DIRICHLET PROBLEM IN A CYLINDRICAL AREA FOR ONE CLASS OF MULTIDIMENSIONAL ELLIPTIC-PARABOLIC EQUATIONS

Abstract. Boundary-value problems for degenerate elliptic-parabolic equations on the plane are studied quite well ([11]). The correctness of the Dirichlet problem for degenerate multidimensional elliptic-parabolic equations with degeneration of type and order was established in [3]. In the work for multidimensional elliptic-parabolic equations with degeneration of type and order, the solvability is shown and an explicit form of the classical solution of the Dirichlet problem is obtained.

Keywords: solvability, mixed problem, multidimensional elliptic-parabolic equations, Bessel function.

Problem statement and result
Let \( \Omega_{\alpha \beta} \) – the cylindrical area of the Euclidean space of \( \mathbb{E}_{m+1} \) points \((x_1, ..., x_m, t)\) bounded by a cylinder \( \Gamma = \{(x, t) : |x| = 1\} \), planes \( t = \alpha > 0 \) and \( t = \beta < 0 \), where \(|x|\) is the length of a vector \( x = (x_1, ..., x_m) \).

Denote by \( \Omega_{\alpha} \) and \( \Omega_{\beta} \) parts \( \Omega_{\alpha \beta} \) – of the area and \( \Gamma_{\alpha}, \Gamma_{\beta} \) – through parts of the surface \( \Gamma \), lying in the half-spaces \( t > 0 \) and \( t < 0 \), \( \sigma_{\alpha} \) – the upper and \( \sigma_{\beta} \) – lower base area \( \Omega_{\alpha \beta} \).

Let \( S \) – further the common part of the borders of the areas \( \Omega_{\alpha} \) and \( \Omega_{\beta} \), representing the \( \{t = 0, 0 \leq |x| < 1\} \) set in \( \mathbb{E}_m \).

In the area \( \Omega_{\alpha \beta} \), we consider degenerate multidimensional hyperbolic-parabolic equations

\[
0 = \begin{cases} 
  p(t) \Delta_x u - p_2(t) u_t + \sum_{i=1}^{m} a_i(x, t) u_{x_i} + b(x, t) u_t + c(x, t) u = 0, & t > 0, \\
  g(t) \Delta_x u - u_t + \sum_{i=1}^{m} d_i(x, t) u_{x_i} + e(x, t) u_t = 0, & t < 0,
\end{cases}
\]

where \( p_i(t) > 0 \) at \( t > 0 \), \( p_i(0) = 0 \), \( p_i(t) \in C([0, \alpha]) \), \( g(t) > 0 \) at \( t < 0 \), and may vanish when \( t = 0 \), \( g(t) \in C(\beta, 0] \), a \( \Delta_x \) – Laplace operator with variables \( x_1, ..., x_m \), \( m \geq 2 \).

In the future, it is convenient for us to move from the Cartesian coordinates \( x_1, ..., x_m, t \) to spherical \( r, \theta_1, ..., \theta_{m-1}, t, r \geq 0, 0 \leq \theta_i \leq \pi, i = 1, 2, ..., m - 2, \theta = \theta_1, ..., \theta_{m-1} \).

Problem 1 (Dirichlet). Find a solution to the equation (1) in the area of \( \Omega_{\alpha \beta} \) at \( t \neq 0 \), from the class \( C^1(\overline{\Omega_{\alpha \beta}}) \cap C^2(\Omega_{\alpha} \cup \Omega_{\beta}) \), satisfying boundary conditions

\[
u_{x_\alpha} = \varphi(r, \theta), \quad u_{x_\beta} = \psi_1(t, \theta),
\]

89
\[ u|_{\sigma} = \psi_{e}(t, 0), \quad u|_{\gamma} = \psi_{e}(t, \theta). \] \tag{3}

wherein \( \psi_{i}(1, \theta) = \psi_{i}(\alpha, \theta), \psi_{i}(0, \theta) = \psi_{i}(0, \theta), \psi_{i}(\beta, \theta) = \psi_{i}(1, \theta). \)

Let \( \{Y^{r}_{n, \alpha}(\theta)\} \) - system of linearly independent spherical functions of order \( n, 1 \leq k \leq k_{n}, (m - 2)!n!k_{n} = (n + m - 3)!(2n + m - 2), W_{2}^{l}(S), l = 0, 1, \ldots \) - Sobolev space. Takes place (14).

