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D.S.Dzhumabaev[1,2], E.A. Bakirova[1,2], Zh.M. Kadirbayeva[1,2]

[1] Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan;
[2]Institute of Information and Computational Technologies, Almaty, Kazakhstan
E-mail: bakirova1974@mail.ru, apelman86pm@mail.ru

AN ALGORITHM FOR SOLVING A CONTROL PROBLEM FOR A DIFFERENTIAL EQUATION WITH A PARAMETER

Abstract. On a finite interval, a control problem for a linear ordinary differential equations with a parameter is considered. By partitioning the interval and introducing additional parameters, considered problem is reduced to the equivalent multipoint boundary value problem with parameters. To find the parameters introduced, the continuity conditions of the solution at the interior points of partition and boundary condition are used. For the fixed values of the parameters, the Cauchy problems for ordinary differential equations are solved. By substituting the Cauchy problem's solutions into the boundary condition and the continuity conditions of the solution, a system of linear algebraic equations with respect to parameters is constructed. The solvability of this system ensures the existence of a solution to the original control problem. The system of linear algebraic equations is composed by the solutions of the matrix and vector Cauchy problems for ordinary differential equations on the subintervals. A numerical method for solving the origin control problem is offered based on the Runge-Kutta method of the 4-th order for solving the Cauchy problem for ordinary differential equations.

Key words: boundary value problem with parameter, differential equation, solvability, algorithm.

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In the present paper we consider the control problem for the linear ordinary differential equations with parameter

$$\frac{dx}{dt} = A(t)x + B(t)\mu + f(t), \quad x, \mu \in R^n, \quad t \in (0,T), \tag{1}$$

$$x(0) = x^0, \quad x^0 \in R^n, \tag{2}$$

$$x(T) = x^1, \quad x^1 \in R^n, \tag{3}$$

where the $(n \times n)$-matrices $A(t)$, $B(t)$ and n-vector-function $f(t)$ are continuous on $[0,T]$.

Let $C([0,T],R^n)$ denote the space of continuous functions $x:[0,T] \to R^n$ with the norm $\|x\|_1 = \max_{t\in[0,T]} \|x(t)\|$.

Solution to problem (1), (2) is a pair $(\mu^*, x^*(t))$, where the function $x^*(t) \in C([0,T],R^n)$ is continuously differentiable on $(0,T)$ and satisfies Eq. (1) with $\mu = \mu^*$ and additional conditions (2), (3).

Qualitative properties of problem (1), (2) and methods for solving boundary value problems with parameters studied by many authors (see [1-16, 19,20] and references cited therein).

In the present paper problem (1), (2) is solved by parametrization's method [17, 18].

Given the points: $t_0 = 0 < t_1 < \ldots < t_{N-1} < t_N = T$, and let Δ_N be the partition of interval $[0,T)$ into N subintervals: $[0,T) = \bigcup_{r=1}^{N} [t_{r-1}, t_r)$.

By $C([0,T],\Delta_N,R^{nN})$ we denote the space of function systems $x[t] = (x_1(t), x_2(t), \ldots, x_N(t))$, where $x_r : [t_{r-1}, t_r) \to R^n$ are continuous and have finite left-hand limits $\lim_{t \to t_r - 0} x_r(t)$ for all $r = \overline{1,N}$, with the norm $\|x\|_2 = \max_{r=\overline{1,N}} \sup_{t \in [t_{r-1}, t_r)} \|x_r(t)\|$.

Denote by $x_r(t)$ the restriction of function $x(t)$ to the r-th interval $[t_{r-1}, t_r)$ and reduce problem (1)-(3) to the equivalent multipoint boundary-value problem

$$\frac{dx_r}{dt} = A(t)x_r + B(t)\mu + f(t), \qquad t \in [t_{r-1}, t_r),\ r = \overline{1,N}, \tag{4}$$

$$x_1(0) = x^0, \tag{5}$$

$$\lim_{t \to T-0} x_N(t) = x^1, \tag{6}$$

$$\lim_{t \to t_s - 0} x_s(t) = x_{s+1}(t_s), \quad s = \overline{1, N-1}, \tag{7}$$

where (7) are the continuity conditions of the solution at the interior points of the partition.

