N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2019.2518-1726.54**
Volume 5, Number 327 (2019), 5 – 10

UDC 524.8

L.M. Chechin[1,2], E.B. Kurmanov[2], T.K. Konysbayev[1]

[1]V.G. Fesenkov Astrophysical Institute “NCSRT” NSA RK, 050020, Observatory 23, Kamenskoe plato, Almaty, Kazakhstan;
[2]Al-Farabi Kazakh National University, Physics and Technology Department, Almaty, Kazakhstan
chechin-lm@mail.ru, ergaly_90@mail.ru, talgar_777@mail.ru

LIGHT RAYS IN THE EPOCH OF DARK MATTER DOMINATION

Abstract. The cosmological Friedmann model has been generalized for the epoch of dark matter domination. In doing this its equation of state was chosen in the new - non-stationary form. The process of light propagation in such metric was explored and its refractive index was found.

Key words: Friedmann cosmology, dark matter, non-stationary equation of state, Mendeleev – Clapeyron equation, gravitational lenses.

Introduction

One of the actual problems of modern cosmology, as well as particle physics, is understanding the physical properties of dark matter. For this, in particular, the astronomical observations can be used. They demonstrate that dark matter is concentrated mainly around epy large-scale space objects such as galaxies and their clusters, and form the corresponding halos [1,2].

Dark matter is also described in global aspect, since in the substantial structure of the Universe it holds the second place (after dark energy) and amounts to 26.8% [3, 4]. Moreover, it dominates during the period up to six and a half - seven billion years. At a later time, the main role in the Universe evolution takes the cosmic vacuum. The dynamics of light in the era of cosmic vacuum is quite well studied [4]. Therefore, an essentially important problem of modern cosmology is study the process of light propagation at the epoch of dark matter domination.

Talking about the physical properties of dark matter, we will declare about two important aspects of it – the carriers of dark matter and the corresponding equations of state of a medium.

The aim of present article is studying the process of light propagation in the era of domination of dark matter, described by the non-stationary equation of state.

1 Non-stationary equation of state of dark matter

According [5], they are similar to neutrinos and antineutrinos, but should be more massive. Such hypothetical heavy particles (with a mass of order $1.0\,TeV$ and more) are called WIMPs. Their peculiarity is the absence of the effect of annihilation, so that they can appear after freezing at an early time. Therefore, all our calculations can be applied to the gas of WIMPs [6,7].

We emphasize that the equation of state of dark matter was even measured in [7] and found that it corresponds to a medium with vanishingly low pressure, for example, nonrelativistic or relativistic gases.

So, we consider a medium of WIMPs as a relativistic ideal gas described by the Mendeleyev-Clapeyron equation of state. By virtue of thermodynamic equilibrium of such particles with cosmic plasma particles, the approximate condition $T_{DM} \approx T_{BM}$ holds. Therefore, the Mendeleev – Clapeyron equation takes the form

$$P_{DM} = \rho_{DM} \frac{R}{\mu} T_{BM}, \qquad (1)$$

with explicit dependency of gas density ρ_{DM}.

For the standard Friedmann cosmological model filled by relativistic gas, the approximate relation linking the temperature of such gas with the age of the Universe holds. It, as shown in [8], is as follows

$$T_{BM} \sim t^{-1/2}. \qquad (2)$$

where t - current time. Therefore, it follows from (1) and (2) that the equation of state of an ideal gas takes on the form $P_{DM} = \bar{\omega}_{DM}(t)\rho_{DM} = \rho_{DM} \frac{R}{\mu} T_{BM}(t)$. So, taking into account (2), its state parameter depends on time in the similar way, i.e.

$$\bar{\omega}_{DM}(t) = \frac{R}{\mu} T_{BM}(t) \sim t^{-1/2}. \qquad (3)$$

Let us consider the case in which the Universe is filled with real gas, consisting of N molecules and described by the van der Waals equation of state. If the temperature is measured in degrees, then, according to [9], the equation takes the form

$$P_{DM}\left(1 + v^2 \frac{\tilde{a}}{P_{DM} V^2}\right)\left(1 - v\frac{\tilde{b}}{V}\right) = \rho_{DM} \frac{R}{\mu} T_{BM}, \qquad (4)$$

in which $\tilde{a}$ and $\tilde{b}$ are constant quantities describing the properties of WIMP's gas, k is the Boltzmann constant. Recall that the physical meaning of parameter $\tilde{a}$ is in describing the interaction of matter's molecules, parameter $\tilde{b}$ is responsible for accounting their sizes. In addition, here μ is the molar mass of a particular matter, and R is the universal gas constant.

