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DIRICHLET PROBLEM IN A CYLINDRICAL AREA FOR ONE CLASS OF MULTIDIMENSIONAL ELLIPTIC-PARABOLIC EQUATIONS

Abstract. Boundary-value problems for degenerate elliptic-parabolic equations on the plane are studied quite well ([1]). The correctness of the Dirichlet problem for degenerate multidimensional elliptic-parabolic equations with degeneration of type and order was established in [3]. In the work for multidimensional elliptic-parabolic equations with degeneration of type and order, the solvability is shown and an explicit form of the classical solution of the Dirichlet problem is obtained.
Keywords: solvability, mixed problem, multidimensional elliptic-parabolic equations, Bessel function.

Problem statement and result

Let $\Omega_{\alpha\beta}$ – the cylindrical area of the Euclidean space of E_{m+1} points $(x_1,...,x_m,t)$ bounded by a cylinder $\Gamma=\{(x,t):|x|=1\}$, planes $t=\alpha>0$ and $t=\beta<0$, where $|x|-$ is the length of a vector $x=(x_1,...,x_m)$.

Denote by Ω_α and Ω_β parts $\Omega_{\alpha\beta}$ – of the area and $\Gamma_\alpha,\Gamma_\beta$ – through parts of the surface Γ, lying in the half-spaces $t>0$ and $t<0$, σ_α – the upper and σ_β – lower base area $\Omega_{\alpha\beta}$.

Let $S-$ further the common part of the borders of the areas Ω_α and Ω_β representing the $\{t=0,\ 0<|x|<1\}$ set in E_m.

In the area $\Omega_{\alpha\beta}$, we consider degenerate multidimensional hyperbolic-parabolic equations

$$0=\begin{cases} p_1(t)\Delta_x u-p_2(t)u_{tt}+\sum_{i=1}^{m}a_i(x,t)u_{x_i}+b(x,t)u_t+c(x,t)u=0,\ t>0,\\ g(t)\Delta_x u-u_t+\sum_{i=1}^{m}d_i(x,t)u_{x_i}+e(x,t)u,t<0,\end{cases} \tag{1}$$

where $p_i(t)>0$ at $t>0, p_i(0)=0, p_i(t)\in C([0,\alpha])g(t)>0$ at $t<0$, and may vanish when $t=0$, $g(t)\in C[\beta,0])$, a Δ_x – Laplace operator with variables $x_1,\dots,x_m$, $m\geq 2$.

In the future, it is convenient for us to move from the Cartesian coordinates $x_1,\dots,x_m$,t to spherical $r,\theta_1,\dots,\theta_{m-1},t,r\geq 0,0\leq\theta_{m-1}<2\pi,\ 0\leq\theta_i\leq\pi, i=1,2,\dots,m-2,\ \theta=(\theta_1,\dots,\theta_{m-1})$.

Problem 1 (Dirichlet). Find a solution to the equation (1) in the area of $\Omega_{\alpha\beta}$ at $t\neq 0$, from the class $C^1(\overline{\Omega}_{\alpha\beta})\cap C^2(\Omega_\alpha\cup\Omega_\beta)$, satisfying boundary conditions

$$u|_{\sigma_\alpha}=\varphi_1(r,\theta),\quad u|_{\Gamma_\alpha}=\psi_1(t,\theta), \tag{2}$$

$$u|_{\Gamma_\beta} = \psi_2(t,0), \ \ u|_{\sigma_\beta} = \varphi_2(t,\theta). \quad (3)$$

wherein $\varphi_1(1,\theta) = \psi_1(\alpha,\theta), \psi_1(0,\theta) = \psi_2(0,\theta), \psi_2(\beta,\theta) = \varphi_2(1,\theta)$.

Let $\{Y_{n,m}^k(\theta)\}$ - system of linearly independent spherical functions of order n, $1 \le k \le k_n$, $(m-2)!n!k_n = (n+m-3)!(2n+m-2), W_2^l(S), l = 0,1,...$ - Sobolev space. Takes place ([4]).

Lemma 1. Let $f(r,\theta) \in W_2^l(S)$. If $l \ge m-1$, that row

$$f(r,\theta) = \sum_{n=0}^{\infty}\sum_{k=1}^{k_n} f_n^k(r) Y_{n,m}^k(\theta), \quad (4)$$

as well as series derived from it by order differentiation $p \le l - m + 1$, converge absolutely and evenly.