A pair $(\mu^*, x^*[t])$ with $\mu^* \in R^n$ and $x^*[t] = (x_1^*(t), x_2^*(t), \ldots, x_N^*(t)) \in C([0,T],\Delta_N,R^{nN})$ is called a solution to problem (4)-(7), if it satisfies the system of differential equations (4) and conditions (5)-(7).

Introducing the additional parameters $\lambda_1 = \mu$, $\lambda_p = x_p(t_{p-1})$, $p = \overline{2,N}$, and performing the substitutions $u_1(t) = x_1(t) - x^0$, $u_p(t) = x_p(t) - \lambda_p$, $t \in [t_{p-1}, t_p)$, $p = \overline{2,N}$, we obtain the boundary value problem with parameters

$$\frac{du_1}{dt} = A(t)(u_1 + x^0) + B(t)\lambda_1 + f(t), \qquad t \in [t_0, t_1), \tag{8}$$

$$u_1(t_0) = 0, \tag{9}$$

$$\frac{du_p}{dt} = A(t)(u_p + \lambda_p) + B(t)\lambda_1 + f(t), \quad t \in [t_{p-1}, t_p), \quad p = \overline{2,N}, \tag{10}$$

$$u_p(t_{p-1}) = 0,\ p = \overline{2,N}, \tag{11}$$

$$\lambda_N + \lim_{t \to T-0} u_N(t) = x^1, \tag{12}$$

$$x^0 + \lim_{t \to t_1 - 0} u_1(t) = \lambda_2, \tag{13}$$

$$\lambda_s + \lim_{t \to t_s - 0} u_s(t) = \lambda_{s+1}, \quad s = \overline{2, N-1}. \tag{14}$$

Solution to problem (8)-(14) is a pair $(\lambda, u[t])$ with $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_N) \in R^{nN}$, $u[t] = (u_1(t), u_2(t), \ldots, u_N(t)) \in C([0,T],\Delta_N,R^{nN})$. If $(\lambda, u[t])$ is a solution to problem (8)-(14), then the pair $(\mu, x(t))$ with parameter μ and the function $x(t)$, defined by the equalities: $\mu = \lambda_1$, $x(t) = x^0 + u_1(t)$, $t \in [t_0, t_1)$, $x(t) = \lambda_p + u_p(t)$, $t \in [t_{p-1}, t_p)$, $p = \overline{2,N}$, $x(T) = \lambda_N + \lim_{t \to T-0} u_N(t)$, is a solution to problem (1)- (3). Conversely, if $(\tilde{\mu}, \tilde{x}(t))$ is a solution to

problem (1)-(3), then the pair $(\widetilde{\lambda}, \widetilde{u}[t])$, with $\widetilde{\lambda} = (\widetilde{\mu}, \widetilde{x}(t_1), \ldots, \widetilde{x}(t_{N-1}))$, $\widetilde{u}[t] = (\widetilde{x}(t) - \widetilde{x}^0, \widetilde{x}(t) - \widetilde{x}(t_1), \ldots, \widetilde{x}(t) - \widetilde{x}(t_{N-1}))$, is a solution to problem (8)-(14).

Let $\Phi(t)$ be a fundamental matrix to the ordinary differential equation

$$\frac{dx}{dt} = A(t)x, \quad t \in [0,T].$$

Then a unique solution of the Cauchy problem for the system of ordinary differential equations (8)-(11) at the fixed values $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_N)$ has the following form

$$u_1(t) = \Phi(t)\int_{t_0}^{t}\Phi^{-1}(\tau)B(\tau)d\tau \cdot \lambda_1 + \Phi(t)\int_{t_0}^{t}\Phi^{-1}(\tau)\left[A(\tau)x^0 + f(\tau)\right]d\tau, \ t \in [t_0, t_1), \quad (15)$$

$$u_p(t) = \Phi(t)\int_{t_{p-1}}^{t}\Phi^{-1}(\tau)A(\tau)d\tau\lambda_p + \Phi(t)\int_{t_{p-1}}^{t}\Phi^{-1}(\tau)B(\tau)d\tau \cdot \lambda_1 +$$

$$+ \Phi(t)\int_{t_{p-1}}^{t}\Phi^{-1}(\tau)f(\tau)d\tau, \quad t \in [t_{p-1}, t_p), \quad p = \overline{2,N}. \quad (16)$$