**Lemma 1.** Let \( f(r, \theta) \in W_{2}^{l}(S). \) If \( l \geq m - 1, \) that row
\[ f(r, \theta) = \sum_{n=0}^{\infty} \sum_{k=1}^{k_{n}} f^{k}_{n}(r) Y^{r}_{n, \alpha}(\theta), \] \tag{4}
as well as series derived from it by order differentiation \( p \leq l - m + 1, \) converge absolutely and evenly.

**Lemma 2.** In order to \( f(r, \theta) \in W_{2}^{l}(S), \) it is necessary and sufficient that the coefficients of the series (4) satisfy the inequalities
\[ |f_{n}(r)| \leq c_{1}, \sum_{n=0}^{\infty} \sum_{k=1}^{k_{n}} n^{2p} |f^{k}_{n}(r)|^{2} \leq c_{2}, c_{1}, c_{2} = \text{const}. \]

Through \( \tilde{d}_{n}(r, t), d_{m}(r, t), \tilde{e}_{n}(r, t), \tilde{e}_{m}(r, t), e_{n}(r, t), \tilde{e}_{m}(r, t), e_{n}(r, t), e_{m}(r, t), \) denote the coefficients of the series (4), respectively functions \( d_{l}(r, \theta, t) \rho(\theta), d_{l} \int \rho(\theta), e(r, \theta, t) \rho, d(r, \theta, t) \rho, \rho(\theta), i = 1, \ldots, m, \varphi_{1}(r, \theta), \varphi_{2}(r, \theta), \psi_{1}(t, \theta), \psi_{2}(t, \theta), \) and \( \rho(\theta) \in C^{r}(H), \) \( H \) - unit sphere in \( E_{3}. \)

Let \( a_{n}(r, \theta, t), b_{n}(r, \theta, t), c(r, \theta, t) \in W_{2}^{l}(\Omega_{\alpha}) \subset C(\overline{\Omega_{\alpha}}), d_{l}(r, \theta, t), c(r, \theta, t) \in W_{2}^{l}(\Omega_{\beta}), i = 1, \ldots, m, l \geq m + 1, c(r, \theta, t) \leq 0, \forall (r, \theta, t) \in \Omega_{\alpha}, e(r, \theta, t) \in \Omega_{\beta}. \)

Then fair

Thorem.

If \( \varphi_{1}(r, \theta), \varphi_{2}(r, \theta) \in W_{2}^{l}(S), \psi_{1}(t, \theta) \in W_{2}^{l}(\Gamma_{\alpha}), \psi_{2}(t, \theta) \in W_{2}^{l}(\Gamma_{\beta}), l > \frac{3m}{2}, \) then

problem 1 is solvable.

**Proof of the theorem.** First, let us rock the solvability of problem (1), (3). In spherical coordinates of equation (1) in the area \( \Omega_{\beta} \) has the appearance
\[ Lu = g(t)(u_{rr} + \frac{m+1}{r} u_{r} - \frac{1}{r^{2}} \partial u) - u_{rr} + \sum_{i=1}^{\infty} d_{i}(r, \theta, t) u_{x_{i}} + e(r, \theta, t) u = 0, \] \tag{5}

\[ \delta = -\sum_{j=1}^{\infty} \frac{1}{g_{j}} \sin^{m-j-1} \theta \frac{\partial}{\partial \theta_{j}} \frac{\partial}{\partial \theta_{j}} (\sin^{m-j-1} \theta \frac{\partial}{\partial \theta_{j}}) g_{1} = 1, g_{j} = (\sin \theta_{1} \ldots \sin \theta_{j-1})^{2}, j > 1. \]