Lemma 2. In order to $f(r,\theta) \in W_2^l(S)$, it is necessary and sufficient that the coefficients of the series (4) satisfy the inequalities.

$$|f_0^1(r)| \le c_1, \sum_{n=1}^{\infty}\sum_{k=1}^{k_n} n^{2l} |f_n^k(r)|^2 \le c_2, \ c_1, c_2 = const.$$

Through $\tilde{a}_{in}^k(r,t),\ d_{in}^k(r,t),\ \tilde{e}_n^k(r,t),\ \tilde{d}_n^k(r,t), \rho_n^k, \overline{\varphi}_{1n}^k(r), \overline{\varphi}_{2n}^k(r), \psi_{1n}^k(t), \psi_{2n}^k(t)$, denote the coefficients of the series (4), respectively functions $d_i(r,\theta,t)\rho(\theta),\ d_i \frac{x_i}{r}\rho$, $e(r,\theta,t)\rho, d(r,\theta,t)\rho,\ \rho(\theta), i = 1,...,m, \varphi_1(r,\theta), \varphi_2(r,\theta), \psi_1(t,\theta), \psi_2(t,\theta)$, and $\rho(\theta) \in C^\infty(H)$, H-unit sphere in E_m.

Let $\frac{a_i(r,\theta,t)}{g_2(t)}, \frac{b(r,\theta,t)}{g_2(t)}, \frac{c(r,\theta,t)}{g_2(t)} \in W_2^l(\Omega_\alpha) \subset C(\bar{\Omega}_\alpha), d_i(r,\theta,t)$,
$c(r,\theta,t) \in W_2^l(\Omega_\beta), i = 1,...,m, l \ge m+1, c(r,\theta,t) \le 0, \forall (r,\theta,t) \in \Omega_\alpha, e(r,\theta,t) \in \Omega_\beta$.

Then fair
Theorem.

If $\varphi_1(r,\theta), \varphi_2(r,\theta) \in W_2^l(S),\ \psi_1(t,\theta) \in W_2^p(\Gamma_\alpha), \psi_2(t,\theta) \in W_2^l(\Gamma_\beta), l > \frac{3m}{2}$, then problem 1 is solvable.

Proof of the theorem. First, let us rock the solvability of problem (1), (3). In spherical coordinates of equation (1) in the area Ω_β has the appearance

$$Lu \equiv g(t)(u_{rr} + \frac{m-1}{r}u_r - \frac{1}{r^2}\delta u) - u_{tt} + \sum_{i=1}^{m} d_i(r,\theta,t)u_{x_i} + e(r,\theta,t)u = 0, \quad (5)$$

$$\delta \equiv -\sum_{j=1}^{m-1} \frac{1}{g_j \sin^{m-j-1}\theta_j} \frac{\partial}{\partial\theta_j}(\sin^{m-j-1}\frac{\partial}{\partial\theta_j}), g_1 = 1, g_j = (\sin\theta_1 ... \sin\theta_{j-1})^2, j > 1.$$

It is known [4] that the spectrum of the operator δ consists of own numbers $\lambda_n = n(n+m-2), n = 0,1,...$ each of which corresponds k_n orthonormal functions $Y_{n,m}^k(\theta)$.

The desired solution to problem 1 in the field Ω_β we will look in the form

$$u(r,\theta,t)=\sum_{n=0}^{\infty}\sum_{k=1}^{k_n}\overline{u}_n^k(r,t)Y_{n,m}^k(\theta), \tag{6}$$

where $\overline{u}_n^k(r,t)$ - functions to be defined.

Substituting (6) в (5), then multiplying the resulting expression by $\rho(\theta)\neq 0,$ and integrating over a single sphere H, for $\overline{u}_n^k$ will get [5-7]

$$\begin{gathered}
g(t)\rho_0^1\overline{u}_{0rr}^1+\rho_0^1\overline{u}_{0tt}^1+(\frac{m-1}{r}g(t)\rho_0^1+\sum_{i=1}^{m}d_{i0}^1)u_{0r}^1+\\
+\sum_{n=1}^{\infty}\sum_{k=1}^{k_n}\{g(t)\rho_n^k\overline{u}_{nrr}^k+\rho_n^k\overline{u}_{ntt}^k+(\frac{m-1}{r}g(t)\rho_n^k+\sum_{i=1}^{m}d_{in}^k)\overline{u}_{nr}^k\\
+[\tilde{c}_n^k-\lambda_n\frac{\rho_n^k}{r^2}g(t)+\sum_{i=1}^{m}(\tilde{d}_{in-1}^k-nd_{in}^k)]\overline{u}_n^k\}=0.
\end{gathered} \tag{7}$$

