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MATHEMATICAL MODELING OF THE PROBLEM OF OPTIMAL CONTROL OF ELECTRIC POWER SYSTEMS

Abstract: The problems of optimal control of nonlinear ordinary differential equations systems are considered in this paper. The considered mathematical model describes the transient processes in the electric power system. And the problem of optimal control of electric power systems is considered in more detail. Numerical experiments have shown that the control found is optimal for the given problem.

Keywords: mathematical model, electric power system, optimal control.

1 INTRODUCTION

The proper functioning of electric power systems (EPS) as an important component of large energy systems forms one of the successful development foundations of the country's economy as a whole. The presence of not only technical but also economic aspects of reliability predetermines the complexity of studying the above objects and their interaction with other components of the economy and the social sphere in order to determine the best control actions to achieve economic benefits and to maintain a constant readiness of the energy systems to overcome the threats to their normal functioning arising in periods of economic, political crises, in case of catastrophes, disasters, etc.

The importance of the problem of optimal control of processes in various fields of science and technology is well known[1-6]. It also has great importance for electric power systems. Without a reliable solution to this problem, reliable and high-quality supply of electricity to consumers in virtually all sectors of the national economy is impossible.

2 Mathematical model of unsteady processes in the electrical system

One of the mathematical models that describes unsteady processes in an electrical system is the following system of differential equations[1-3]:

$$\begin{aligned} \frac{d\delta_i}{dt} &= S_i, \\ H_i \frac{dS_i}{dt} &= -D_i S_i - E_i^2 Y_{ii} \sin \alpha_{ii} - P_i \sin(\delta_i - \alpha_i) - \sum_{j=1, j \neq i}^l P_{ij} \sin(\delta_{ij} - \alpha_{ij}) + u_i \\ \delta_{ij} &= \delta_i - \delta_j, \quad P_i = E_i U Y_{i,n+1}, \quad P_{ij} = E_i E_j Y_{i,j}, \end{aligned} \quad (1)$$

where – angle of rotor rotation of the i-th generator concerning some synchronous rotation axis; - slide of the i-th generator; - the inertia constant of the i-th machine; mechanical power, which are brought to the generator; - EMF of the i-th synchronous machine; - mutual conductance of the i-th and j-th branches of the system; U=const - bus voltage of constant voltage; - characterizes the connection (conductivity) of the i-th generator with the buses of constant voltage; - mechanical damping; – constant values that consider the effect of active resistance in the stator generator circuits. .

Let the state variables and control in the steady-state post-emergency condition have the following values:

$$S_i = 0, \quad \delta_i = \delta_i^f, \quad u_i = u_i^f, \quad i = 1, l \quad (2)$$

In order to obtain a perturbed motion system we proceed to equations in deviations, assuming

$$u_i = u_i^F + \Delta u_i, \delta_i = \delta_i^F + \Delta \delta_i, S_i = \Delta S_i, i = 1, l \quad (3)$$

Further, for convenience, we again denote by and use the formula

$$\sin(\delta_{ij} - \alpha_{ij}) = \cos \alpha_{ij} \sin \delta_{ij} - \sin \alpha_{ij} \cos \delta_{ij}$$

from the system (1) we obtain

$$\begin{aligned} \frac{d\delta_i}{dt} &= S_i \\ \frac{dS_i}{dt} &= \frac{1}{H_i} (-D_i S_i - f_i(\delta_i) - N_i(\delta) + M_i(\delta) + u_i) \\ i &= 1, l, t \in [0, T] \end{aligned} \quad (4)$$

Where

$$\begin{aligned} f_i(\delta_i) &= P_i [\sin(\delta_i + \delta_i^F - \alpha_i) - \sin(\delta_i^F - \alpha_i)] \\ N_i(\delta) &= \sum_{j=1, i \neq j}^l \Gamma_{ij}^1 [\sin(\delta_{ij} + \delta_{ij}^F) - \sin(\delta_{ij}^F)] \\ M_i(\delta) &= \sum_{j=1, i \neq j}^l \Gamma_{ij}^2 [\cos(\delta_{ij} + \delta_{ij}^F) - \cos(\delta_{ij}^F)] \\ \Gamma_{ij}^1 &= P_{ij} \cos \alpha_i, \quad \Gamma_{ij}^2 = P_{ij} \sin \alpha_i, \end{aligned}$$

