N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2019.2518-1726.65**
Volume 5, Number 327 (2019), 111 – 119

UDC 531, 53(075). 622.732.62-9. 53.072
IRSTI 29.03.77

K.A.Kabylbekov[1], Kh.K.Abdrakhmanova[2], P.A.Saidakhmetov[1], Ye.B. Issayev[1], B.Sh.Kedelbayev[1]

[1]M.Auezov South-Kazakhstan State University, Shymkent, Kazakhstan;
[2]South-Kazakhstan State Pedagogical University, Shymkent, Kazakhstan
kenkab@mail.ru, khadi_kab@mail.ru, timpf_ukgu@mail.ru, erzhanisaev@mail.ru, b.sh.kedelbaev@mail.ru

RESEARCH OF A CAR COLLISION WITH AN OBSTACLE

Abstract. The article considers the calculation and visualization of collision of a car with an obstacle by using the Matlab software. It contains the formulation of the problem, car and obstacle models, modeling parameters (the mass of the car, the speed of the car, parameters of the suspension: width, height, profile) and mathematical model of the collision profile. There are the calculation and visualization of the profile of the road roughness. The *m*-file titled "polizei.m" is created for calculation and visualization of the car's motion. Calculation results are presented in graphs of displacement around a vertical axis and speed along the vertical axis at a various mass of the car, coefficients of elasticity and damping of the suspension. There is also the discussion of the results of the calculation and visualization. The program allows making experiments with a change of width and height of roughness of the road, of the mass of the car, of suspension parameters.

Keywords. Car, mass, suspension, profile of the obstacle, width and height, displacement around and along the vertical axis.

Introduction

Nowadays all educational institutions of Kazakhstan are provided with computer hardware and software, interactive boards and internet. Almost all teachers have completed language and computer courses for professional development. Hence the educational institutions have all conditions for using computer training programs and models for performing computer laboratory works. In recent years the new computer system Matlab for performing mathematical and engineering calculations is widely used in university and engineering researches throughout the world [1-7]. Unfortunately, the numerical calculations which are carried out by students often are done by means of the calculator that is almost manually. Modern computers are frequently used only for presentation of the work. Actually, students should be able not only to solve these or other engineering problems, but also do them by using modern methods, that is, using personal computers.

Students of the physics specialties 5B060400 and 5B011000 successfully master the discipline "Computer modeling of physical phenomena" which is the logical continuation of the disciplines "Information technologies in teaching physics" and "Use of electronic textbooks in teaching physics". The aim of this discipline is to study and learn the MATLAB program language, acquaintance with its huge opportunities for modeling and visualization of physical processes.

In our early works [8-28] we have shown the potentials of the Matlab software for modeling and visualization of physical processes in mechanics, molecular physics, electromagnetism and quantum physics where it have been used for solving the ordinary differential equations (ODE), for visualization of the equipotential lines of the systems of charged conductors and of the motion of charged particles in electric, magnetic and gravitational fields.

The present article is devoted to calculation and visualization of the car's collision with the obstacle by using the MATLAB software.

Formulation of the problem. The car (fig.1) moves on the flat road and collides with an artificial obstacle, the so called “sleeping policeman” or “speed bump”. Let us consider the kinematics and dynamics of the car’s motion.

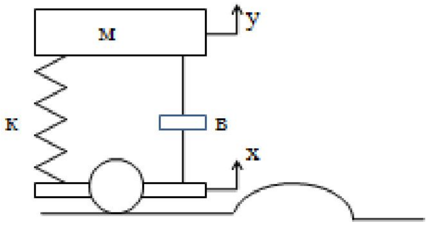

Figure 1 - The car collides with an artificial obstacle

The car model. Let's consider the collision only of one wheel of the car with The horizontal component of the car’s velocity doesn’t change. The action of the obstacle causes only the production of the vertical motion of the car. The wheel when driving completely repeats the obstacle’s profile. The suspension consists of an elastic spring (K) and a damper (B).

Parameters of the modeling. The mass of the car, the velocity of the car. The parameters of the suspension: the coefficient of elasticity and the damping coefficient. The parameters of the obstacle: its width, height and profile, elastic properties and the damping parameters.

Mathematical model. When the wheel collides with the obstacle the wheel moves along the vertical direction. This displacement is described by a variable x. The action of the road is transferred to the car body by the aid of suspension (the spring with rigidity k and the damper with damping coefficient B). Power action by means of spring is defined by the relative motion of the car body described by the displacements x and y. Power action of the damper is defined by the relative velocities of these displacements dx/dt and dy/dt. In the equation of motion we will neglect the continuous action on the car body which is compensated by equal in magnitude and oppositely directed elastic force of the spring $Mg = k\Delta x$. By taking into account the above made suggestions and using the Newton’s second law we get the following equation describing the motion of the car:

$$M\frac{d^2y}{dt^2} = B\left(\frac{dx}{dt} - \frac{dy}{dt}\right) + k\left(x - y\right) \tag{1}$$

$$\frac{d^2y}{dt^2} + \frac{B}{M}\frac{dy}{dt} = \frac{B}{M}\frac{dx}{dt} + \frac{k}{M} \tag{2}$$

The vertical component of the car acceleration $\frac{d^2y}{dt^2}$ is the function of the car horizontal velocity. At the same time the profile of the road makes a significant contribution to this acceleration. Let us take the following function as the mathematical model of the road roughness:

$$x(s) = \frac{H}{2}\left(1 - \cos\left(\frac{2\pi s}{L}\right)\right) \quad 0 \le s \le L$$

Visualization of the profile of the road roughness at H = 10 cm and L = 100 cm.

```
>> clc
>> H=10;
>> L=100; s=0:1:L;
>> plot(s,H)
>> x=(H/2)*(1-cos(2*pi*s/L));
>> plot(s,x)
```

```
>> grid on
>> xlabel('s, sm')
>> ylabel('H,sm')
>> title('x(s)')
```

The diagram of this function for H = 10 cm and L = 100cm is presented in the fig.2

It should be noted that the radius of the wheel mustn't be greater than the radius of the curvature fitting to each point of the trajectory.

