the system of hyperbolic equations with respect to \( v(t, x) \) on \( \Omega \). The integral relations (8) allow us to determine the unknown functions \( u(t, x) \) and its partial derivative \( \frac{\partial u(t, x)}{\partial t} \) for all \( (t, x) \in \Omega \).

4. Algorithm for finding of the approximate solution to problem (5)–(8).

The unknown function \( v(t, x) \) will be determined from nonlocal problem with integral conditions for system of hyperbolic equations (5)–(7). The unknown function \( u(t, x) \) and its partial derivative \( \frac{\partial u(t, x)}{\partial t} \) will be found from integral relations (8). If we known the function \( u(t, x) \) and its partial derivative \( \frac{\partial u(t, x)}{\partial t} \), then from nonlocal problem with integral conditions (5)–(7) we find the function \( v(t, x) \).

Conversely, if we known the function \( v(t, x) \) and its partial derivative \( \frac{\partial v(t, x)}{\partial t} \), then from integral relations (8) we find the function \( u(t, x) \) and its partial derivative \( \frac{\partial u(t, x)}{\partial t} \). Since the functions \( u(t, x) \) and \( v(t, x) \) are unknowns together for finding of the solution to problem (5)–(8) we use an iterative method.

The solution to problem (5)–(8) is the pair functions \( (v^*, u^* (t, x)) \) we defined as a limit of sequence of pairs \( (v^{(k)}(t, x), u^{(k)}(t, x)) \), \( k = 0, 1, 2, \ldots \), according to the following algorithm:

**Step 0.** 1) Suppose in the right-hand part of system (5) \( u(t, x) = \psi_0(t), \quad \frac{\partial u(t, x)}{\partial t} = \psi_0(t) \), from nonlocal problem with integral conditions (5)–(7) we find the initial approximation \( v^{(0)}(t, x) \) and its partial derivatives for all \( (t, x) \in \Omega \);

2) From integral relations (8) under \( v(t, x) = v^{(0)}(t, x) \), \( \frac{\partial v(t, x)}{\partial t} = \frac{\partial v^{(0)}(t, x)}{\partial t} \), we find the functions \( u^{(0)}(t, x) \) and \( \frac{\partial u^{(0)}(t, x)}{\partial t} \) for all \( (t, x) \in \Omega \).
Step 1. 1) Suppose in the right-hand part of system (5) \( u(t,x) = u^{(0)}(t,x), \frac{\partial u(t,x)}{\partial t} = \frac{\partial u^{(0)}(t,x)}{\partial t}, \) from nonlocal problem with integral conditions (5)--(7) we find the first approximation \( v^{(1)}(t,x) \) and its partial derivatives for all \((t,x) \in \Omega\).

2) From integral relations (8) under \( v(t,x) = v^{(1)}(t,x), \) \( \frac{\partial v(t,x)}{\partial t} = \frac{\partial v^{(1)}(t,x)}{\partial t}, \) we find the functions \( u^{(1)}(t,x) \) and \( \frac{\partial u^{(1)}(t,x)}{\partial t} \) for all \((t,x) \in \Omega\).

And so on.

Step \( k \). 1) Suppose in the right-hand part of system (5) \( u(t,x) = u^{(k-1)}(t,x), \) \( \frac{\partial u(t,x)}{\partial t} = \frac{\partial u^{(k-1)}(t,x)}{\partial t}, \) from nonlocal problem with integral conditions (5)--(7) we find the \( k \)-th approximation \( v^{(k)}(t,x) \) and its partial derivatives for all \((t,x) \in \Omega: \)

\[
\frac{\partial^2 v^{(k)}}{\partial t \partial x} = A(t,x) \frac{\partial v^{(k)}}{\partial x} + B(t,x) \frac{\partial v^{(k)}}{\partial t} + C(t,x)v^{(k)} + D(t,x) \frac{\partial u^{(k-1)}}{\partial t} + E(t,x)u^{(k-1)} + f(t,x), (t,x) \in \Omega, \quad (9)
\]

\[
\int_0^T L(\tau,x)v^{(k)}(\tau,x)d\tau = \phi(x), \quad x \in [0,\omega], \quad (10)
\]

\[
\int_0^\omega P(t,\xi)v^{(k)}(t,\xi)d\xi = \psi_1(t), \quad t \in [0,T]. \quad (11)
\]