Substituting (15) and (16) into (12)-(14) yields the system of algebraic equations for finding the unknown parameters $\lambda_1, \lambda_2, \ldots, \lambda_N$:

$$\lambda_N + \Phi(T)\int_{t_{N-1}}^{T}\Phi^{-1}(\tau)A(\tau)d\tau\lambda_N + \Phi(T)\int_{t_{N-1}}^{T}\Phi^{-1}(\tau)B(\tau)d\tau \cdot \lambda_1 + \Phi(T)\int_{t_{N-1}}^{T}\Phi^{-1}(\tau)f(\tau)d\tau = x^1, \quad (17)$$

$$x^0 + \Phi(t_1)\int_{t_0}^{t_1}\Phi^{-1}(\tau)A(\tau)x^0 d\tau + \Phi(t_1)\int_{t_0}^{t_1}\Phi^{-1}(\tau)B(\tau)\lambda_1 d\tau + \Phi(t_1)\int_{t_0}^{t_1}\Phi^{-1}(\tau)f(\tau)d\tau = \lambda_2, \quad (18)$$

$$\lambda_s + \Phi(t_s)\int_{t_{s-1}}^{t_s}\Phi^{-1}(\tau)\{A(\tau)\lambda_s + B(\tau)\lambda_1\}d\tau + \Phi(t_s)\int_{t_{s-1}}^{t_s}\Phi^{-1}(\tau)f(\tau)d\tau = \lambda_{s+1}, \ s = \overline{2, N-1}. \quad (19)$$

Let $Q_*(\Delta_N)$ denote the matrix corresponding to the left-hand side of system (17)- (19). Then the system can be written as

$$Q_*(\Delta_N)\lambda = -F_*(\Delta_N), \ \lambda \in R^{nN}, \quad (20)$$

where

$$F_*(\Delta_N) = \Bigg(-x^1 + \Phi(T)\int_{t_{N-1}}^{T}\Phi^{-1}(\tau)f(\tau)d\tau, x^0 + \Phi(t_1)\int_{t_0}^{t_1}X^{-1}(\tau)A(\tau)x^0 d\tau + \Phi(t_1)\int_{t_0}^{t_1}X^{-1}(\tau)f(\tau)d\tau,$$

$$\Phi(t_2)\int_{t_1}^{t_2}X^{-1}(\tau)f(\tau)d\tau, \ldots, \Phi(t_{N-1})\int_{t_{N-2}}^{t_{N-1}}\Phi^{-1}(\tau)f(\tau)d\tau\Bigg).$$

Lemma 1. The following assertions hold:

(a) The vector $\lambda^* = (\lambda_1^*, \lambda_2^*, \ldots, \lambda_N^*) \in R^{nN}$, consisting of $\lambda_1^* = \mu^*$ and the values of the solution $x^*(t)$ to problem (1)-(3) at the partition points $\lambda_p^* = x^*(t_{p-1})$, $r = \overline{2,N}$, satisfies system (20).

(b) The pair $(\widetilde{\mu}, \widetilde{x}(t))$ defined by the equalities $\widetilde{\mu} = \widetilde{\lambda}_1$, $\widetilde{x}(t) = \widetilde{u}_1(t) + x^0$, $t \in [t_0, t_1)$, $\widetilde{x}(t) = \widetilde{u}_p(t) + \widetilde{\lambda}_p$, $t \in [t_{p-1}, t_p)$, $p = \overline{2,N}$, $\widetilde{x}(T) = \widetilde{\lambda}_N + \lim\limits_{t \to T-0}\widetilde{u}_N(t)$ is a solution to problem (1)-(3), where $\widetilde{\lambda} = (\widetilde{\lambda}_1, \widetilde{\lambda}_2, \ldots, \widetilde{\lambda}_N) \in R^{nN}$ is a solution to system (20) and the system of functions $\widetilde{u}[t] = (\widetilde{u}_1(t), \widetilde{u}_2(t), \ldots, \widetilde{u}_N(t)) \in C([0,T], \Delta_N, R^{nN})$ is solution to the Cauchy problem (8)- (11) for $\widetilde{\lambda}_1 = \mu$, $\lambda_p = \widetilde{\lambda}_p$, $p = \overline{2,N}$.