Note that since $P_{ij} = P_{ji}$, then

$$\Gamma_{ij}^1 = \Gamma_{ji}^1, \quad \Gamma_{ij}^2 = \Gamma_{ji}^2.$$

Controls $u_i, i = 1, l$ are chosen so as to compensate for the non-conservative term - $M_i(\delta), i = 1, l$ i.e.

$$u_i = v_i - M_i(\delta), i = 1, l \quad (5)$$

2.1 The problem of optimal control of electric power systems

We consider the following problem of minimizing the functional:

$$J(v) = J(v_1, \dots, v_l) = 0.5 \sum_{i=1}^l \int_0^T (w_{S_i} S_i^2 + w_{v_i} v_i^2) * * \exp\{\gamma_i t\} dt + \Lambda(\delta(T), S(T)), \quad (6)$$

under conditions

$$\begin{aligned} \frac{d\delta_i}{dt} &= S_i, \\ H_i \frac{dS_i}{dt} &= -D_i S_i - f_i(\delta_i) - N_i(\delta_i) + v_i, \\ \delta &= (\delta_1, \dots, \delta_l), S = (S_1, S_2, \dots, S_l) \end{aligned} \quad (7)$$

where

$$f_i(\delta_i) = P_i [\sin(\delta_i + \delta_i^F - \alpha_i) - \sin(\delta_i^F - \alpha_i)], \quad N_i(\delta) = \sum_{j=1, i \neq j}^l P_{ij} \cos \alpha_i [\sin(\delta_{ij} + \delta_{ij}^F) - \sin(\delta_{ij}^F)], \quad \text{and}$$

w_{S_i}, w_{v_i} - positive constant weight coefficients, $F_i(\delta_i) - 2\pi$ periodic continuously differentiable

functions, $N_i(\delta) - 2\pi$. Periodic continuously differentiable functions with respect to δ_{ij} , with respect to the summand $N_i(\delta)$ the integrability condition is fulfilled

$$\frac{\partial N_i(\delta)}{\partial \delta_k} = \frac{\partial N_k(\delta)}{\partial \delta_i} (\forall i \neq k); \quad (8)$$

T - the transient period is considered given.

The equations system (7) is supplemented by the initial conditions

$$\delta_i(0) = \delta_{i0}, \quad S_i(0) = S_{i0}, \quad i = 1, \dots, l. \quad (9)$$

Terminal values $\delta(T), S(T)$ are not known in advance, so they are also subject to definition.

The following theorem is valid.

Theorem 1. In order that the controls $v_i^0(S_i) = -\frac{1}{w_{v_i}}S_i$, $i = \overline{1, l}$ and the corresponding solution of

the system (7)-(9) be optimal, it is necessary and sufficient that

$$\begin{aligned} \Lambda(\delta(T), S(T)) &= K(\delta(T), S(T)), \\ w_{S_i}(t) &= 2D_i + \frac{1}{w_{v_i}} > 0, \quad i = \overline{1, l}, \end{aligned}$$

Where

$$K(\delta, S) = \frac{1}{2} H_i S_i^2 + \sum_{i=1}^l \int_0^{\delta_i} f_i(\delta_i) d\delta_i + \sum_{\substack{i=1, \\ \delta_j=0, \\ j>i}}^l \int_0^{\delta_i} N_i(\delta_1, \dots, \delta_{i-1}, \delta_{i+1}, \dots, \delta_l) d\xi_i$$

the Bellman-Krotov function, and

$$J(v^0) = \min_v J(v) = K(\delta^0, S^0). \quad (10)$$

Proof. For the continuously differentiable function $K(\delta(t), S(t))$ the functional (6) has the representation:

$$J(u) = J(\delta(t), S(t), v(t)) = \int_0^T R[\delta(t), S(t), v(t)] dt + m_0(\delta(0)S(0)) + m_1(\delta(T)S(T)) \quad (11)$$

Where

$$R(\delta, S, v) = \sum_{i=1}^l \left[K_{\delta_i} S_i + \frac{1}{H_i} K_{S_i} (-D_i S_i - f_i(\delta_i) - N_i(\delta) + v_i) + \right] + \frac{1}{2} (w_{S_i} S_i^2 + w_{v_i} v_i^2) \quad (12)$$

$$K_{\delta_i} = \frac{\partial K}{\partial \delta_i}, \quad K_{S_i} = \frac{\partial K}{\partial S_i} \quad (13)$$

$$m_0(\delta, S) = K(\delta, S), \quad m_1(\delta, S) = -K(\delta, S) + \Lambda(\delta, S).$$

