

NEWS

OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN

PHYSICO-MATHEMATICAL SERIES

ISSN 1991-346X

<https://doi.org/10.32014/2018.2518-1726.9>

Volume 5, Number 321 (2018), 68 – 74

S.N. Kharin^{1,2}, S.A. Kassabek^{2,3}, M. Slyamkhan³

¹ Institute of Mathematics of the National Academy of Sciences of Kazakhstan;

² Kazakh-British Technical University, Kazakhstan;

³ Suleyman Demirel University, Kazakhstan,

staskharin@yahoo.com, kassabek@gmail.com

PROBLEM FROM THE THEORY OF BRIDGE EROSION

Abstract. In this paper, we represent the exact solution of a two phase Stefan problem. Radial heat polynomials and integral error function are used for solving bridge problem. The recurrent expressions for the coefficients of these series are presented. The mathematical models describe the dynamics of contact opening and bridging.

Keywords: radial heat polynomials, Stefan problem.

Introduction

In consideration the heat transfer, the shape of the liquid bridge plays an important role. The overwhelming majority of researchers proceed from the fact that the visible part of the bridge has the shape of a cylinder whose axis is directed perpendicular to the plane of the electrodes [1]. In the general case, in the result of the action of surface tension and the pinch effect, the bridge takes the form of a certain surface of revolution about the z-axis. In this problem, we consider a symmetric model of the bridge, where the shape of the bridge is described by a surface $y(z, t) = z^{\nu/2}$. For a liquid bridge in we consider the generalized heat equation for and for solid contact we use the spherical Holm model [2].

Preliminaries

The fundamental solution for the equation

$$\frac{\partial \theta}{\partial t} = a^2 \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\nu}{x} \frac{\partial \theta}{\partial x} \right) \quad (1)$$

can be obtained by the solution of this equation with the initial condition containing delta-function using the Laplace transform in the form [3]

$$G(x, y, t) = \frac{C_\nu}{2t} (xy)^{-\beta} e^{-\frac{x^2+y^2}{4t}} I_\beta \left(\frac{xy}{2t} \right), \quad \beta = \frac{\nu-1}{2}, \quad C_\nu = 2^{-\beta} \Gamma(\beta+1) \quad (2)$$

If we consider the corresponding heat potentials for this solution

$$Q_{n,\nu}(x, t) = 2^{-\beta} \Gamma(\beta+1)^{-1} \int_0^\infty G(x, y, t) y^{2n+\nu} dy \quad (3)$$

and integrating by parts we obtain the explicit formula for the heat polynomials

$$Q_{n,\nu}(x, t) = \sum_{k=0}^n 2^{2k} \frac{n! \Gamma(\beta+1)}{k!(n-k)! \Gamma(\beta+1+n-k)} x^{2n-2k} t^k \quad (4)$$

It is more convenient for applications to multiply both sides of this formula by $\frac{\Gamma(\beta+1+n)}{\Gamma(\beta+1)}$.

$$\text{Then } R_{n,\nu}(r,t) = \frac{\Gamma(\beta+1+n)}{\Gamma(\beta+1)} Q_{n,\nu}(x,t) = \sum_{k=0}^n 2^{2k} \frac{n! \Gamma(\beta+1)}{k!(n-k)! \Gamma(\beta+1+n-k)} x^{2n-2k} t^k \quad (5)$$

Mathematical model

The heat equations for each zone are

$$\frac{\partial \theta_1}{\partial t} = a_1^2 \left(\frac{\partial^2 \theta_1}{\partial x^2} + \frac{\nu}{x} \frac{\partial \theta_1}{\partial x} \right) \quad \alpha(t) < x < 0 \quad (6)$$

$$\frac{\partial \theta_2}{\partial t} = a_1^2 \left(\frac{\partial^2 \theta_2}{\partial r^2} + \frac{2}{r} \frac{\partial \theta_2}{\partial r} \right) \quad r_0 < r < \beta(t) \quad (7)$$

$$\frac{\partial \theta_3}{\partial t} = a_2^2 \left(\frac{\partial^2 \theta_3}{\partial r^2} + \frac{2}{r} \frac{\partial \theta_3}{\partial r} \right) \quad \beta(t) < r < \infty \quad (8)$$

with boundary and initial conditions:

$$\alpha(0) = 0, \beta(0) = r_0, \theta_1(0,0) = \theta_2(r_0,0) = \theta_m, \theta_3(r,0) = f(r), f(r_0) = \theta_m \quad (9)$$