It is known [4] that the spectrum of the operator \( \delta \) consists of own numbers \( \lambda_{n} = n(n + m - 2), n = 0, 1, \ldots \) each of which corresponds \( k_{n} \) orthonormal functions \( Y^{r}_{n, \alpha}(\theta). \)

The desired solution to problem 1 in the field \( \Omega_{\beta} \) we will look in the form
\[ u(r, \theta, t) = \sum_{n=0}^{\infty} \sum_{k=1}^{k_n} \bar{u}^k_n(r, t) Y_{n,m}^k(\theta), \]  

(6)

where \( \bar{u}^k_n(r, t) \) - functions to be defined.

Substituting (6) in (5), then multiplying the resulting expression by \( \rho(\theta) \neq 0 \), and integrating over a single sphere \( H \), for \( \bar{u}^k_n \) will get [5-7]

\[ g(t) \rho^k_0 \bar{u}^k_{0r} + \rho^k_0 \bar{u}^k_{0t} + \left( \frac{m-1}{r} \right) g(t) \rho^k_0 + \sum_{i=1}^{m} d_{i0}^k \bar{u}^k_{i0r} + \]
\[ + \sum_{n=1}^{\infty} \sum_{k=1}^{k_n} \{ g(t) \rho^k_n \bar{u}^k_{nr} + \rho^k_n \bar{u}^k_{nt} + \left( \frac{m-1}{r} \right) g(t) \rho^k_n + \sum_{i=1}^{m} d_{in}^k \bar{u}^k_{nr} \} + \]
\[ + \left[ \bar{\varepsilon}^k_n - \lambda_n \frac{L_n^k}{r^2} g(t) + \sum_{i=1}^{m} (\bar{d}^k_{i-1} - nd^k_{i}) \bar{u}^k_{nt} \right] = 0. \]  

(7)

Now consider the infinite system of differential equations

\[ g(t) \rho^k_0 \bar{u}^k_{0r} + \rho^k_0 \bar{u}^k_{0t} + \frac{m-1}{r} g(t) \rho^k_0 \bar{u}^k_{0r} = 0, \]  

(8)

\[ g(t) \rho^k_1 \bar{u}^k_{1r} - \rho^k_1 \bar{u}^k_{1t} + \frac{m-1}{r} g(t) \rho^k_1 \bar{u}^k_{1r} - \frac{\lambda_1}{r^2} g(t) \rho^k_1 \bar{u}^k_{1t} = \]
\[ = - \frac{1}{k_1} \left( \sum_{i=1}^{m} d_{i0}^k \bar{u}^k_{i0r} + \bar{\varepsilon}^k_0 \bar{u}^k_{0r} \right), \quad n = 1, k = \bar{1}, k_1, \]  

(9)

\[ g(t) \rho^k_n \bar{u}^k_{nr} - \rho^k_n \bar{u}^k_{nt} + \frac{m-1}{r} g(t) \rho^k_n \bar{u}^k_{nr} - \frac{\lambda_n}{r^2} g(t) \rho^k_n \bar{u}^k_{nt} = \]
\[ = - \frac{1}{k_n} \sum_{k=1}^{k_n} \left\{ \sum_{i=1}^{m} d_{in-1}^k \bar{u}^k_{i(n-1)r} + [\bar{\varepsilon}^k_{n-1} + \sum_{i=1}^{m} (\bar{d}^k_{i(n-2)} - (n-1) \bar{d}^k_{i(n-1)})] \bar{u}^k_{n-1r} \right\}, \]
\[ k = \bar{1}, k_n. \quad n = 2, 3, \ldots. \]  

(10)

Summing up the equation (8) from 1 before \( k_1 \), and the equation (9) from 1 before \( k_n \), and then adding the resulting expressions together with (7), come to the equation (6).