2) From integral relations (8) under \( v(t,x) = v^{(k)}(t,x), \) \( \frac{\partial v(t,x)}{\partial t} = \frac{\partial v^{(k)}(t,x)}{\partial t}, \) we find the function \( u^{(k)}(t,x) \) and \( \frac{\partial u^{(k)}(t,x)}{\partial t} \) for all \((t,x) \in \Omega: \)

\[
u^{(k)}(t,x) = \psi_0(t) + \int_0^x v^{(k)}(t,\xi)d\xi, \quad \frac{\partial u^{(k)}(t,x)}{\partial t} = \psi_0(t) + \int_0^x \frac{\partial v^{(k)}(t,\xi)}{\partial t}d\xi, \quad (t,x) \in \Omega. \quad (12)
\]

\( k = 1,2,3,... \)

5. The main result.

Consider nonlocal problem with integral conditions (5)-(7) at fixed \( u(t,x). \) Then nonlocal problem with integral conditions for system of hyperbolic equations can be have the following form

\[
\frac{\partial^2 v}{\partial t \partial x} = A(t,x) \frac{\partial v}{\partial x} + B(t,x) \frac{\partial v}{\partial t} + C(t,x)v + F(t,x), (t,x) \in \Omega, \quad (13)
\]

\[
\int_0^T L(\tau,x)v(\tau,x)d\tau = \phi(x), \quad x \in [0,\omega], \quad (14)
\]

\[
\int_0^\omega P(t,\xi)v(t,\xi)d\xi = \psi_1(t), \quad t \in [0,T]. \quad (15)
\]

where \( n \)-vector function \( F(t,x) \) is continuous on \( \Omega \).

The following theorem gives conditions of feasibility and convergence of the constructed algorithm and the conditions of the existence unique solution to problem (5)--(8).

**Theorem 1.** Suppose that

i) the \( n \times n \)-matrices \( A(t,x), B(t,x), C(t,x), D(t,x), E(t,x), \) and \( n \)-vector function \( f(t,x) \)
are continuous on $\Omega$;

ii) the $n \times n$-matrix $L(t,x)$ is continuous and continuously differentiable by $x$ on $\Omega$; and the $n$-vector-function $\varphi(x)$ continuously differentiable on $[0, \omega]$;

iii) the $n \times n$-matrix $P(t,x)$ is continuous and continuously differentiable by $t$ on $\Omega$; the $n$-vector-functions $\psi_0(t)$ and $\psi_1(t)$ are continuously differentiable on $[0, T]$;

iv) Nonlocal problem with integral conditions for system of hyperbolic equations (13)–(15) has a unique classical solution.

Then equivalent nonlocal problem for system of hyperbolic equations with integral conditions and parameters (5)–(8) has a unique solution.

**Proof.** Let the conditions i) - iv) of Theorem 1 are fulfilled. By the algorithm, proposing above, on $0^\text{th}$ step we have

$$
\frac{\partial^2 v^{(0)}}{\partial t \partial x} = A(t,x) \frac{\partial v^{(0)}}{\partial x} + B(t,x) \frac{\partial v^{(0)}}{\partial t} + C(t,x) v^{(0)} + D(t,x) \bar{y}_0(t) + E(t,x) \bar{y}_0(t) + f(t,x), \quad (t,x) \in \Omega, \quad (16)
$$

$$
\int_0^T L(t,x) v^{(0)}(t,x) dt = \varphi(x), \quad x \in [0, \omega], \quad (17)
$$

$$
\int_0^T P(t,x) v^{(0)}(t,x) d\xi = \psi_1(t), \quad t \in [0, T]. \quad (18)
$$