$$-\lambda \pi \alpha^\nu(t) \frac{\partial \theta_1}{\partial x} \Big|_{x=\alpha(t)} = Q(t) \quad (10)$$

$$\theta_1(0,t) = \theta_2(r_0,t) \quad (11)$$

$$\lambda_1 \frac{\partial \theta_1(0,t)}{\partial x} = 2 \lambda_2 \frac{\partial \theta_2(r_0,t)}{\partial r} \quad (12)$$

$$\theta_2(\beta(t),t) = \theta_m \quad (13a)$$

$$\theta_3(\beta(t),t) = \theta_m \quad (13b)$$

The Stefan's condition

$$-\lambda_1 \frac{\partial \theta_2}{\partial r} \Big|_{r=\beta(t)} = -\lambda_2 \frac{\partial \theta_3}{\partial r} \Big|_{r=\beta(t)} + L \gamma \frac{d\beta}{dt} \quad (14)$$

$$\theta_3(\infty,t) = 0 \quad (15)$$

Method of solution

We represent solution of the problem (6)-(15) in the form

$$\theta_1(x,t) = \sum_{n=0}^{\infty} A_n \sum_{k=0}^n \zeta_{n,k} x^{2n-2k} t^k \quad (16)$$

$$\theta_2(r,t) = \sum_{n=0}^{\infty} C_n \sum_{k=0}^n \zeta_{n,k,2} r^{2n-2k} t^k + \sum_{n=0}^{\infty} D_n \frac{(2a_1 \sqrt{t})^{2n+1}}{r} (i^{2n+1} erfc \frac{-(r-r_0)}{2a_1 \sqrt{t}} - i^{2n+1} erfc \frac{(r-r_0)}{2a_1 \sqrt{t}}) \quad (17)$$

$$\theta_3(r,t) = \sum_{n=0}^{\infty} E_n \sum_{k=0}^n \zeta_{n,k,2} r^{2n-2k} t^k + \sum_{n=0}^{\infty} G_n \frac{(2a_2 \sqrt{t})^{2n+1}}{r} (i^{2n+1} erfc \frac{-(r-r_0)}{2a_2 \sqrt{t}} - i^{2n+1} erfc \frac{(r-r_0)}{2a_2 \sqrt{t}}) \quad (18)$$

$$\text{Where } \zeta_{n,k} = \frac{2^{2k} n! \Gamma\left(\frac{\nu-1}{2} + n + 1\right)}{k!(n-k)! \Gamma\left(\frac{\nu-1}{2} + n + 1 - k\right)} \text{ and } \zeta_{n,k,2} = \frac{2^{2k} n! \Gamma\left(\frac{3}{2} + n\right)}{k!(n-k)! \Gamma\left(\frac{3}{2} + n - k\right)},$$

and coefficients A_n, C_n, D_n, E_n, G_n have to be determined. Free boundary $\beta(t)$ represent in the form

$$[4] \quad \beta(t) = \sum_{n=0}^{\infty} \beta_n t^{\frac{n}{2}}.$$

Using the boundary condition (10) we get

$$\sum_{n=0}^{\infty} A_n \sum_{k=0}^n \zeta_{n,k} (n-k) \alpha(t)^{2n-2k-1} t^k = F(t) \quad (19)$$

where

$$F(t) = -\frac{Q(t)}{2\lambda\pi\alpha^\nu(t)}$$

Taking into account that $\alpha(t)$ is given and using properties of raising the power series to a power [5]

$$\alpha(t)^{2n-2k-1} = \sum_{m=0}^{\infty} \beta(\alpha)_m t^m \quad (20)$$

Where coefficients $\beta_m(\alpha)$ determined by

$$\begin{aligned} \beta_0(\alpha) &= \alpha_0^{2n-2k-1} \\ \beta(\alpha)_i &= \frac{1}{i\alpha_0} \sum_{m=1}^i [2m(n-k)-i] \alpha_m \beta(\alpha)_{i-m} \quad i \geq 1 \end{aligned}$$

Substituting the formula (20) into (19) we obtain

$$\sum_{n=0}^{\infty} A_n \sum_{k=0}^n \zeta_{n,k} (n-k) \sum_{m=0}^{\infty} \beta(\alpha)_m t^{m+k} = F(t) \quad (21)$$