Definition. Problem (1)-(3) is called uniquely solvable if it has a unique solution for any $x^0, x^1 \in R^n$ and $f(t) \in C([0,T], R^n)$.

Theorem 1. Problem (1)-(3) is uniquely solvable if and only if the matrix $Q_*(\Delta_N): R^{nN} \to R^{nN}$ has an inverse one.

Proof. Necessity. Assume the opposite, i.e., that $Q_*(\Delta_N)$ is not invertible. Then the homogeneous system of equations

$$Q_*(\Delta_N)\lambda = 0 \tag{22}$$

has a nontrivial solution $\tilde{\lambda} = (\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_N) \in R^{nN}$. In the case of a homogeneous boundary value problem for the ordinary differential equation, i.e., for problem (1)-(3) with $x^0 = 0,\ x^1 = 0,\ f(t) = 0$, system (20) becomes (22). Therefore, by Lemma 1, the pair $(\tilde{\mu}, \tilde{x}(t))$ defined by the equalities $\tilde{\mu} = \tilde{\lambda}_1, \quad \tilde{x}(t) = \tilde{u}_1(t) + x^0, \quad t \in [t_0, t_1), \quad \tilde{x}(t) = \tilde{u}_p(t) + \tilde{\lambda}_p,\ t \in [t_{p-1}, t_p), \quad p = \overline{2,N},$ $\tilde{x}(T) = \tilde{\lambda}_N + \lim\limits_{t \to T-0} \tilde{u}_N(t),$ where the system of functions $\tilde{u}[t] = (\tilde{u}_1(t), \tilde{u}_2(t), \ldots, \tilde{u}_N(t))$ is solution to following problem

$$\frac{du_1}{dt} = A(t)u_1 + B(t)\lambda_1, \qquad t \in [t_0, t_1),\ u_1(t_0) = 0,$$

$$\frac{du_p}{dt} = A(t)(u_p + \lambda_p) + B(t)\lambda_1, \quad t \in [t_{p-1}, t_p), \quad u_p(t_{p-1}) = 0,\ p = \overline{2,N},$$

is a nontrivial solution of the homogeneous boundary value problem (1)-(3) ($x^0 = 0,\ x^1 = 0,\ f(t) = 0$). Since, problem (1)-(3) has the trivial solution $\tilde{x}(t) = 0,\ \tilde{\mu} = 0$. This contradicts to a unique solvability of problem (1)-(3).

Sufficiency. Due to the invertibility of the matrix $Q_*(\Delta_N)$, we can find unique solution of system equation (20) $\lambda^* = -[Q_*(\Delta_N)]^{-1} F_*(\Delta_N),\ \lambda^* = (\lambda_1^*, \lambda_2^*, \ldots, \lambda_N^*) \in R^{nN}$. Solving Cauchy problem (8)-(11) for $\lambda_r = \lambda_r^*,\ r = \overline{1,N},$ we obtain the system of functions $\tilde{u}[t] = (\tilde{u}_1(t), \tilde{u}_2(t), \ldots, \tilde{u}_N(t))$. According to Lemma 1, the pair $(\mu^*, x^*(t))$ defined by equalities $\mu^* = \lambda_1^*,\ x^*(t) = u_1^*(t) + x^0,$ $t \in [t_0, t_1),\ x^*(t) = \lambda_p^* + u_p^*(t),\ t \in [t_{p-1}, t_p),\ p = \overline{2,N}, \quad x^*(T) = \lambda_N^* + \lim\limits_{t \to T-0} u_N^*(t),$ is a solution to problem (1)-(3).

Establish the uniqueness of the solution. Suppose that the problem (1)-(3) has another solution $(\tilde{\mu}, \tilde{x}(t))$ except $(\mu^*, x^*(t))$. Then the pair $(\tilde{\lambda}, \tilde{u}[t])$ composed by the parameter $\tilde{\mu}$ and $\tilde{x}(t)$ is also a solution of the boundary value problem with a parameter (8)-(14). By Lemma 1, λ^* and $\tilde{\lambda}$ satisfy the system of equations (20):

$$Q_*(\Delta_N)\lambda^* = -F_*(\Delta_N),\ Q_*(\Delta_N)\tilde{\lambda} = -F_*(\Delta_N).$$

Invertibility of the matrix $Q_*(\Delta_N)$, yields: $\lambda^* = \tilde{\lambda}$. Unique solvability of the Cauchy problem (8)-(11) provides fulfillment of relationships $u_r^*(t) = \tilde{u}_r(t), \quad t \in [t_{r-1}, t_r),\ r = \overline{1,N},$ $\lim\limits_{t \to T-0} u_N^*(t) = \lim\limits_{t \to T-0} \tilde{u}_N(t)$. Therefore, $\mu^* = \tilde{\mu}$ and $x^*(t) = \tilde{x}(t)$ for all $t \in [0,T]$. The proof is complete.