Since condition iv) is valid, problem (16)–(18) has a unique classical solution $v^{(0)}(t,x)$ and the following inequality holds

$$
\max_{(t,x) \in \Omega} \left\{ \max_{\frac{\partial v^{(0)}}{\partial x}, \frac{\partial^2 v^{(0)}}{\partial t \partial x}} \max_{(t,x) \in \Omega} \left\{ \frac{\partial v^{(0)}}{\partial x}, \frac{\partial^2 v^{(0)}}{\partial t \partial x} \right\} \leq K(x) \max_{(t,x) \in \Omega} \left\{ \max_{f(t,x)} \max_{\psi_0(t)} \max_{\varphi(x)} \max_{\psi_1(t)} \max_{F(t,x)} \right\},
$$

where the function $K(x)$ is continuous on $[0, \omega]$, positive and independent on functions $f(t,x)$, $\psi_1(t)$, $\varphi(x)$, $F^{(0)}(t,x) = D(t,x) \bar{y}_0(t) + E(t,x) \bar{y}_0(t)$.

Initial approximations $u^{(0)}(t,x)$ and $\frac{\partial u^{(0)}}{\partial t}(t,x)$ are determined by the following form

$$
u^{(0)}(t,x) = \psi_0(t) + \int_0^x v^{(0)}(t,\xi) d\xi, \quad \frac{\partial u^{(0)}}{\partial t}(t,x) = \bar{y}_0(t) + \int_0^x \frac{\partial v^{(0)}}{\partial t}(t,\xi) d\xi, \quad (t,x) \in \Omega. \quad (19)
$$

Then functions $u^{(0)}(t,x)$ and $\frac{\partial u^{(0)}}{\partial t}(t,x)$ satisfy the estimate

$$
\max_{t \in [0,T]} \left\{ \max_{x \in [0,T]} \left\{ \frac{\partial u^{(0)}}{\partial x}, \frac{\partial^2 u^{(0)}}{\partial t \partial x} \right\} \right\} \leq \max_{t \in [0,T]} \max_{x \in [0,T]} \left\{ \frac{\partial \psi_0(t)}{\partial t}, \frac{\partial \bar{y}_0(t)}{\partial t} \right\} +
$$

$$
+ \int_0^T \max_{t \in [0,T]} \left\{ \max_{x \in [0,T]} \left\{ \frac{\partial v^{(0)}}{\partial x}, \frac{\partial^2 v^{(0)}}{\partial t \partial x} \right\} \right\} d\xi.
$$

For $k$ th approximations determined by relations (9)–(12), we have

$$
\max_{(t,x) \in \Omega} \left\{ \max_{(t,x) \in \Omega} \left\{ \frac{\partial v^{(0)}}{\partial x}, \frac{\partial^2 v^{(0)}}{\partial t \partial x} \right\} \right\} \leq \max_{(t,x) \in \Omega} \left\{ \max_{(t,x) \in \Omega} \left\{ \frac{\partial v^{(0)}}{\partial x}, \frac{\partial^2 v^{(0)}}{\partial t \partial x} \right\} \right\} \leq
$$

37.
\[
\begin{align*}
&\leq K(x) \max_{(x,y) \in \Omega} \left\{ \max_{t \in [0,T]} \|f(t,x)\|, \max_{t \in [0,T]} \|v_x(t)\|, \max_{(x,y) \in \Omega} \|\phi(x)\|, \max_{(x,y) \in \Omega} \|\psi^{(k-1)}(t,x)\| \right\}, \\
&\max_{t \in [0,T]} \left\{ \max_{x \in [0,a]} \|u^{(k)}(t,x)\|, \max_{x \in [0,a]} \left\| \frac{\partial u^{(k)}(t,x)}{\partial t} \right\| \right\} \leq \max_{t \in [0,T]} \left\{ \max_{x \in [0,a]} \|v_0(t)\|, \max_{x \in [0,a]} \|\psi_0(t)\| \right\} + \\
&\quad + \int_0^x \max_{x \in [0,a]} \left\{ \max_{x \in [0,a]} \|v^{(k)}(t,\xi)\|, \max_{x \in [0,a]} \left\| \frac{\partial v^{(k)}(t,\xi)}{\partial t} \right\| \right\} d\xi, \\
\end{align*}
\]
where \( F^{(k-1)}(t,x) = D(t,x) \frac{\partial u^{(k-1)}(t,x)}{\partial t} + E(t,x)u^{(k-1)}(t,x) \).