Now our task is to combine (4) with (3) and finding the explicit dependency on time the state parameter of the real gas.

For discussion, we assume that $N\tilde{b}/V << 1$. Such condition describes the real property of WIMP's gas, whose volume is significantly larger than the size of all molecules themselves. In addition, assume that the interaction of molecules is not too large. This corresponds to the situation when the requirement $N^2\tilde{a}/V^2 P_{DM} << 1$ takes the place.

Having in mind these considerations and taking into account the equation of state of an ideal gas (1), we have

$$P_{DM} = \rho_{DM} \frac{R}{\mu}\left(1 - v^2 P_{DM}\tilde{a} \Big/ \frac{m^2}{\mu^2} R^2 T_{BM}{}^2\right) T_{BM}. \qquad (5)$$

In an ultra-relativistic hot gas – baryonic matter, the pressure is proportional to its temperature in fourth power [10], i.e.

$$P_{DM}(T) = \frac{\pi^2}{90} n(T_{DM}) T_{DM}{}^4 \approx \frac{\pi^2}{90} n(T_{BM}) T_{BM}{}^4, \qquad (6)$$

where $n(T_{BM})$ is the effective number of types of particles (bosons and fermions) in different quantum states. Moreover, in realistic theories of elementary particles, as is known, it has an upper limit - $n(T_{BM}) = n_0 \le 10^4$. So the relation (6) can be rewritten in the form of a power-law function with a constant coefficient $\zeta = (\pi^2/90) n_0$. Thus,

$$P_{DM}(T) = \zeta T_{BM}{}^4. \qquad (7)$$

Applying (7) to (5), we find the state parameter as a function of temperature

$$\omega_{DM}\left(T_{BM}\right)=\bar{\omega}_{DM}\left(T_{BM}\right)+\omega'_{DM}\left(T_{BM}\right)=\frac{R}{\mu}\left(1-v^2\tilde{a}\zeta\frac{\mu^2}{m^2R^2}T_{BM}{}^2\right)T_{BM}\ , \tag{8}$$

or as a function of time with two different terms

$$\omega_{DM}\left(t\right)=\begin{matrix}\bar{\omega}_{DM}\left(t\right)\sim t^{-1/2}>0\\ \omega'_{DM}\left(t\right)\sim t^{-3/2}>0\end{matrix}\ . \tag{9}$$

2 The solution of Friedmann equations for the end of domination of dark matter

Recall that the Friedmann equations connect the expansion parameter of the Universe (namely, Hubble parameter $H=\dot{a}/a$) with a density of matter contained in it (evolution equation)

$$\frac{1}{2}\left(\frac{da}{dt}\right)^2-\frac{4\pi G}{3}\rho\left(t\right)a^2=0, \tag{10}$$

and their time with a similar expression,

$$\dot{\rho}+3\frac{\dot{a}}{a}\left(\rho+P\right)=\dot{\rho}+3\frac{\dot{a}}{a}\rho\left(1+\omega\left(t\right)\right)=0, \tag{11}$$

represented the law of energy conservation.

However, the question arises here: is it possible to use a non-static medium in Friedmann's static equations? For example the medium with non-stationary equation of state.

According to Tolman [11] with reference to Lemetre's research, the non-static spherical interval differs from the static case by the presence of mixed terms in the energy-momentum tensor of the gravitating medium. In the general case, they correspond to the appearance of transverse waves related with radial mass flows. How this conclusion relates with properties of the gas of WIMPs considering by us?

We emphasize once again that they represent massive particles, a priori moving with velocity much lower than the velocity of light, i.e. $v_{WIMP}<<c$. This factor gives possibility to neglect the flows of matter in the gas of WIMPs and, therefore, to confidently use the proposed energy-momentum tensor to study the cold (ideal) dark matter.