$F(t)$ is given, can be expanded in Maclaourin series thus to derive A_m , we take both sides of (21), l -times derivative at $t = 0$ we have

$$\sum_{n=0}^l A_{n+1} \hbar_{1,n} + \sum_{n=l}^{\infty} A_{n+1} \hbar_{2,n} = l! F_n \quad (22)$$

where

$$\begin{aligned} \hbar_{2,n} &= \sum_{i=0}^l \zeta_{n+1,i} \beta(\alpha)_{l-i,i} l!(n-i+1) \\ \hbar_{1,n} &= \sum_{i=0}^n \zeta_{n+1,i} \beta(\alpha)_{l-i,i} l!(n-i+1) \end{aligned}$$

from (22) we can find A_n .

Satisfying the boundary conditions of conjugations of temperature (11) and heat flux (12) we get

$$\sum_{n=0}^{\infty} A_n \zeta_{n,n} t^n = \sum_{n=0}^{\infty} C_n \sum_{k=0}^n \zeta_{n,k,2} r_0^{2n-2k} t^k \quad (23)$$

$$\sum_{n=0}^{\infty} C_n \sum_{k=0}^n \zeta_{n,k,2} r_0^{2n-2k-1} t^k + \frac{2}{r_0} \sum_{n=0}^{\infty} D_n (2a_1 \sqrt{t})^{2n} i^{2n} erfc 0 = 0 \quad (24)$$

from taking the m -times derivative of (23) and (24) we get

$$A_m \zeta_{m,m} = \sum_{i=m}^{\infty} C_i \zeta_{i,m,2} r_0^{2n-2m} \quad (23^*)$$

$$\sum_{i=0}^{\infty} C_n \zeta_{i+m+1,m,2} r_0^{2i+1} (i+1) = -\frac{1}{r_0} D_m (2a_1)^{2m} i^{2m} erfc 0 \quad (25)$$

From expression (13a) when we put $r = \beta(t)$ we have

$$\sum_{n=0}^{\infty} C_n \sum_{k=0}^n \zeta_{n,k,2} \beta(\tau)^{2n-2k} \tau^{2k} + \sum_{n=0}^{\infty} \frac{1}{\beta(\tau)} (2a_1 \tau)^{2n+1} (i^{2n+1} erfc(-\gamma(\tau)) - i^{2n+1} erfc(\gamma(\tau))) = \theta_m \quad (26)$$

$$\text{where } \tau = \sqrt{t} \text{ and } \frac{\beta(\tau) - \beta_0}{2a_1 \tau} = \frac{1}{2a_1} \sum_{n=0}^{\infty} \gamma_{n+1} \tau^n$$

Multiplying both sides of (26) by $\beta(\tau)$ we get

$$\sum_{n=0}^{\infty} C_n \sum_{k=0}^n \zeta_{n,k,2} \sum_{m=0}^{\infty} \eta(\beta)_m \tau^{m+2k} + \sum_{n=0}^{\infty} D_n (2a_1 \tau)^{2n+1} (i^{2n+1} erfc(-\gamma(\tau)) - i^{2n+1} erfc(\gamma(\tau))) = \theta_m \beta(\tau) \quad (27)$$

To comparing coefficients in (27) we apply Leibniz, Faa Di Bruno's formula and Bell polynomials. Using Leibniz we have

$$\left. \frac{\partial^l \left[(2a_1)^{2n+1} \tau^{2n+1} i^{2n+1} erfc(\gamma) \right]}{\partial \tau^l} \right|_{\tau=0} = \begin{cases} 0 & , \quad \text{for } l < 2n+1 \\ \frac{(2a_1)^{2n+1} l!}{(l-2n-1)!} [i^{2n+1} erfc(\delta)]^{(l-2n-1)} & \end{cases}$$