Let \( \Delta v^{(k)}(t,x) = v^{(k)}(t,x) - v^{(k)}(t,x) \), \( \Delta_x v^{(k)}(t,x) = \frac{\partial v^{(k)}(t,x)}{\partial x} - \frac{\partial v^{(k)}(t,x)}{\partial x} \),
\[\Delta_t v^{(k)}(t,x) = \frac{\partial v^{(k)}(t,x)}{\partial t} - \frac{\partial v^{(k)}(t,x)}{\partial t}, \Delta u^{(k)}(t,x) = u^{(k-1)}(t,x) - u^{(k)}(t,x), \Delta_x u^{(k)}(t,x) = \frac{\partial u^{(k)}(t,x)}{\partial x} - \frac{\partial u^{(k)}(t,x)}{\partial x}, \Delta_t u^{(k)}(t,x) = \frac{\partial u^{(k)}(t,x)}{\partial t} - \frac{\partial u^{(k)}(t,x)}{\partial t}, \]
\[k = 1, 2, 3, \ldots. \]

Then, for differences \( \Delta v^{(k)}(t,x) \), \( \Delta_x v^{(k)}(t,x) \), \( \Delta_t v^{(k)}(t,x) \), \( \Delta u^{(k)}(t,x) \), \( \Delta_x u^{(k)}(t,x) \), \( \Delta_t u^{(k)}(t,x) \) are valid the following inequalities
\[
\begin{align*}
&\max_{(x,y) \in \Omega} \left\{ \max_{(x,y) \in \Omega} \|\Delta v^{(k)}(t,x)\|, \max_{(x,y) \in \Omega} \|\Delta_x v^{(k)}(t,x)\|, \max_{(x,y) \in \Omega} \|\Delta_t v^{(k)}(t,x)\| \right\} \leq \\
&\leq K(x) \max_{(x,y) \in \Omega} \left\{ \max_{(x,y) \in \Omega} \|\Delta u^{(k-1)}(t,x)\|, \max_{(x,y) \in \Omega} \|\Delta_x u^{(k)}(t,x)\|, \max_{(x,y) \in \Omega} \|\Delta_t u^{(k)}(t,x)\| \right\}, \\
&\max_{t \in [0,T]} \left\{ \max_{x \in [0,a]} \|\Delta u^{(k)}(t,x)\|, \max_{x \in [0,a]} \|\Delta_x u^{(k)}(t,x)\| \right\} \leq \int_0^x \max_{t \in [0,T]} \left\{ \max_{x \in [0,a]} \|\Delta v^{(k)}(t,\xi)\|, \max_{x \in [0,a]} \|\Delta_x v^{(k)}(t,\xi)\| \right\} d\xi. \\
\end{align*}
\]

From estimates (20)—(23) is follows
\[
\max_{t \in [0,T]} \left\{ \max_{x \in [0,a]} \|\Delta u^{(k)}(t,x)\|, \max_{x \in [0,a]} \|\Delta_x u^{(k)}(t,x)\| \right\} \leq \frac{\hat{K}(x)}{k!} \cdot \max_{(x,y) \in \Omega} \|\Delta v^{(k)}(t,x)\|, \max_{(x,y) \in \Omega} \|\Delta_x v^{(k)}(t,x)\|, \\
where \( \hat{K} = \max_{x \in [0,a]} K(x) \).
\]

Hence, we obtain the uniform convergence of the sequences \( \left\{ u^{(k)}(t,x) \right\} \) and \( \left\{ \frac{\partial u^{(k)}(t,x)}{\partial t} \right\} \) to functions \( u^*(t,x) \) and \( \frac{\partial u^*(t,x)}{\partial t} \) on \( \Omega \), respectively, as \( k \to \infty \).