It follows that if \( \{ \bar{u}^k_n \}, k = \bar{1}, k_n, n = 0, 1, \ldots \) system solution (7)-(9), then it is a solution to the equation (6).

It is easy to see that each equation of system (7)-(9) can be represented as

\[ g(t)(\bar{u}^k_{nr} + \frac{m-1}{r} \bar{u}^k_{nt} - \frac{\lambda_n}{r^2} \bar{u}^k_n) - \bar{u}^k_{nt} = \tilde{f}^k_n(r, t), \]  

(11)

where \( \tilde{f}^k_n(r, t) \) are determined from the previous equations of this system, while \( \tilde{f}^0_n(r, t) \equiv 0 \).

Further, from the boundary condition (3), by virtue of (6), we will have

\[ \bar{u}^k_n(r, \beta) = \bar{g}^k_{2n}(r), \quad \bar{u}^k_n(1, t) = \psi^k_{2n}(t), \quad k = \bar{1}, k_n, n = 0, 1, \ldots. \]  

(12)

In (11), (12) replacing \( \bar{g}^k_n(r, t) = \bar{u}^k_n(r, t) - \psi^k_{2n}(t), \) will get

\[ g(t)(\bar{g}^k_{nr} + \frac{m-1}{r} \bar{g}^k_{nt} - \frac{\lambda_n}{r^2} \bar{g}^k_n) - \bar{g}^k_{nt} = \tilde{f}^k_n(r, t), \]  

(13)
\[
\begin{align*}
\tilde{G}^k_n(r, \beta) &= \phi^k_n(r), \quad \tilde{G}^k_n(1, t) = 0, \quad k = 1, n, \quad n = 0, 1, \ldots \\

f_n^k(r, t) &= \tilde{f}_n^k(r, t) + \psi_n^k + \frac{\lambda_n g(t)}{r^2} \psi_n^k, \quad \phi_n^k(r) = \tilde{\phi}_n^k(r) - \psi_n^k, (\beta).
\end{align*}
\]

Replacing the variable \( \tilde{G}^k_n(r, t) = r^{(1-m) \frac{(1-m)}{2}} g_n^k(r, t) \) we will lead to the following problem

\[
L_n \phi_n^k = g(t)\left(\phi^k + \frac{\lambda_n}{r^2} \phi_n^k\right) - \phi_n^k = \tilde{f}^k_n(r, t),
\]

\[
\tilde{G}_n^k(r, \beta) = \tilde{\phi}_n^k(r), \quad \tilde{G}_n^k(1,t) = 0, \quad \tilde{G}_n^k(1,t) = 0,
\]

\[
\tilde{\lambda}_n = \frac{[(m-1)(3-m) - 4\lambda_n]}{4}, \quad \tilde{f}_n^k(r, t) = r^{(m-1) \frac{(m-1)}{2}} f_n^k(r, t),
\]

\[
\tilde{\phi}_n^k(r) = r^{(m-1) \frac{(m-1)}{2}} \phi_n^k(r).
\]

The solution of the problem (15), (16) is sought in the form

\[
G_n^k(r, t) = G_{1n}^k(r, t) + G_{2n}^k(r, t),
\]

where \( G_{1n}^k(r, t) \) the solution of the problem

\[
L_n \phi_{1n}^k = \tilde{f}_n^k(r, t),
\]

\[
G_{1n}^k(r, \beta) = 0, \quad G_{1n}^k(1,t) = 0,
\]

where \( G_{2n}^k(r, t) \) the solution of the problem

\[
L_n \phi_{2n}^k = 0,
\]

\[
G_{2n}^k(r, \beta) = \tilde{\phi}_{2n}^k(r), \quad G_{2n}^k(1,t) = 0,
\]