Using Faa Di Bruno's formula and Bell polynomials for a derivative of a composite function we have

$$\left. \frac{\partial^{l-2n-1}}{\partial \tau^{l-2n-1}} \left\{ i^{2n+1} erfc(\pm \delta) \right\} \right|_{\tau=0} = \left. \sum_{m=1}^{l-2n-1} \left(i^{2n+1} erfc(\pm \delta) \right)^{(m)} \right|_{\delta=0} B_{l-2n-1,m}$$

where

$$B_{l-2n-1,m} = \sum \frac{(l-2n-1)!}{j_1! j_2! \dots j_{l-2n-m}!} \zeta_1^{j_1} \zeta_2^{j_2} \zeta_3^{j_3} \dots \zeta_{l-2n-m}^{j_{l-2n-m}}$$

and j_1, j_2, \dots satisfy the following equations

$$\begin{aligned} j_1 + j_2 + \dots + j_{l-2n-m} &= m \\ j_1 + 2j_2 + \dots + (l-2n-m)j_{l-2n-m} &= l-2n-1 \end{aligned}$$

by taking both sides of (27) l -times derivative at $\tau = 0$ we get

$$\begin{aligned} \sum_{n=0}^{\left[\frac{l}{2}\right]-1} C_n \sum_{i=0}^n \zeta_{n,i,2} \eta(\beta)_{l-2i,i,2} + \sum_{n=\left[\frac{l}{2}\right]}^{\infty} C_n \sum_{i=0}^{\left[\frac{l}{2}\right]} \zeta_{n,i,2} \eta(\beta)_{l-2i,i,2} + \\ + D_n \frac{2^{2n+1} l!}{(l-2n-1)!} \sum_{m=1}^{l-2n-1} \left(i^{2n+1-m} erfc(-\gamma_1) + (-1)^{2n+1-m} erfc(\gamma_1) \right) \beta_{l-2n-1,m} = \theta_m l! \beta_l \end{aligned} \quad (28)$$

from (28), (23*) and (25) we can find C_n and D_n .

By the properties of Integral error functions [6] and condition (9) we get

$$\sum_{n=0}^{\infty} \left\{ E_n r^{2n} + \frac{G_n}{r} \frac{2}{(2n+1)!} r^{2n+1} \right\} = f(r)$$

Suggesting that the initial function can be expanded in $f(r) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} r^n$

we have

$$E_n + G_n \frac{2}{(2n+1)!} = \frac{f^{(2n)}(0)}{2n!} \quad (29)$$

From condition (13b) we have

$$\sum_{n=0}^{\infty} E_n \sum_{k=0}^n \zeta_{n,k,2} \beta(\tau)^{2n-2k} \tau^{2k} + \sum_{n=0}^{\infty} G_n \frac{(2a_1\tau)^{2n+1}}{\beta(\tau)} (i^{2n+1} \operatorname{erfc}(-\xi(\tau)) - i^{2n+1} \operatorname{erfc}(\xi(\tau))) = \theta_m \quad (30)$$

where

$$\xi(\tau) = \frac{\beta(\tau)}{2a_2}$$

As previously by taking by both sides of (30), l – times derivatives at $\tau = 0$ by using Leibniz, Faa Di Bruno's formulas and Bell polynomials we have

$$\begin{aligned} & \sum_{n=0}^{\left[\frac{l}{2}\right]-1} E_n \sum_{i=0}^n \zeta_{n,i,2} \eta(\beta)_{l-2i,i,2} + \sum_{n=\left[\frac{l}{2}\right]}^{\infty} E_n \sum_{i=0}^{\left[\frac{l}{2}\right]} \zeta_{n,i,2} \eta(\beta)_{l-2i,i,2} + \\ & + G_n \frac{2^{2n+1} l!}{(l-2n-1)!} \sum_{m=1}^{l-2n-1} (i^{2n+1-m} \operatorname{erfc}(-\xi_1) + (-1)^{2n+1-m} \operatorname{erfc}(\xi_1)) \beta_{l-2n-1,m} = \theta_m l! \beta_l \end{aligned} \quad (31)$$