The Cauchy problems for ordinary differential equations on the subintervals

$$\frac{dz}{dt} = A(t)z + P(t), \quad z(t_{r-1}) = 0, \quad r = \overline{1,N} \tag{23}$$

are a significant part of the proposed algorithm. Here $P(t)$ is either $(n \times n)$ matrix or n vector, continuous on $[t_{r-1}, t_r)$, $r = \overline{1,N}$. Consequently, solution to problem (23) is a square matrix or a vector of dimension n. Denote by $a_r(P,t)$ the solution to the Cauchy problem (23). Obviously,

$$a_r(P,t) = \Phi(t)\int_{t_{r-1}}^{t} \Phi^{-1}(\tau)P(\tau)d\tau, \quad [t_{r-1}, t_r), \quad r = \overline{1,N}, \tag{24}$$

where $\Phi(t)$ is a fundamental matrix of differential equation (23) on the r-th interval.

We offer the following numerical implementation of algorithm for finding solution of problem (1)-(3) based on solving Cauchy problems by the Runge-Kutta method of 4-th order.

I. Suppose we have a partition $0 = t_0 < t_1 < \ldots < t_{N-1} < t_N = T$. Divide each $r-$th interval $[t_{r-1}, t_r)$, $r = \overline{1,N}$, into N_r parts with the step $h_r = (t_r - t_{r-1})/N_r$. Assume that on each interval $[t_{r-1}, t_r)$, the variable $\hat{t}$ takes its discrete values: $\hat{t} = t_{r-1}$, $\hat{t} = t_{r-1} + h_r$, ..., $\hat{t} = t_{r-1} + (N_r - 1)h_r$, $\hat{t} = t_r$, and denote by $\{t_{r-1}, t_r\}$ the set of such points.

II. Solving the Cauchy problem for ordinary differential equations

$$\frac{dz}{dt} = A(t)z + A(t), \ z(t_{r-1}) = 0, \ r = \overline{1,N},$$

$$\frac{dz}{dt} = A(t)z + B(t), \ z(t_{r-1}) = 0, \ r = \overline{1,N},$$

$$\frac{dz}{dt} = A(t)z + f(t), \ z(t_{r-1}) = 0, \ r = \overline{1,N},$$

by using Runge-Kutta method of the 4-th order, we find the values of the $(n \times n)$-matrix $a_r(A,\hat{t})$, $a_r(B,\hat{t})$, and n-vector $a_r(f,\hat{t})$ on $\{t_{r-1}, t_r\}$, $r = \overline{1,N}$.

III. Construct the system of linear algebraic equations with respect to parameters

$$Q_*^{\tilde{h}}(\Delta_N)\lambda = -F_*^{\tilde{h}}(\Delta_N), \ \lambda \in R^{nN}, \ \tilde{h} = (h_1, h_2, \ldots, h_N). \tag{25}$$

Solving the system of algebraic equations (25) we find $\lambda^{\tilde{h}} \in R^{nN}$.

IV. To define the values of the approximate solution at the remaining points of the set $\{t_{r-1}, t_r\}$, we solve the following Cauchy problems by applying the Runge-Kutta method of the 4-th order

$$\frac{dx}{dt} = A(t)x + B(t)\lambda_1^{\tilde{h}} + f(t), \ t \in [t_0, t_1), \ x(t_0) = x^0, \tag{26}$$

$$\frac{dx}{dt} = A(t)x + B(t)\lambda_1^{\tilde{h}} + f(t), \ t \in [t_{r-1}, t_r), \ x(t_{r-1}) = \lambda_r^{\tilde{h}}, \ r = \overline{2,N}. \tag{27}$$

To illustrate the proposed method for control problem (1)-(3) we consider the following example.