Now consider the infinite system of differential equations

$$g(t)\rho_0^1\overline{u}_{0rr}^1+\rho_0^1u_{ott}^1+\frac{m-1}{r}g(t)\rho_0^1\overline{u}_{0r}^1=0, \tag{8}$$

$$\begin{gathered}
g(t)\rho_1^k\overline{u}_{1rr}^k-\rho_1^k\overline{u}_{1t}^k+\frac{m-1}{r}g(t)\rho_1^k\overline{u}_{1r}^k-\frac{\lambda_1}{r^2}g(t)\rho_1^k\overline{u}_1^k=\\
=-\frac{1}{k_1}(\sum_{i=1}^{m}d_{i0}^1\overline{u}_{0r}^1+\tilde{e}_0^1\overline{u}_o^1),\ n=1,\ k=\overline{1,k_1},
\end{gathered} \tag{9}$$

$$\begin{gathered}
g(t)\rho_n^k\overline{u}_{nrr}^k-\rho_n^k\overline{u}_{nt}^k+\frac{m-1}{r}g(t)\rho_n^k\overline{u}_{nr}^k-\frac{\lambda_n}{r^2}g(t)\rho_n^k\overline{u}_n^k=\\
=-\frac{1}{k_n}\sum_{k=1}^{k_{n-1}}\{\sum_{i=1}^{m}d_{in-1}^k\overline{u}_{n-1r}^k+[\tilde{e}_{n-1}^k+\sum_{i=1}^{m}(\tilde{d}_{in-2}^k-(n-1)d_{in-1}^k)]\overline{u}_{n-1}^k\},\\
k=\overline{1,k_n}.\quad n=2,3,\ldots .
\end{gathered} \tag{10}$$

Summing up the equation (8) from 1 before k_1, and the equation (9)- from 1 before k_n, and then adding the resulting expressions together with (7), come to the equation (6).

It follows that if $\{\overline{u}_n^k\},k=\overline{1,k_n},n=0,1,\ldots.$ system solution (7)-(9), then it is a solution to the equation (6).

It is easy to see that each equation of system (7) - (9) can be represented as

$$g(t)(\overline{u}_{nrr}^k+\frac{m-1}{r}\overline{u}_{nr}^k-\frac{\lambda_n}{r^2}\overline{u}_n^k)-u_{nt}^k=\overline{f}_n^k(r,t), \tag{11}$$

where $\overline{f}_n^k(r,t)$ are determined from the previous equations of this system, while $\overline{f}_0^1(r,t)\equiv 0$.

Further, from the boundary condition (3), by virtue of (6), we will have

$$\overline{u}_n^k(r,\beta)=\overline{\varphi}_{2n}^k(r),\ \overline{u}_n^k(1,t)=\psi_{2n}^k(t),\ k=\overline{1,k_n},\ n=0,1,\ldots. \tag{12}$$

In (11), (12) replacing $\overline{\vartheta}_n^k(r,t)=\overline{u}_n^k(r,t)-\psi_{2n}^k(t),$ will get

$$g(t)(\overline{\vartheta}_{nrr}^k+\frac{m-1}{r}\overline{\vartheta}_{nr}^k-\frac{\lambda_n}{r^2}\overline{\vartheta}_n^k)-\overline{\vartheta}_{ntt}^k=\overline{f}_n^k(r,t), \tag{13}$$

$$\bar{\vartheta}_n^k(r,\beta)=\varphi_{2n}^k(r),\ \bar{\vartheta}_n^k(1,t)=0,\quad k=\overline{1.k_n},\ n=0,1,.... \tag{14}$$

$$f_n^k(r,t)=\bar{f}_n^k(r,t)+\psi_{2nt}^k+\frac{\lambda_n g(t)}{r^2}\psi_{2n}^k,\varphi_{2n}^k(r)=\bar{\varphi}_{2n}^k(r)-\psi_{2n}^k(\beta).$$