So, we pass directly to the solution of the Friedmann equations. In our case, based on expressions (9), we will operate with the non-stationary equation of state in the form $P/\rho\sim\left(t/t_{DM}\right)^n$ where t_{DM} - the time of the end of domination of dark matter. Substituting it into (11), and putting $t>t_{DM}$, $\xi>1$ anyone can find a solution in the following representation

$$\ln\rho/\rho_0=-3H_0t_{DM}\left(1-\frac{1}{\xi^2}\right)\xi\approx-3H_0t_{DM}\cdot\xi,$$

$$\rho\left(t\right)/\rho_0\approx\exp\left(-3t/t_{DM}\right)\ . \tag{12}$$

So, it follows that the density of dark matter even with the non-stationary equation of state decreases with time during the evolution of the Universe.Then the evolutionary Friedmann equation takes on the form

$$\frac{\dot{a}}{a}=\sqrt{\frac{8\pi G}{3}}\sqrt{\rho\left(t\right)} \tag{13}$$

with the standard expression of the Hubble parameter. But in contrast to the main vacuum model of the Universe $\left(p=-\rho\right)$, the density of a substance consideration depends on time.

Introducing (12) into (13), we obtain

$$\frac{\dot{a}}{a} = \chi \exp\left(-\frac{3}{2} t/t_{DM}\right), \tag{14}$$

where $\chi = \sqrt{\frac{8\pi G}{3}}$. Thus, an additive to the Hubble parameter is expressed as follows

$$\square \mathrm{H} \propto \exp\left(-\frac{3}{2} t/t_{DM}\right). \tag{15}$$

An analysis of this expression shows that find additive (15), in contrast to the purely vacuum model of the Universe, decreases with time. But despite the one-sidedness of model constructed by us, the full expression of Hubble constant, under considering the overall substances, especially the cosmic vacuum, will increase. Moreover, as shown above

$$a(t)/a_0 = \exp\left(\chi t_{DM}\left(-\frac{2}{3}\exp\left(-\frac{3}{2} t/t_{DM}\right)\right)\right), \tag{16}$$

where a_0 - some constant quantity.

One of such models, a homogeneous and isotropic flat Universe filled with non-relativistic matter and a scalar field with potential, can provide not only accelerated, but also slowed down the expansion of the Universe.

3 On the theory of gravitational lenses in the Universe with a domination of dark matter

Gravitational lenses are massive galaxies or clusters of galaxies that act as a collecting object when light is refracted in their gravitational field.

Although today more than 400 such lenses are known, it is believed that at photographing review of the sky (for example, in the Sloan Digital review [12]) they were captured significantly more, but many of them have not yet been identified.

One of the very distant galaxies in the Universe is MACS0647-JD, located 13.3 billion light-years from us. We see its how it was about 420 million years after the Big Bang. A very important factor in its discovery is that it has changed significantly under the influence of the intermediate galaxy MACSJ0647 +7015 (gravitational lens) at a distance of about five billion light-years.

Another example is the discovery of the supernova PS1-10afx. It originated in the galaxy about nine billion light-years ago, which also makes it one of the farthest type Ia supernovae.

Recently [13] galaxy (J1000 0221) with a pronounced effect of gravitational lensing was discovered. This galaxy is extremely distant and giving four images. One more distant IRC 0218 lens was discovered by researchers from the Keck Observatory, but it has a double images.

The amount of images in the gravitational lens is theoretically can calculate, using algebraic aberration equations. Its justification is given in the monograph [14], and application to some gravitational lenses was proposed in [15,16]. The amount of images is determined by the order of such an equation. Besides, in [17] gives an overview of some theoretical researches of gravitational lensing, including the results of local research.

In general, dark matter can produce a refractive index which differs from vacuum. Its presence, as noted in [18], is described by frequency-dependent effects at the propagation and attenuation of light. Other characteristics of light propagation in the Universe have been considered in [19].

So, we write the standard expression for the Friedman metric –

$$ds^2 = c^2 dt^2 - a^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right], \tag{17}$$

from which it is easy to obtain the law of light propagation and its corresponding velocity in the medium. For simplicity, we'll consider the movement of light along the radial component, so that $\theta = const, \varphi = const$. Therefore, the velocity of light propagation in our case is $\upsilon = dr/dt = c/a(t)$.