Example. Consider the control problem for differential equations with parameter:

$$\frac{dx}{dt} = A(t)x + B(t)\mu + f(t), \ t \in [0,1], \ x \in R^2, \tag{28}$$

$$x(0) = x^0, \tag{29}$$

$$x(1) = x^1, \tag{30}$$

where $A(t) = \begin{pmatrix} t & t+4 \\ 2t^2 & 7 \end{pmatrix}$, $B(t) = \begin{pmatrix} t+1 & 1 \\ t & 2t^2 \end{pmatrix}$, $f(t) = \begin{pmatrix} -t^4 - 4t^3 + t^2 + 2t - 18 \\ -9t^3 - 9t^2 + 12t - 23 \end{pmatrix}$, $x^0 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $x^1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$.

In this example, the matrix of the differential part is variable and construction of a fundamental

matrix breaks down. We use the numerical implementation of the algorithm proposed. We provide the results of the numerical implementation of the algorithm by partitioning the subintervals [0,0.5], [0.5,1] with step $h_1 = h_2 = 0.05$.

Solving the system of equations (20) we obtain the numerical values of the parameters

$$\lambda_1^{\tilde{h}} = \begin{pmatrix} 1.9997698 \\ 5.000474 \end{pmatrix}, \quad \lambda_2^{\tilde{h}} = \begin{pmatrix} 1.500072 \\ 2.1249704 \end{pmatrix}.$$

We find the numerical solutions at the other points of the subintervals applying the Runge-Kutta method of the 4th order to the following Cauchy problems:

$$\frac{dx_1}{dt} = A(t)x_1 + B(t)\lambda_1^{\tilde{h}} + f(t), \quad t \in \left[0, \frac{1}{2}\right), \quad x_1(0) = x^0,$$

$$\frac{dx_2}{dt} = A(t)x_2 + B(t)\lambda_1^{\tilde{h}} + f(t), \quad t \in \left[\frac{1}{2}, 1\right), \quad x_2\left(\frac{1}{2}\right) = \lambda_2^{\tilde{h}}.$$

The exact solution of the problem (28)-(30) is a pair $(\mu^*, x^*(t))$, where $\mu^* = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$,

$$x^*(t) = \begin{pmatrix} t+1 \\ t^3 - 2t + 3 \end{pmatrix}, \quad t \in [0,1].$$

The results of calculations of numerical and exact solutions at the partition points are presented in the following table:

t	$\tilde{x}_1(t)$ (numerical solution)	$x_1^*(t)$	$\tilde{x}_2(t)$ (numerical solution)	$x_2^*(t)$
0.05	1.050012	1.05	2.900125	2.900125
0.1	1.1000233	1.1	2.8009992	2.801
0.15	1.1500338	1.15	2.7033727	2.703375
0.2	1.2000432	1.2	2.6079955	2.608
0.25	1.2500516	1.25	2.5156176	2.515625
0.3	1.3000587	1.3	2.426989	2.427
0.35	1.3500644	1.35	2.3428599	2.342875
0.4	1.4000686	1.4	2.2639803	2.264
0.45	1.4500712	1.45	2.1911004	2.191125
0.5	1.500072	1.5	2.1249704	2.125
0.55	1.5500709	1.55	2.0663403	2.066375
0.6	1.6000679	1.6	2.0159606	2.016
0.65	1.650063	1.65	1.9745814	1.974625
0.7	1.7000563	1.7	1.9429532	1.943
0.75	1.7500478	1.75	1.9218265	1.921875
0.8	1.800038	1.8	1.9119519	1.912
0.85	1.8500272	1.85	1.9140803	1.914125
0.9	1.9000163	1.9	1.9289631	1.929
0.95	1.9500066	1.95	1.957352	1.957375
1	2	2	2	2

$\tilde{\mu}_1 = \lambda_{11}^{\tilde{h}}$ (numerical solution)	μ_1^*	$\tilde{\mu}_2 = \lambda_{12}^{\tilde{h}}$ (numerical solution)	μ_2^*
1.9997698	2	5. 000474	5

For the difference of the corresponding values of the exact and constructed solutions of the problem the following estimate is true:

$$\max \|\mu^* - \tilde{\mu}\| < 0.0002,$$

$$\max_{j=0,20}\left\|x^*(t_j)-\tilde{x}(t_j)\right\|<\varepsilon,\ \ \varepsilon=0.000072.$$