Replacing the variable $\bar{\vartheta}_n^k(r,t)=r^{\frac{(1-m)}{2}}\vartheta_n^k(r,t)$ задачу (13), (14) we will lead to the following problem

$$L\vartheta_n^k=g(t)(\vartheta_{nrr}^k+\frac{\bar{\lambda}_n}{r^2}\vartheta_n^k)-\vartheta_{nt}^k=\tilde{f}_n^k(r,t), \tag{15}$$

$$\vartheta_n^k(r,\beta)=\bar{\varphi}_{2n}^k(r),\ \vartheta_n^k(1,t)=0,\ \vartheta_n^k(1,t)=0, \tag{16}$$

$$\bar{\lambda}_n=\frac{[(m-1)(3-m)-4\lambda_n]}{4},\ \tilde{f}_n^k(r,t)=r^{\frac{(m-1)}{2}}f_n^k(r,t),$$

$$\tilde{\varphi}_{2n}^k(r)=r^{\frac{(m-1)}{2}}\varphi_{2n}^k(r).$$

The solution of the problem (15), (16) is sought in the form

$$\vartheta_n^k(r,t)=\vartheta_{1n}^k(r,t)+\vartheta_{2n}^k(r,t), \tag{17}$$

where $\vartheta_{1n}^k(r,t)$ the solution of the problem

$$L\vartheta_{1n}^k=\tilde{f}_n^k(r,t), \tag{18}$$

$$\vartheta_{1n}^k(r,\beta)=0,\ \vartheta_{1n}^k(1,t)=0, \tag{19}$$

where $\vartheta_{2n}^k(r,t)$ the solution of the problem

$$L\vartheta_{1n}^k=0, \tag{20}$$

$$\vartheta_{2n}^k(r,\beta)=\tilde{\varphi}_{2n}^k(r)0,\ \vartheta_{2n}^k(1,t)=0, \tag{21}$$

The solution to the above problems, we consider in the form

$$\vartheta_n^k(r,t)=\sum_{s=1}^{\infty}R_s(r)T_s(t), \tag{22}$$

at the same time let

$$\tilde{f}_n^k(r,t)=\sum_{s=1}^{\infty}a_{ns}^k(t)R_s(r),\ \tilde{\varphi}_{2n}^k(r)=\sum_{s=1}^{\infty}b_{ns}^kR_s(r). \tag{23}$$

Substituting (22) into (18), (19), taking into account (23), we obtain

$$R_{srr}+\frac{\bar{\lambda}_n}{r^2}R_s+\mu_{s,n}R_s=0,\ 0<r<1, \tag{24}$$

$$R_s(1)=0,\ |R_s(0)|<\infty, \tag{25}$$

$$T_{st}-\mu_{s,n}g(t)T_s(t)=-a_{ns}^k(t),\ \beta<t<0, \tag{26}$$

$$T_s(\beta)=0. \tag{27}$$

A limited solution to problem (24), (25) is ([8])

$$R_s(r)=\sqrt{r}J_v(\mu_{s,n}r), \tag{28}$$

where $v=\dfrac{n+(m-2)}{2}$, $\mu_{s,n}$ - zeros of the Bessel function of the first kind $J_v(z)$, $\mu=\mu_{s,n}^2$.

The solution to problem (26), (27) is

$$T_{s,n}(t)=(\exp(-\mu_{s,n}^2\int_0^t g(\xi)d\xi))\int_t^\beta g(\xi)(\exp\mu_{s,n}^2\int_0^\xi g(\xi_1)d\xi_1)d\xi). \tag{29}$$

Substituting (28) into (23) we get

$$r^{-\frac{1}{2}}\tilde{f}_n^k(r,t)=\sum_{s=1}^\infty a_{ns}^k(t)J_v(\mu_{s,n}r),\ r^{-\frac{1}{2}}\tilde{\varphi}_{1n}^k(r)=\sum_{s=1}^\infty b_{ns}^k J_v(\mu_{s,n}r), 0<r<1. \tag{30}$$

Rows (30) - Fourier-Bessel series expansions ([9]), if

$$a_{ns}^k(t)=2[J_{v+1}(\mu_{s,n})]^{-2}\int_0^1\sqrt{\xi}\tilde{f}_n^k(\xi,t)J_v(\mu_{s,n}\xi)d\xi. \tag{31}$$

$$b_{ns}^k=2[J_{v+1}(\mu_{s,n})]^{-2}\int_0^1\sqrt{\xi}\tilde{\varphi}_{2n}^k(\xi)J_v(\mu_{s,n}\xi)d\xi, \tag{32}$$

where $\mu_{s,n}$ $s=1,2,...$ - positive zeros of the Bessel function $J_v(z)$, located in ascending order of magnitude.