The corresponding variable refractive index referred to it, as follows from (16) in the approximation t/t_{DM} , will be described by a quasi-constant quantity

$$n\left(t_{DM}\right)=\sqrt{\frac{8\pi G}{3}}\left(1-\frac{2}{3}\right)t_{DM}. \qquad (18)$$

Further, it is important to pay attention to the following possible cosmological effect. For time $\Delta t \approx \frac{1}{3}t_{DM}\left[1-\left(\frac{\rho_v}{\rho_0}\right)^{1/2}\right]$, $\rho_v<\rho_0$, $t_{DM}<t_{Un}$ the vacuum expansion the vacuum expansion of the Universe will be equal to its deceleration under the influence of non-stationary dark matter. So the light will move almost in empty space. Therefore, in the specified period of time astronomer must detect the radiation splash from galaxies. But such the splash, as it easy to see, will be determined as time t_{DM} and density ρ_0 by these poorly defined quantities.

Conclusion

We constructed a model of the Friedmann Universe with a non-stationary equation of state. It is shown that the density of dark matter decreases with time, and the addition to the Hubble constant increases with time. But this result does not violate the general conclusion about the evolution of the Universe (its expansion) with all the set of matter included in it. The refractive index of our model was calculated, which turned out to be a constant value. The refractive index of our model was calculated, which turned out to be a constant value (more precisely, depending on the era of the end of domination of dark matter) and a possible burst effect of incoming radiation was predicted.

Acknowledgments

The authors (T.K. and L.Ch.) express their sincere gratitude to "NCSRT" NSA RK for supporting this work in the framework of financing the scientific project AP 05134454 "Evolution of perturbations in the density of dark matter in the very early Universe" under the budget program 217 "Development of science", subprogram 102 "Grant financing of scientific research", priority - information, telecommunications and space technologies, scientific research in the field of natural sciences ", The Republic of Kazakhstan.

Author (E.K.) expresses sincere gratitude to his supervisor Corresponding Member of the National Academy of Sciences of the Republic of Kazakhstan, Doctor of Physical and Mathematical Sciences, Professor L.M. Chechin for his assistance in writing articles.

УДК 524.8

Л.М.Чечин[1,2], Е.Б. Курманов[2], Т.К. Конысбаев[1]

[1]В.Г. Фесенков атындағы Астрофизикалық институт "ҰҒЗТО" ҚР ҚАӨМ АҒК, 050020 Обсерватория 23, Алматы, Қазақстан

[2]Физика-техникалық факультет, Әл-Фараби атындағы. Қазақ Ұлттық Университеті, Алматы, Қазақстан; Обсерватория 23, Алматы, Қазақстан

ҚАРАҢҒЫ МАТЕРИЯНЫҢ ҮСТЕМ БОЛУ ДӘУІРІНДЕГІ ЖАРЫҚ СӘУЛЕЛЕРІ

Аннотация. Қараңғы материяның үстемдік ету дәуіріне жалпыланған Фридманның космологиялық моделі қарастырылды. Оның үстіне оның күй теңдеуі жаңа - стационарлық емес формада таңдалды. Осындай метрикада жарықтың таралу процесі зерттелді және оның сыну көрсеткіші табылды.

Түйін сөздер: Фридман космологиясы, қараңғы материя, стационар емес күй теңдеуі, Менделеев – Клапейрон теңдеуі, гравитациялық линзалар.

УДК 524.8

Л.М. Чечин[1,2], Е.Б. Курманов[2], Т.К. Конысбаев[1]

[1]Астрофизический институт В.Г. Фесенкова НЦКИТ (МЦРИАП) РК, 050020,
Обсерватория 23, Каменское плато, Алматы, Казахстан;
[2]Казахский национальный университет имени аль-Фараби,
Физико-технический факультет, Алматы, Казахстан

ЛУЧИ СВЕТА В ЭПОХУ ДОМИНИРОВАНИЯ ТЕМНОЙ МАТЕРИИ

Аннотация. Рассмотрена космологическая модель Фридмана, обобщенная на эпоху доминирования темной материи. При этом ее уравнение состояния выбрано в новой - нестационарной форме. Исследован процесс распространения света в такой метрике, найден ее показатель преломления.

Ключевые слова: космология Фридмана, темная материя, нестационарное уравнение состояния, уравнение Менделеева – Клапейрона, гравитационные линзы.