From expression (29) and (31) we can find E_n and G_n

Satisfying Stefan's condition (14) and substituting $\sqrt{t} = \tau$ we have

$$\begin{aligned} & -\lambda_1 \left\{ \sum_{n=0}^{\infty} C_n \sum_{k=0}^n \zeta_{n,k,2} \beta(\tau)^{2n-2k-1} \tau^{2k} - \frac{1}{\beta^2(\tau)} \sum_{n=0}^{\infty} D_n (2a_1 \tau)^{2n+1} (i^{2n+1} \operatorname{erfc}(-\gamma(\tau)) \right. \\ & \left. - i^{2n+1} \operatorname{erfc}(\gamma(\tau))) - \frac{\lambda_1}{\beta(\tau)} \sum_{n=0}^{\infty} D_n (2a_1 \tau)^{2n} (i^{2n} \operatorname{erfc}(-\gamma(\tau)) + i^{2n} \operatorname{erfc}(\gamma(\tau))) \right\} = \\ & = -\lambda_2 \left\{ \sum_{n=0}^{\infty} E_n \sum_{k=0}^n \zeta_{n,k,2} \beta(\tau)^{2n-2k-1} \tau^{2k} - \frac{1}{\beta^2(\tau)} \sum_{n=0}^{\infty} G_n (2a_2 \tau)^{2n+1} (i^{2n+1} \operatorname{erfc}(-\xi(\tau)) - \right. \\ & \left. - i^{2n+1} \operatorname{erfc}(\xi(\tau))) + \frac{1}{\beta(\tau)} \sum_{n=0}^{\infty} D_n (2a_2 \tau)^{2n} (i^{2n} \operatorname{erfc}(-\xi(\tau)) + i^{2n} \operatorname{erfc}(\xi(\tau))) \right\} + L\gamma\beta'(\tau) \text{If} \end{aligned} \quad (32)$$

multiply both sides of (32) by $\beta(\tau)$ and using conditions (13a), (13b) we have the following expression

$$\begin{aligned} & -2\lambda_1 \sum_{n=0}^{\infty} C_n \sum_{k=0}^n \zeta_{n,k,2} \beta(\tau)^{2n-2k} \tau^{2k} - \lambda_1 \sum_{n=0}^{\infty} D_n (2a_1 \tau)^{2n} (i^{2n} \operatorname{erfc}(-\gamma(\tau)) + i^{2n} \operatorname{erfc}(\gamma(\tau))) \\ & = -2\lambda_2 \sum_{n=0}^{\infty} E_n \sum_{k=0}^n \zeta_{n,k,2} \beta(\tau)^{2n-2k} \tau^{2k} - \lambda_2 \sum_{n=0}^{\infty} G_n (2a_2 \tau)^{2n} (i^{2n} \operatorname{erfc}(-\xi(\tau)) + i^{2n} \operatorname{erfc}(\xi(\tau))) \\ & \quad + (\lambda_2 - \lambda_1) \theta_m + L\gamma'\psi(\tau) \end{aligned} \quad (33)$$

where

$$\psi(\tau) = \beta'(\tau) \beta(\tau) = \frac{1}{2} \frac{d}{d\tau} \beta^2(\tau)$$

$$\begin{aligned} (\beta(\tau))^2 &= \sum_{n=0}^{\infty} \mu(\beta)_n \tau^n \\ \mu(\beta) = \beta_0^2 &\quad \mu(\beta)_m = \frac{1}{m\beta_0} \sum_{k=1}^m (3k-m)\beta_k \cdot \mu(\beta)_{m-k} \quad m \geq 1 \end{aligned}$$

Previously by taking both sides of (33) l -times derivatives $\tau = 0$ and for $l \geq 1$ we have

$$\begin{aligned} -2\lambda_1 \left(\sum_{n=0}^{\left[\frac{l}{2}\right]-1} C_n \chi_{1,n} + \sum_{n=\left[\frac{l}{2}\right]}^{\infty} C_n \chi_{2,n} \right) - \lambda_1 D_n \frac{2^{2n} l!}{(l-2n)!} \sum_{m=1}^{l-2n} (i^{2n-m} \operatorname{erfc}(-\gamma_1) + (-1)^m i^{2n-m} \operatorname{erfc}(\gamma_1)) \beta_{l-2n,m} \\ = -2\lambda_2 \left(\sum_{n=0}^{\left[\frac{l}{2}\right]-1} E_n \chi_{21n} + \sum_{n=\left[\frac{l}{2}\right]}^{\infty} E_n \chi_{2,n} \right) - \lambda_2 G_n \frac{2^{2n} l!}{(l-2n)!} \sum_{m=1}^{l-2n} (i^{2n-m} \operatorname{erfc}(-\xi_1) + (-1)^m i^{2n-m} \operatorname{erfc}(\xi_1)) \beta_{l-2n,m} \\ + \frac{L\gamma'}{2} l! \mu(\beta)_{l+1} \end{aligned} \quad (34)$$