The solution to the above problems, we consider in the form

\[
G_n^k(r, t) = \sum_{s=1}^{\infty} R_s(T_s(t)),
\]

at the same time let

\[
\tilde{f}_n^k(r, t) = \sum_{s=1}^{\infty} \alpha_n^k(T_s(r)), \quad \tilde{\phi}_n^k(r) = \sum_{s=1}^{\infty} \beta_n^k R_s(r).
\]

Substituting (22) into (18), (19), taking into account (23), we obtain

\[
R_{sr} + \frac{\lambda_n}{r^2} R_s + \mu_{sr} R_s = 0, \quad 0 < r < 1,
\]

\[
R_s(1) = 0, \quad |R_s(0)| < \infty,
\]

\[
T_{sr} - \mu_{sr} g(t) T_s(t) = -\alpha_n^k(t), \quad \beta < t < 0,
\]

\[
T_s(\beta) = 0.
\]
A limited solution to problem \((24), (25)\) is \((28)\)

\[
R_n(r) = \sqrt{r} J_v(\mu, r),
\]

(28)

where \(v = \frac{n + (m - 2)}{2}\), \(\mu_{s,n}\) - zeros of the Bessel function of the first kind \(J_v(z)\), \(\mu = \mu_{s,n}^2\).

The solution to problem \((26), (27)\) is

\[
T_{s,n}(t) = (\exp(-\mu_{s,n}^2 \int_0^t g(\xi) d\xi))^{1/2} \sum_{i=1}^{\infty} a_i^{s,n}(t) J_v(\mu_{s,n} r), \quad T_{s,n}(t) = (\exp(-\mu_{s,n}^2 \int_0^t g(\xi) d\xi))^{1/2} \sum_{i=1}^{\infty} b_i^{s,n} J_v(\mu_{s,n} r), 0 < r < 1.
\]

(29)

(30)

Rows \((30)\) - Fourier-Bessel series expansions \((9)\), if

\[
a_i^{s,n}(t) = 2 [J_{v+1}(\mu_{s,n})]^2 \int_0^1 \sqrt{\xi} f_{s,n}(\xi) J_v(\mu_{s,n} \xi) d\xi,
\]

(31)

\[
b_i^{s,n} = 2 [J_{v-1}(\mu_{s,n})]^2 \int_0^1 \sqrt{\xi} \tilde{f}_{s,n}(\xi) J_v(\mu_{s,n} \xi) d\xi,
\]

(32)

where \(\mu_{s,n}, s = 1, 2, \ldots\) positive zeros of the Bessel function \(J_v(z)\), located in ascending order of magnitude.

Of \((22), (28), (29)\) get the solution to the problem \((18), (19)\)

\[
f_n(t) = \sum_{s=1}^{\infty} \sqrt{r} T_{s,n}(t) J_v(\mu_{s,n} r),
\]

(33)

where \(a_i^{s,n}(t)\) determined from \((31)\).

Next, substituting \((22)\) \& \((20), (21)\), taking into account \((23)\), will have

\[
T_{s,n} - \mu_{s,n}^2 g(t)T_s = 0, \quad \beta < t < 0, T_s(\beta) = b_n^{s,n},
\]

which solution is

\[
T_{s,n}(t) = b_n^{s,n} \exp(\mu_{s,n}^2 \int_0^t g(\xi) d\xi).
\]

(34)

From \((28), (34)\) we get

\[
f_n(t) = \sum_{s=1}^{\infty} \sqrt{r} (\exp(\mu_{s,n}^2 \int_0^t g(\xi) d\xi)) J_v(\mu_{s,n} r),
\]

(35)

where \(b_n^{s,n}\) are from \((32)\).

Therefore, first solving the problem \((8), (12)\) \((n=0)\), and then \((9), (12)\) \((n=1)\) etc. let's find everything \(f_n(t)\) \((17)\), where \(f_n(t)\) are determined from \((33), (35)\).