Information about authors:

Chechin L.M. - Doctor of physical-mathematical science, professor, corresponding member of NAS RK, Chief Researcher, Fesenkov Astrophysical Institute "NCSRT" NSA RK, al-Farabi Kazakh National University. The author formulated the problem statement, checked the calculations. E-mail.ru: chechin-lm@mail.ru;

Kurmanov E.B. – PhD student at Faculty of Physics and Technology, Al-Farabi Kazakh National University. The author carried out calculations for solving the Friedmann equations with a variable equation of state of dark matter. E-mail.ru: ergaly_90@mail.ru;

Konysbayev T.K. – junior researcher, Fesenkov Astrophysical Institute "NCSRT" NSA RK, Master of Science (Astronomy), Author conducted a review of gravitational lenses. E-mail.ru: talgar_777@mail.ru

REFERENCES

[1] Bertone G., Hooper D. Silk J. Particle dark matter: evidence, candidates and constraints, Phys. Rep. 405, DOI: 10.1016/j.physrep.2004.08.031

[2] Iocco F., Pato M., Bertone G. (2011) Dark Matter distribution in the Milky Way: microlensing and dynamical constraints, JCAP11 029, DOI: 10.1088/1475-7516/2011/11/029

[3] Byrd G.G., A.D.Chernin, M.J.Valtonen. (2007) Cosmology. Foundations, and frontiers. M. URSS

[4] P. A. R. Ade et al. Planck Collaboration. (2013) Planck 2013 results. I. Overview of products and scientific results, Astronomy and Astrophysics J., V.1303, DOI: 10.1051/0004-6361/201321529

[5] Green A.M., Hofmann S., Schwarz D.J., MNRAS, 353, L23–L27 (2004). DOI: 10.1111/j.1365-2966.2004.08232.x

[6] Chechin L.M., Kairatkyzy D., Konysbayev T.K. (2018) Toward a theory of the evolution of perturbations of the dark matter density in the very early universe, R. Phys. J., Vol. 61, No. 5, DOI 10.1007/s11182-018-1472-9

[7] Amangeldyieva A., Kairatkyzy D., Konysbayev T.K. (2018) On the nonstationary parameter of the state for dark matter, V. 6, No. 322, 44-48 https://doi.org/10.32014/2018.2518-1726.16

[8] Dolgov A.D., Sazhin M.V., Zeldovich Ya.B. (1990) Basics of Modern Cosmology, Editions Frontiers, ISBN-10: 2863320734

[9] Landau L.D., Lifshitz E.M. (1980) Statistical Physics, Butterworth-Heinemann, London ISBN 10: 0750633727

[10] Linde A. D., (1990) Elementary Particle Physics and Inflationary Cosmology, CRC Press, Boca Raton ISBN-10: 3718604906

[11] Tolman R. (1974) Relativity, thermodynamics and cosmology, Nauka (in Russian).

[12] http://www.astronet.ru/db/msg/1202878/index.html

[13] A. van der Wel , G. van der Ven , M. Maseda , et al. (2018) Astrophysical J. Letters Discovery of a quadruple lens in candels with a record lens redshift z = 1.53 DOI: 10.1088/2041-8205/777/1/L17

[14] Blioh P.V., Minakov A.A. (1989) Gravitational lenses [Gravitacionnye linzy], Kiev, Nauka, P. 41(in Russian)

[15] Chechin L.M., Avkhunbaeva G.M., (2013) Two component gravitational lens, Rus. Phys. J., No. 56, P. 30 – 35. (in Russian)

[16] Chechin L.M., Kairatkyzy D. (2014) Differential aberration equation in gravitational optics [Differencial'noe aberracionnoe uravnenie v gravitacionnoj optike], No 3, Reports of NAS RK, (in Russian)

[17] Chechin L.M., Kurmanov E.B. (2019) On the new direction in the theory of gravitational lensing [O novom napravlenii v teorii gravitacionnogo linzirovanija], No.1(68), Rec. Cont. Phys. DOI: 10.26577/RCPh-2019-1-1115

[18] Gardner S., Latimer D.C. (2010) Dark Matter Constraints from a Cosmic Index of Refraction DOI: 10.1103/PhysRevD.82.063506

[19] Linder, E. V. (1989) Light propagation through gravitationally perturbed Friedmann universes Thesis Stanford Univ., CA.