We use the Cauchy-Bellman problem to find the Bellman function $K(\delta, S)$:

$$\begin{aligned} \inf_v R[\delta, S, v] &= 0, \quad t \in [0, T], \\ K(\delta(T), S(T)) &= \Lambda(\delta(T), S(T)). \end{aligned} \quad (14)$$

From the necessary condition of extremum of the function $R(\delta, S, v)$ we obtain:

$$R_{vj} \equiv \frac{1}{H_i} K_{S_i} + w_{S_i} v_i = 0, \quad i = \overline{1, l}$$

$$\nu_i^0 \equiv -\frac{1}{H_i w_{vi}} K_{Si}, i = \overline{1, l}, \quad (15)$$

We find the function $K(\delta, S)$ and weight coefficients w_{Si}, w_{vi} from the condition (14) i.e.

$$\begin{aligned} \bar{R} = \min R(\delta, S, \nu) &= \sum_{i=1}^l K_{\delta_i} S_i - \frac{1}{H_i} K_{S_i} (D_i S_i + f_i(\delta_i) + \\ &+ N_i(\delta)) - \frac{1}{2H_i^2 w_{vi}} K_{Si}^2 + \frac{1}{2} w_{Si}^2 S_i^2 = 0 \end{aligned} \quad (16)$$

For this, we set

$$K_{\delta_i} S_i = \frac{K_{S_i}}{H_i} (f_i(\delta_i) + N_i(\delta)), i = \overline{1, l}$$

i.e.

$$K_{S_i} = H_i S_i, K_{\delta_i} = f_i(\delta_i) + N_i(\delta), i = \overline{1, l} \quad (17)$$

Then taking into account (17), we obtain from (16) that

$$\sum_{i=1}^l \left[-D_i S_i - \frac{1}{w_{vi}} S_i^2 + \frac{1}{2} w_{Si} \nu_i^2 \right] = 0$$

or

$$w_{Si} = 2D_i + \frac{1}{2w_{vi}} > 0, w_{vi} > 0, i = \overline{1, l} \quad (18)$$

In addition, from (15) we find that the optimal controls $\nu_i^0, i = \overline{1, l}$ have the form

$$\nu_i^0(S_i) = -\frac{1}{w_{vi}} S_i, i = \overline{1, l} \quad (19)$$

2.2 Numerical solution

The implicit Adams method (Adams-Moulton) has been used for the numerical solution of the differential equations system (7)-(9).

Table 1: Adams-Moulton formulas in different orders

Order	Formula	Error order
1	$y_{n+1} = y_n + hf_{n+1}$	$-\frac{h^2}{2} y''(\eta)$
2	$y_{n+2} = y_{n+1} + \frac{h}{2} [f_{n+2} - f_{n+1}]$	$-\frac{h^3}{12} y'''(\eta)$
3	$y_{n+3} = y_{n+2} + \frac{h}{12} [5f_{n+3} + 8f_{n+2} - f_{n+1}]$	$-\frac{h^4}{24} y^{(4)}(\eta)$
4	$y_{n+4} = y_{n+3} + \frac{h}{24} [9f_{n+4} + 19f_{n+3} - 5f_{n+2} + f_{n+1}]$	$-\frac{19h^5}{720} y^{(5)}(\eta)$
5	$y_{n+5} = y_{n+4} + \frac{h}{720} [251f_{n+5} + 646f_{n+4} - 264f_{n+3} + 106f_{n+2} - 19f_{n+1}]$	$-\frac{3h^6}{160} y^{(6)}(\eta)$

We have used the Adams-Moulton method of the 4th order to solve this problem:

$$y_{n+4} = y_{n+3} + \frac{h}{24}(9f(t_{n+4}, y_{n+4}) + 19f(t_{n+3}, y_{n+3}) - \\ - 5f(t_{n+2}, y_{n+2}) + f(t_{n+1}, y_{n+1})) - \frac{19}{720}h^5(\eta).$$

And we also have used the Runge-Kutta method of the 4-th order:

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = f(t, y),$$

$$k_2 = f\left(t + \frac{h}{2}, y + \frac{hk_1}{2}\right),$$

$$k_3 = f\left(t + \frac{h}{2}, y + \frac{hk_2}{2}\right),$$

$$k_4 = f(t + h, y + hk_3).$$

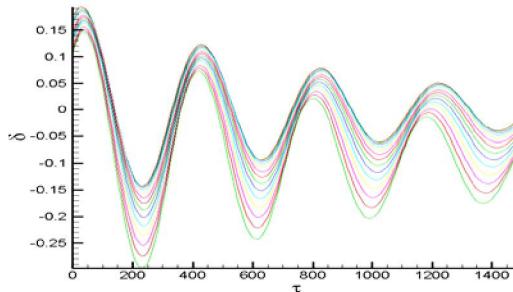


Figure 1 - Angle of rotor rotation at $l = 15$.

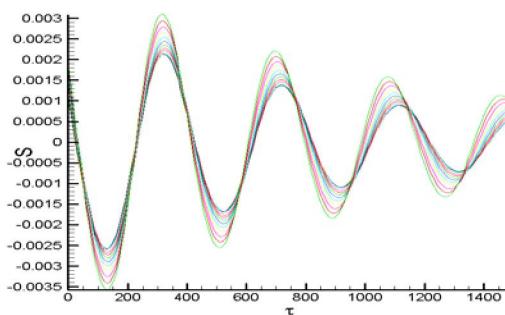


Figure 2 - Generator slide at $l=15$.

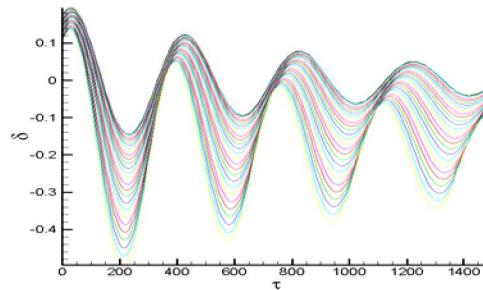
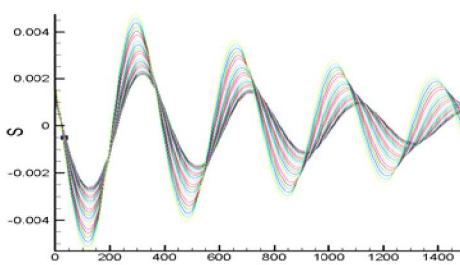


Figure 3 - Angle of rotor rotation at $l=30$.

Figure 4 - Generator slide at $l=30$

3 CONCLUSIONS

The process of optimal control of complex electric power systems is described in this paper. The continuous Bellman-Krotov function, that has continuous partial derivatives everywhere by all its arguments, has been found. The numerical solution of this problem is obtained at $l = 15$ and $l = 30$. When solving it the implicit Adams method (Adams-Moulton) has been used.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ЗАДАЧИ ОПТИМАЛЬНОГО УПРАВЛЕНИЯ ЭЛЕКТРОЭНЕРГЕТИЧЕСКИМИ СИСТЕМАМИ

Аннотация: В данной статье рассматривается задача оптимального управления системами нелинейных обыкновенных дифференциальных уравнений. Рассматриваемая математическая модель описывает переходные процессы в электроэнергетической системе. Проблема оптимального управления электроэнергетическими системами рассматривается более подробно. Численные эксперименты показали, что найденное управление является оптимальным для данной задачи.

Ключевые слова: математическая модель, электроэнергетическая система, оптимальное управление.

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ЭЛЕКТР ЭНЕРГЕТИКАЛЫҚ ЖҮЙЕЛЕРДІ ТИІМДІ БАСҚАРУ МӘСЕЛЕСІН МАТЕМАТИКАЛЫҚ МОДЕЛЬДЕУ

Аннотация: Бұл макалада сзықты емес қарапайым дифференциалдық тендеулер жүйелерін тиімді басқару мәселесі қарастырылады. Қарастырылған математикалық модель электроэнергетикалық жүйедегі өтпелі процестерді сипаттайты. Электроэнергетикалық жүйелерді тиімді басқару мәселесі толығырақ қарастырылады. Сандақ эксперименттер табылған басқарудың бұл мәселе үшін тиімді екендігін көрсетті.

Түйін сөздер: математикалық модель, электроэнергетикалық жүйе, тиімдібасқару.

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