So, in the area \(D_n\), takes place

\[
\int_{H} \rho(\theta) L u dH = 0.
\]

(36)

Let \(f(r, \theta, t) = R(r, \rho(\theta) T(t))\), and \(R(r) \in V_0, V_0 - \text{tight in } L_2((0, 1))\), \(\rho(\theta) \in C^\infty(H) - \text{tight in } \mathcal{L}_2(H)\), \(T(t) \in \mathcal{L}_1, V_1 - \text{tight in } \mathcal{L}_2((\beta, 0))\). Then \(f(r, \theta, t) \in V, V = V_0 \otimes H \otimes V_1 - \text{tight in } \mathcal{L}_2(\Omega_p)\) \([10]\).
From here and from (36), it follows that

$$\int_{\Omega_{\beta}} f(r, \theta, t) L u d\Omega = 0$$

and

$$L u = 0, \quad \forall (r, \theta, t) \in \Omega_{\beta}.$$ 

Thus, by solving the problem (1), (3) in the field $\Omega_{\beta}$ is the function

$$u(r, \theta, t) = \sum_{n=0}^{\infty} \sum_{k=1}^{k_{n}} \left\{ \psi_{2n}^{k}(t) + r^{\frac{1-m}{2}} \left[ \mathcal{G}_{1n}(r,t) + \mathcal{G}_{2n}(r,t) \right] \right\} Y_{n,m}^{k}(\theta),$$

(37)

where $\mathcal{G}_{1n}(r,t), \mathcal{G}_{2n}(r,t)$ are from (33), (35).

Given the formula ([9]):

$$2J_{v}(z) = J_{v-1}(z) - J_{v+1}(z), \quad \text{ratings} \ [11,4]$$

$$J_{v}(z) = \frac{2}{\pi z} \cos \left( z - \frac{v}{2} \right) + \frac{1}{z^{2}}, \quad v \geq 0,$$

$$|k_{n}| \leq c_{n} r^{m-2} \left| \frac{\partial^{l}}{\partial \theta^{l}} Y_{n,m}^{k}(\theta) \right| \leq c_{n} r^{m-1}, \quad j = 1, m-1, l = 0, 1,...,$$

(38)

as well as lemmas, restrictions on the coefficients of equation (1) and on given functions $\varphi_{1}(r, \theta), \varphi_{2}(r, \theta), \psi_{1}(t, \theta), \psi_{2}(t, \theta)$ can be shown that received solution (37) belongs to the class $C^{1}(\Omega_{\beta}) \cap C^{2}(\Omega_{\beta}).$

$$u(r, \theta, 0) = \tau(r, \theta) = \sum_{n=0}^{\infty} \sum_{k=1}^{k_{n}} \tau_{n}^{k}(r) Y_{n,m}^{k}(\theta),$$

(39)

$$\tau_{n}^{k}(r) = \psi_{2n}^{k}(0) + \sum_{j=3}^{\infty} r^{2j-2} \left[ a_{n,j}(\xi) \exp \left( \mu_{n,j}(\xi) \right) \int_{0}^{\mu} g(\xi) d\xi \right] J_{j}(\mu_{n,j} r).$$

From (30) - (33), (35), and also from the lemmas, it follows that $\tau(r, \theta) \in W^{l}(S), \ l \geq \frac{3m}{2}.$

Thus, taking into account the boundary conditions (2) and (39), we arrive at $\Omega_{\beta}$ to the Dirichlet problem for an elliptic equation.

$$L u = p_{1}(t) \Delta u + p_{2}(t) u_{t} + \sum_{i=1}^{m} a_{i}(r, \theta, t) u_{t_{i}} + b(r, \theta, t) u_{t} + c(r, \theta, t) u = 0,$$

(40)

with data

$$u|_{\sigma_{1}} = \tau(r, \theta), \quad u|_{\Gamma_{u}} = \psi_{1}(t, \theta), \quad u|_{\sigma_{a}} = \varphi_{1}(r, \theta),$$

(41)

having a solution ([12]).