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Д.С. Джумабаев[1,2], Э.А. Бакирова[1,2], Ж.М. Кадирбаева[1,2]

[1,2]Математика және математикалық модельдеу институты, Алматы, Қазақстан;
[1,2] Ақпараттық және есептеуіш технологиялар институты, Алматы, Қазақстан

ПАРАМЕТРІ БАР ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕР ҮШІН БАСҚАРУ ЕСЕБІН ШЕШУДІҢ БІР АЛГОРИТМІ ТУРАЛЫ

Аннотация. Шектелген аралықта параметрі бар сызықты жәй дифференциалдық теңдеу үшін басқару есебі қарастырылады. Аралықты бөлу мен қосымша параметрлер енгізу арқылы қарастырылып отырған сызықты басқару есебі параметрлері бар эквивалентті көпнүктелі шеттік есепке келтіріледі. Енгізілген параметрлерді анықтау үшін шешімнің ішкі бөліктеу нүктелерінің үзіліссіздік шарттарымен шеттік шарт

пайдаланылады. Параметрлердің бекітілген мәнінде жәй дифференциалдық теңдеулер үшін Коши есептері шешіледі. Коши есептерінің шешімдерін шеттік шартқа және шешімнің үзіліссіздік шарттарына қою арқылы енгізілген параметрлерге қатысты сызықтық алгебралық теңдеулер жүйесі құрылады. Осы жүйенің шешілімділігі бастапқы басқару есебінің шешімінің бар болуын қамтамасыз етеді. Сызықтық алгебралық теңдеулер жүйесінің шешімі жәй дифференциалдық теңдеулер үшін ішкі интервалдағы матрицалық және векторлық Коши есептерінің шешімдері көмегімен жүзеге асырылады. Бастапқы басқару есебінің шешімін табудың жәй дифференциалдық теңдеулер үшін Коши есептерін шешуге арналған төртінші ретті Рунге-Куттаның әдісіне негізделген сандық әдісі ұсынылады.

Түйін сөздер: параметрі бар шеттік есеп, дифференциалдық теңдеу, шешілімділік, алгоритм.

Д.С. Джумабаев[1,2], Э.А. Бакирова[1,2], Ж.М. Кадирбаева[1,2]

[1,2]Институт математики и математического моделирования, Алматы, Казахстан;
[1,2]Институт информационных и вычислительных технологий, Алматы, Казахстан

ОБ ОДНОМ АЛГОРИТМЕ РЕШЕНИЯ ЗАДАЧИ УПРАВЛЕНИЯ ДЛЯ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С ПАРАМЕТРОМ

Аннотация. На ограниченном отрезке рассматривается задача управления для линейного обыкновенного дифференциального уравнения, содержащего параметр. Разбиением интервала и введением дополнительных параметров рассматриваемая линейная задача управления сводится к эквивалентной многоточечной краевой задаче с параметрами. Для определения введенных параметров используются условия непрерывности решения во внутренних точках разбиения и краевое условие. При фиксированных значениях параметров решаются задачи Коши для обыкновенных дифференциальных уравнений. Подставляя решения задач Коши в краевое условие и условия непрерывности решения составляется система линейных алгебраических уравнений относительно введенных параметров. Разрешимость этой системы обеспечивает существование решения исходной задачи управления. Нахождение системы линейных алгебраических уравнений осуществляется с помощью решений матричных и векторных задач Коши для обыкновенных дифференциальных уравнений на подинтервалах. Предлагается численный метод решения исходной задачи управления, основанный на методе Рунге-Кутта четвертого порядка для решения задач Коши обыкновенных дифференциальных уравнений.

Ключевые слова: краевая задача с параметром, дифференциальное уравнение, разрешимость, алгоритм.

Information about the authors:

Dzhumabaev D.S. - Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan; Institute of Information and Computational Technologies, Almaty, Kazakhstan , https: //orcid.org/0000-0002-6143-1563;

Bakirova E.A. - Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan; Institute of Information and Computational Technologies, Almaty, Kazakhstan , https://orcid.org/0000-0002-3820-5373;

Kadirbayeva Zh.M. - Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan; Institute of Information and Computational Technologies, Almaty, Kazakhstan, https://orcid.org/0000-0001-8861-4100