For $l = 0$ we have

$$\theta_m - D_0 (\operatorname{erfc}(-\gamma_1) + \operatorname{erfc}(\gamma_1)) - 2 \sum_{n=0}^{\infty} C_n \zeta_{n,0,2} r_0^{2n} = \frac{\lambda_2}{\lambda_1} \left(\theta_m - G_0 (\operatorname{erfc}(-\xi_1) + \operatorname{erfc}(\xi_1)) - 2 \sum_{n=0}^{\infty} E_n \zeta_{n,0,2} r_0^{2n} \right)$$

where

$$\begin{aligned} \chi_{2,n} &= \sum_{i=0}^{\left|\frac{l}{2}\right|} \zeta_{n,i,2} v(\beta)_{l-2i,i,2} \\ \chi_{1,n} &= \sum_{i=0}^n \zeta_{n,i,2} v(\beta)_{l-2i,i,2} \end{aligned}$$

From this recurrent formula we can express β_n .

Conclusion

To summarize, the coefficients A_n, C_n, D_n, E_n, G_n are determined from equations (22), (23*), (28), (29), (31) and (25), the moving boundary $\beta(t)$ obtained from equation (34). For the convergence of temperature functions $\theta_1, \theta_2, \theta_3$, it is possible to follow the idea proposed in [6].

REFERENCES

- [1] Davidson P. M. "Proc. IEE", 1949, v. 96, p. 1.
- [2] R. Holm, Electrical Contacts, fourth edition, Springer –Verlag, 1981.
- [3] V. I. Kudrya, A. E. Pudy, S. N. Kharin, Fundamental solution and heat potentials of the heat equation for a rod with a variable cross – section, Equations with discontinuous coefficients and their applications, Nauka, Alma-Ata, 1985, pp 76-81.
- [4] Kharin S. N. Ob odnomobobshenii funktsii shibokie prilozhenii v zadacha teploprovodnosti // V. Sb. Differencial'nye uravneniya i ikh prilozheniya. Alma-Ata: Nauka. 1982. C. 51-59.
- [5] Градштейн И. С., Рыжик И. М. Таблицы интегралов, сумм, рядов и произведений, Москва: государственное издательство физико-математической литературы, 1963.
- [6] Kharin S.N. The analytical solution of the two-face Stefan problem with boundary flux condition, Matematicheskiy zhurnal, № 1, 2014, pp. 55-75.

С. Н. Харин^{1,2}, С. А. Қасабек^{2,3}, М. Слямхан³

¹ Математика және математикалық модельдеу институты, Алматы Қазақстан,

² Қазақстан-БританТехникалықУниверситеті, Алматы Қазақстан,

³ Сулейман Демирелуниверситеті, Қакелен, Қазақстан

КӨПІР ЭРОЗИЯСЫНЫҢ ТЕОРИЯ ЕСЕБІ

Аннотация. Осы мақалада біз екі фазалық Стефан мәселесінің дәл шешімін ұсынамыз. Көпір мәселесін шешу үшін радиалды жылу полиномы және интегралдың қателілфункциясы қолданылады. Осы құтарлардың коэффициенттері үшін қайталанатын өрнектер ұсынылған. Математикалық модельдер байланыстың ашылу және көпіршікті динамикасын сипаттайды.

Түйін сөздер: радиалды жылу полиномы, Стефан проблемасы, интеграл

С. Н. Харин^{1,2}, С. А. Қасабек^{2,3}, М. Слямхан³

¹Институт математики и математического моделирования, Алматы Казахстан,

²Казахстанско-Британский Технический Университет, Алматы Казахстан,

³ Университет имени Сулеймана Демиреля, Каскелен, Казахстан

ЗАДАЧА ИЗ ТЕОРИИ МОСТИКОВОЙ ЭРОЗИИ

Аннотация. В настоящей работе мы представляем точное решение двухфазной задачи Стефана. Для решения данной задачи использовали решение в виде радиальных тепловых полиномов и интегральной функции ошибок. Приводятся рекуррентные выражения для коэффициентов ряда. Математические модели описывают динамику размыкания металлических контактов.

Ключевые слова: радиальные тепловые полиномы, проблема Стефана.

Information about authors:

Kharin S.N. - Institute of Mathematics of the National Academy of Sciences of Kazakhstan; Kazakh-British Technical University, Kazakhstan;

Kassabek S.A. – Kazakh-British Technical University, Kazakhstan; SuleymanDemirelUniversity, Kazakhstan, <https://orcid.org/0000-0002-1714-5850>;

Slyamkhan M. - SuleymanDemirelUniversity, Kazakhstan, <https://orcid.org/0000-0002-4297-7958>