Hence the solvability of the problem 1 is established.

**The uniqueness of the solution to problem 1.** First, we consider problem (1), (3) in the area $\Omega_{\beta}$ and prove its uniqueness of the solution. To do this, we first construct the solution of the first boundary value problem for the equation
\[ L^* \mathcal{G} = g(t) \Delta \mathcal{G} + \mathcal{G}_t - \sum_{i=1}^{m} d_i \mathcal{G}_{\xi_i} + d \mathcal{G} = 0, \]  

(5*)

with data

\[ \mathcal{G} |_{x} = \tau(r, \theta) \sum_{n=1}^{k} \sum_{m=1}^{k} \tilde{\tau}^k_n(r) V^k_n(\theta), \quad \mathcal{G} |_{\Gamma_p} = 0, \]  

(42)

where \( d(x,t) = e - \sum_{i=1}^{m} d_i \), \( \tilde{\tau}^k_n(r) \in G, G \) - many functions \( \tau(r) \) from the class \( C([0,1]) \cap C^1(0,1) \). Lots of \( G \) tight everywhere in \( L_2((0,1)) \) [10]. The solution to the problem (5*), (42) will be sought in the form (6), where the functions \( \mathcal{G}^k_n(r,t) \) will be defined below. Then, similarly to item 2. functions \( \mathcal{G}^k_n(r,t) \) satisfy the system of equations (8)-(10), where \( \tilde{d}^k_m, \tilde{d}_m^k \) replaced respectively by \( -\tilde{d}^k_m, -\tilde{d}_m^k \), a \( \tilde{\tau}^k_n \) on \( \tilde{d}^k_m, i = 1, ..., m, k = 1, k_n, n = 0, 1, ... \).

Further, from the boundary condition (42), by virtue of (6), we obtain

\[ \mathcal{G}^k_n(r, \theta) = \tilde{\tau}^k_n(r), \quad \mathcal{G}^k_n(1,t) = 0, \quad k = 1, k_n, n = 0, 1, ... . \]  

(43)

As previously noted, each equation of system (8) - (10) is represented as (11). Problem (11), (43) we will result in the following problem.

\[ L \mathcal{G}^k_n = g(t) (\mathcal{G}^k_n + \frac{\bar{\tau}^k_n}{r^2} \mathcal{G}^k_n) + \mathcal{G}^k_n \mathcal{G}_t = \tilde{f}^k_n(r,t), \]  

(15*)

\[ \mathcal{G}^k_n(r, 0) = \tilde{\tau}^k_n(r), \quad \mathcal{G}^k_n(1,t) = 0, \]  

(44)

\[ \mathcal{G}^k_n(r) = \frac{(m-1)}{2} \mathcal{G}^k_n(r,t), \quad \tilde{f}^k_n(r,t) = \frac{(m-1)}{2} \tilde{f}^k_n(r,t), \quad \tilde{\tau}^k_n(r) = \frac{(m-1)}{2} \tilde{\tau}^k_n(r). \]

The solution to problem (15), (44) will be sought in the form (17), where \( \mathcal{G}^k_n(1,t) \) - solution of the problem for equation (18) with the data \( \mathcal{G}^k_n(1,t) \)

\[ \mathcal{G}^k_{ln}(r, 0) = 0, \quad \mathcal{G}^k_{ln}(1,t) = 0, \]  

(45)

and - \( \mathcal{G}^k_{ln}(r,t) \) solution of the problem for equation (20) with the condition

\[ \mathcal{G}^k_{ln}(r, 0) = 0, \quad \mathcal{G}^k_{ln}(1,t) = 0, \]  

(46)