Of (22), (28), (29) get the solution to the problem (18), (19)

$$\vartheta_{1n}^k(r,t)=\sum_{s=1}^\infty\sqrt{r}T_{s,n}(t)J_v(\mu_{s.n}r), \tag{33}$$

where $a_{ns}^k(t)$ determined from (31).

Next, substituting (22) в (20), (21), taking into account (23), will have

$$T_{st}-\mu_{s,n}^2 g(t)T_s=0,\ \beta<t<0, T_s(\beta)=b_{ns}^k,$$

which solution is

$$T_{s,n}(t)=b_{ns}^k\exp(\mu_{s,n}^2\int_t^\beta g(\xi)d\xi. \tag{34}$$

From (28), (34) we get

$$\vartheta_{2n}^k(r,t)=\sum_{s=1}^\infty b_{ns}^k\sqrt{r}(\exp\mu_{s,n}^2\int_t^\beta g(\xi)d\xi)J_v(\mu_{s,n}r), \tag{35}$$

where b_{ns}^k are from (32).

Therefore, first solving the problem (8), (12) (n=0), and then (9) ,(12) (n=1) etc. let's find everything $\vartheta_n^k(r,t)$ of (17), where $\vartheta_{2n}^k(r,t)$ are determined from (33),(35).

So, in the area D_α, takes place

$$\int_H\rho(\theta)L_1 u dH=0. \tag{36}$$

Let $f(r,\theta,t)=R(r)\rho(\theta)T(t)$, and $R(r)\in V_0$, V_0- tight in $L_2((0,1))$, $\rho(\theta)\in C^\infty(H)-$ tight in в $L_2(H)$, $T(t)\in V_1, V_1-$ tight in $L_2((\beta,0))$. Then $f(r,\theta,t)\in V, V=V_0\otimes H\otimes V_1-$ tight in $L_2(\Omega_\beta)$ [10].

From here and from (36), it follows that

$$\int_{\Omega_\beta} f(r,\theta,t) L_1 u d\Omega_\beta = 0$$

and

$$L_1 u = 0, \ \forall (r,\theta,t) \in \Omega_\beta .$$

Thus, by solving the problem (1), (3) in the field Ω_β is the function

$$u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{k=1}^{k_n}\{\psi_{2n}^k(t) + r^{\frac{(1-m)}{2}}[\vartheta_{1n}^k(r,t) + \vartheta_{2n}^k(r,t)]\} Y_{n,m}^k(\theta), \tag{37}$$

where $\vartheta_{1n}^k(r,t), \vartheta_{2n}^k(r,t)$ are from (33), (35).

Given the formula ([9]):

$$2J_v'(z) = J_{v-1}(z) - J_{v+1}(z), \text{ ratings [11,4]}$$

$$J_v(z) = \sqrt{\frac{2}{\pi z}}\cos(z - \frac{\pi}{2}v - \frac{\pi}{4}) + 0(\frac{1}{z^{3/2}}), \ v \geq 0,$$

$$|k_n| \leq c_1 n^{m-2}, \left|\frac{\partial^l}{\partial \theta_j^l} Y_{n,m}^k(\theta)\right| \leq c_2 n^{\frac{m}{2}-1+l}, j = \overline{1,m-1}, l = 0,1,..., \tag{38}$$

as well as lemmas, restrictions on the coefficients of equation (1) and on given functions $\varphi_1(r,\theta), \varphi_2(r,\theta), \psi_1(t,\theta), \psi_2(t,\theta)$ can be shown that received solution (37) belongs to the class $C^1(\bar{\Omega}_\beta) \cap C^2(\Omega_\beta)$.

$$u(r,\theta,0) = \tau(r,\theta) = \sum_{n=0}^{\infty}\sum_{k=1}^{k_n} \tau_n^k(r) Y_{n,m}^k(\theta), \tag{39}$$

$$\tau_n^k(r) = \psi_{2n}^k(0) + \sum_{s=1}^{\infty} r^{\frac{(2-m)}{2}} [\int_0^\beta a_{ns}^k(\xi)(\exp \mu_{s,n}^2 \int_0^\xi g(\xi_1)d\xi_1)d\xi + b_{ns}^k \exp(\mu_{s,n}^2 \int_0^\beta g(\xi)d\xi] J_v(\mu_{s,n} r).$$

From (30) - (33), (35), and also from the lemmas, it follows that $\tau(r,\theta) \in W_2^l(S), \ l > \frac{3m}{2}$.