Then the sequences \( \left\{ v^{(k)}(t,x) \right\}, \left\{ \frac{\partial v^{(k)}(t,x)}{\partial x} \right\}, \) and \( \left\{ \frac{\partial v^{(k)}(t,x)}{\partial t} \right\} \) also will be convergent to functions \( v^*(t,x), \frac{\partial v^*(t,x)}{\partial x}, \) and \( \frac{\partial v^*(t,x)}{\partial t} \) on \( \Omega \), respectively, as \( k \to \infty \).

The uniqueness of solution to problem (5)—(8) is proved by contradiction.

Theorem 1 is proved.

From equivalence of problem (1)—(4) and (5)—(8) follows the following assertion.

**Theorem 2.** Suppose that the conditions i) - iv) of Theorem 1 are fulfilled.

Then nonlocal problem with integral conditions for third order system of partial differential
equations (1)–(4) has a unique classical solution.

Conditions of Theorem 1 are sufficient of the well-posedness to nonlocal problem with integral solutions for third order system of partial differential equations (1)–(4).

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КОРРЕКТНАЯ РАЗРЕШИМОСТЬ НЕЛОКАЛЬНОЙ ЗАДАЧИ
С ИНТЕГРАЛЬНЫМИ ДЛЯ СИСТЕМЫ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ
В ЧАСТНЫХ ПРОИЗВОДНЫХ ТРЕТЬЕГО ПОРЯДКА

Аннотация. Рассматривается нелокальная задача с интегральными условиями для системы дифференциальных уравнений в частных производных третьего порядка. Исследуются вопросы существования и единственности классического решения нелокальной задачи для системы дифференциальных уравнений в частных производных третьего порядка и предлагается методы построения их приближенных решений. Установлены условия однозначной разрешимости нелокальной задачи для системы дифференциальных уравнений в частных производных третьего порядка. Путем введения новой неизвестной функции исследуемая задача сводена к эквивалентной задаче, состоящей из нелокальной задачи для системы гиперболических уравнений второго порядка с интегральными условиями и функциональными параметрами и интегрального соотношения. Предложены алгоритмы нахождения приближенного решения исследуемой задачи и доказана их сходимость. Получены достаточные условия существования единственного решения эквивалентной задачи с параметрами. Корректная разрешимость нелокальной задачи с интегральными условиями для системы дифференциальных уравнений в частных производных третьего порядка получены в терминах корректной разрешимости нелокальной задачи с интегральными условиями для системы гиперболических уравнений второго порядка.

Ключевые слова: дифференциальное уравнение в частных производных третьего порядка, нелокальная задача, интегральное условие, система гиперболических уравнений второго порядка, разрешимость,
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УШІНШІ РЕТТЕ ДЕРБЕС ТУЫНДЫЛЫ ДИФФЕРЕНЦИАЛДЫҚ ТЕНДЕУЛЕР ЖУЙЕСІ УШІН ИНТЕГРАЛДЫҚ ШАРТТАРЫ БАР БЕЙЛОҚАЛ ЕСЕПТІҢ КОРРЕКТІЛІ ШЕШІЛІМДІЛІГІ

Аннотация. Ушінші ретти дербес туындылы дифференциалдық тендеулер жүйесі ушін интегралдық шарттары бар бейлокал есептін корректілі шешілімділігін қалыптастырады. Ушінші ретте сұрып қалғанда, бейлокал есептін қалыптастырудың жағдайларын талқылаймыз. Екінші ретте бейлокал есептін қалыптастырудың жағдайларын талқылаймыз.

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