The solution of problems (18), (45) and (20), (46) respectively I have the form

\[ \mathcal{G}^k_{ln}(r,t) = \sum_{s=1}^{\infty} \sqrt{r} \left( \exp(\mu_\gamma^2) \int_0^1 g(\xi) d\xi \right) \left( \int_0^1 \mu_\gamma^2 \int_0^1 g(\xi) d\xi \right) J_v(\mu_\gamma^2 r), \]  

where

\[ \tau_{s,n} = 2 \left[ J_{\nu+1}(\mu_\gamma^2) \right]^2 \int_0^1 \sqrt{x} \left( \tau^k_n(\xi) J_v(\mu_\gamma^2) \xi \right) d\xi, \quad v = \frac{n + (m-2)}{2}. \]

Thus, solving the problem (5*), (42) in the form of a series

\[ u(r, \theta, t) = \sum_{n=0}^{\infty} \sum_{k=1}^{k_n} r \left( \frac{1}{2} \left[ \mathcal{G}^k_{ln}(r,t) + \mathcal{G}^k_{ln}(r,t) \right] Y^k_{n,m}(0), \right. \]

built, which by virtue of estimates (38) belongs to the class \( C(\Omega_n) \cap C^2(\Omega_n) \).
As a result of integration by area $\Omega_\beta$ identity [13]

$$\partial L_x u - u \partial L_x \theta = -\partial P(u) + u P(\theta) - u \partial Q,$$

where

$$P(u) = g(t) \sum_{i=1}^{m} u_i \cos(N^i, x_j), \quad Q = \cos(N^i, t) - \sum_{i=1}^{m} d_i \cos(N^i, x_j),$$

but $N^\perp$ - internal normal to the border $\partial \Omega_\beta$, according to the Green formula we get

$$\int_S \tau(r, \theta)u(r, \theta, 0)ds = 0. \quad (47)$$

Since the linear span of a system of functions $\{h(r, \theta)Y_{n,m}(\theta)\}$ tight $L_2(H) ([10])$, then from (47) we conclude that $u(r, \theta, 0) = 0, \forall (r, \theta) \in S$. So on the principle of extremum for a parabolic equation (5) [14] $u \equiv 0$ in $\Omega_\beta$.

Next, from the Hopf principle ([15]) $u \equiv 0$ in $\Omega_\beta$.

The theorem is proven completely.

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КОП-ОЛШЕМДІ ЭЛЛИПТИКО-ПАРАБОЛЫҚ ТЕНДЕУЕРІНІҢ БІР КЛАСЫ БОЙЫНША ЦИЛИНДРІК ОБЛЫСЫНДА ДИРИХЛЕ ЕСЕБІ

Анализация. Жазықтықтан эллиптико-параболик тендеулер ушін шеттік есептер әте жақсы зерттелген ([11]). Дирихле есебінің коректілігі тури мен ретті алып тұратын көп олшемді эллиптико-параболик тендеулер ушін орнамыми [3]. Көп-oлшемді эллиптико-параболик тендеулер ушін жақсы істен тури мен ретті озгертумен әрекет етілген жаңа Дирихле есебін классикалық шешуден айырмай ықтималдығы түрі алынады.

Түйін сөз: шешімділігі, аралас есеп, көп олшемді эллиптико-параболик тендеулер, Бессель функциясы.

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ЗАДАЧА ДИРИХЛЕ В ЦИЛИНДРИЧЕСКОЙ ОБЛАСТИ ДЛЯ ОДНОГО КЛАССА МНОГОМЕРНЫХ ЭЛЛИПТИКО-ПАРАБОЛИЧЕСКИХ УРАВНЕНИЙ

Аннотация. Краевые задачи для вырождающихся эллиптико-параболических уравнений на плоскости достаточно хорошо изучены ([11]). Корректность задачи Дирихле для вырожденных многомерных эллиптико-параболических уравнений с вырожденным типом и порядка была установлена в [3]. В работе для многомерных эллиптико-параболических уравнений с вырождением типа и порядка показана разрешимость и получена явный вид классического решения задачи Дирихле.

Ключевые слова: разрешимость, смешанная задача, многомерные эллиптико-параболические уравнения, функция Бесселя.
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