Thus, taking into account the boundary conditions (2) and (39), we arrive at Ω_β to the Dirichlet problem for an elliptic equation.

$$L_2 u \equiv p_1(t)\Delta_x u + p_2(t) u_{tt} + \sum_{i=1}^{m} a_i(r,\theta,t) u_{x_i} + b(r,\theta,t) u_t + c(r,\theta,t) u = 0, \tag{40}$$

with data

$$u\big|_{S_0} = \tau(r,\theta), \ u\big|_{\Gamma_\alpha} = \psi_1(t,\theta), \ u\big|_{\sigma_\alpha} = \varphi_1(r,\theta), \tag{41}$$

having a solution ([12]).

Hence the solvability of the problem 1 is established.

The uniqueness of the solution to problem 1. First, we consider problem (1), (3) in the area Ω_β and prove its uniqueness of the solution. To do this, we first construct the solution of the first boundary value problem for the equation

$$L_1^*\vartheta \equiv g(t)\Delta_x\vartheta + \vartheta_t - \sum_{i=1}^{m} d_i\vartheta_{x_i} + d\vartheta = 0, \qquad (5^*)$$

with data

$$\vartheta\big|_s = \tau(r,\theta)\sum_{n=0}^{\infty}\sum_{k=1}^{k_n}\overline{\tau}_n^k(r)Y_{n,m}^k(\theta), \quad \vartheta\big|_{\Gamma_\beta} = 0, \qquad (42)$$

where $d(x,t) = e - \sum_{i=1}^{m} d_{ix_i}$, $\overline{\tau}_n^k(r) \in G, G$ - many functions $\tau(r)$ from the class $C([0,1]) \cap C^1(0,1))$. Lots of G tight everywhere in $L_2((0,1))$ [10]. The solution to the problem (5 *), (42) will be sought in the form (6), where the functions $\vartheta_n^k(r,t)$ will be defined below. Then, similarly to item 2. functions $\overline{\vartheta}_n^k(r,t)$ satisfy the system of equations (8)-(10), where $\overline{d}_{in}^k, d_{in}^k$ replaced respectively by $-\tilde{d}_{in}^k, -d_{in}^k$, a $\tilde{e}_n^k$ on $\tilde{d}_n^k$, $i = 1,...,m,\ k = \overline{1,k_n},\ n = 0,1,...$.

Further, from the boundary condition (42), by virtue of (6), we obtain

$$\overline{\vartheta}_n^k(r,\theta) = \overline{\tau}_n^k(r),\ \overline{\vartheta}_n^k(1,t) = 0,\ k = \overline{1,k_n},\ n = 0,1,... . \qquad (43)$$

As previously noted, each equation of system (8) - (10) is represented as (11). Problem (11), (43) we will result in the following problem.

$$L\vartheta_n^k = g(t)(\vartheta_{nrr}^k + \frac{\overline{\lambda}_n}{r^2}\vartheta_n^k) + \vartheta_{nt}^k = \tilde{f}_n^k(r,t), \qquad (15')$$

$$\vartheta_n^k(r,0) = \tau_n^k(r), \quad \vartheta_n^k(1,t) = 0, \qquad (44)$$

$$\vartheta_n^k(r) = r^{\frac{(m-1)}{2}}\overline{\vartheta}_n^k(r,t), \quad \tilde{f}_n^k(r,t) = r^{\frac{(m-1)}{2}}\overline{\tilde{f}}_n^k(r,t), \quad \tau_n^k(r) = r^{\frac{(m-1)}{2}}\overline{\tau}_n^k(r).$$

The solution to problem (15), (44) will be sought in the form (17), where $\vartheta_n^k(1,t)$ - solution of the problem for equation (18) with the data $\vartheta_{2n}^k(r,t)$

$$\vartheta_{1n}^k(r,0) = 0,\ \vartheta_{1n}^k(1,t) = 0, \qquad (45)$$

a - $\vartheta_{2n}^k(r,t)$ solution of the problem for equation (20) with the condition

$$\vartheta_{2n}^k(r,0) = 0, \quad \vartheta_{2n}^k(1,t) = 0, \qquad (46)$$

The solution of problems (18), (45) and (20), (46) respectively I have the form

$$\vartheta_{1n}^k(r,t) = \sum_{s=1}^{\infty}\sqrt{r}(\exp(+\mu_{s,n}^2\int_0^t g(\xi)d\xi))(\int_0^t a_{ns}^k(\xi)(\exp(\mu_{s,n}^2\int_0^\xi g(\xi_1)d\xi_1)J_v(\mu_{s,n}r),$$

where

$$\tau_{s,n} = 2[J_{v+1}(\mu_{s,n})]^{-2}\int_0^1\sqrt{\xi}\tau_n^k(\xi)J_v(\mu_{s,n}\xi)d\xi, \quad v = \frac{n+(m-2)}{2}.$$

Thus, solving the problem (5 *), (42) in the form of a series

$$u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{k=1}^{k_n} r^{\frac{(1-m)}{2}}[\vartheta_{1n}^k(r,t) + \vartheta_{2n}^k(r,t)]Y_{n,m}^k(0),$$

built, which by virtue of estimates (38) belongs to the class $C(\overline{\Omega}_\beta) \cap C^2(\Omega_\beta)$.

As a result of integration by area Ω_β identity [13]

$$\vartheta L_1 u - u L_1^* \vartheta = -\vartheta P(u) + u P(\vartheta) - u\vartheta Q,$$

where

$$P(u) = g(t)\sum_{i=1}^{m} u_{x_i}\cos(N^\perp, x_i),\ Q = \cos(N^\perp, t) - \sum_{i=1}^{m} d_i \cos(N^\perp, x_i),$$

but $N^\perp$ - internal normal to the border $\partial\Omega_\beta$, according to the Green formula we get

$$\int_S \tau(r,\theta)u(r,\theta,0)ds = 0. \tag{47}$$

Since the linear span of a system of functions $\{\overline{\tau}_n^k(r)Y_{n,m}^k(\theta)\}$ tight $L_2(S)$ ([10]), hen from (47) we conclude that $u(r,\theta,0) = 0,\ \forall (r,\theta) \in S.$. So on the principle of extremum for a parabolic equation (5) [14] $u \equiv 0$ в $\overline{\Omega}_\beta$.

Next, from the Hopf principle ([15]) $u \equiv 0$ в $\overline{\Omega}_\beta$.

The theorem is proven completely.

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КӨП-ӨЛШЕМДІ ЭЛЛИПТИКО-ПАРАБОЛАЛЫҚ ТЕҢДЕУЛЕРІНІҢ БІР КЛАСЫ БОЙЫНША ЦИЛИНДРЛІК ОБЛЫСЫНДА ДИРИХЛЕ ЕСЕБІ

Аннотация. Жазықтықтағы эллиптико-параболикалық теңдеулер үшін шеттік есептер өте жақсы зерттелген ([1]). Дирихле есебінің корректілігі түрі мен ретті алып тұратын көп өлшемді эллиптико-параболалық теңдеулер үшін орнатылған [3]. Көп-өлшемді эллиптико-параболалық теңдеулер үшін жұмыс істеу түрі мен ретті өзгертумен рұқсат етілген және Дирихле есебін классикалық шешудің айқын түрі алынған.

Түйін сөздер: шешімділігі, аралас есеп, көп өлшемді эллиптико-параболалық теңдеулер, Бессель функциясы.

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ЗАДАЧА ДИРИХЛЕ В ЦИЛИНДРИЧЕСКОЙ ОБЛАСТИ ДЛЯ ОДНОГО КЛАССА МНОГОМЕРНЫХ ЭЛЛИПТИКО-ПАРАБОЛИЧЕСКИХ УРАВНЕНИЙ

Аннотация. Краевые задачи для вырождающихся эллиптико-параболических уравнений на плоскости достаточно хорошо изучены ([1]). Корректность задачи Дирихле для вырожденных многомерных эллиптико-параболических уравнений с вырождением типа и порядка была установлена в [3]. В работе для многомерных эллиптико-параболических уравнений с вырождением типа и порядка показана разрешимость и получен явный вид классического решения задачи Дирихле.

Ключевые слова: разрешимость, смешанная задача, многомерные эллиптико-параболические уравнения, функция Бесселя.

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