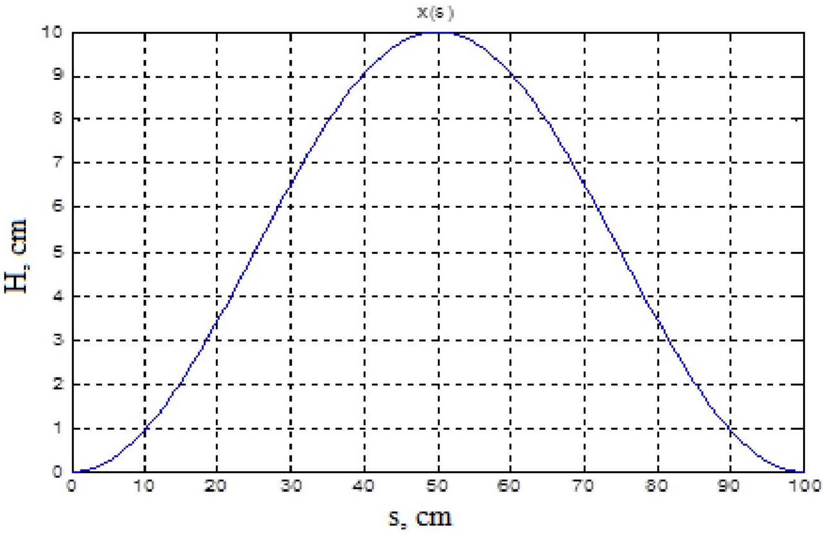

Figure 2 - The model of the road roughness

At the constant horizontal component of the car velocity $S = \upsilon_0 \cdot t$ the function determining the road roughness and its derivative are given by the expressions:

$$x(s) = \frac{H}{2}\left(1 - \cos\left(\frac{2\pi\upsilon_0 t}{L}\right)\right)$$

$$\frac{dx}{dt} = \frac{H}{L}\pi\upsilon_0 \sin\left(\frac{2\pi\upsilon_0 t}{L}\right)$$

By using the change of variables

$$y = z_1; \quad \frac{dy}{dt} = \frac{dz_1}{dt} = z_2; \quad \frac{d^2y}{dt^2} = \frac{dz_2}{dt} = -\frac{B}{M}\frac{dy}{dt} - \frac{k}{M}y + \frac{B}{M}\frac{dx}{dt} + \frac{k}{M}x$$

we reduce the equation of motion to the expression easy for integration with the help of procedure ode45.

For calculation and visualization of the car motion we create the m-file titled “polizei.m”:

```
function dzdt=polizei(t,z)
global M K B V0 H L
g=9.81; % free fall acceleration
dzdt=zeros (2,1); % vector column
s=V0*t;
x=H/2*(1-cos(2*pi*s/L));
xdot=H/L*pi*V0*sin(2*pi*V0*t/L);
% the roughness of the road is in the range of 0 ≤ S ≤ L
if s> L
x=0;
```

```
xdot=0;
end
dzdt(1)=z(2);
dzdt(2)=-B/M*z(2)-K/M*z(1)+B/M*xdot+K/M*x;
```

In the command line we write

```
clc
global M K B V0 H L
M=450;
K=35000;
B=7300;
z10=0;
z20=0;
accel=zeros(1000);
i=1;
V0=20;
H=0.05;
L=0.8;
% L/V0 –the time of motion on the roughness
tmax=2*L/V0;
z0=[z10; z20];
dt=[0 tmax];
dt=0:tmax/500:tmax;
ii=0;
aa=zeros(1000,1);
>> opt=odeset('RelTol', 1e-8);
[t,z]=ode45(@polizei, dt,z0, opt);
subplot(2,2,1); plot(t,z(:,1))
title('displacement')
grid on
subplot(2,2,2); plot(t,z(:2))
grid on
subplot(2,2,3); plot(t,aa(1:size(t)))
grid on
```

The result is presented in the fig.3

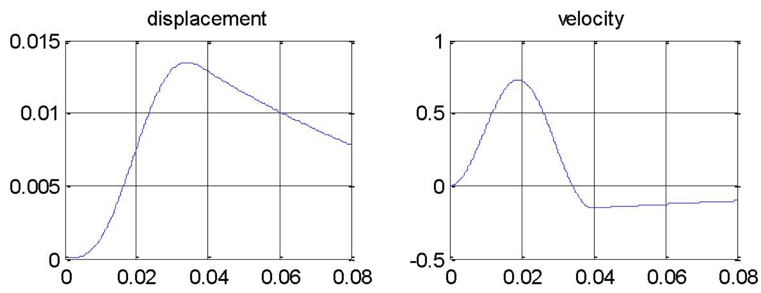

Figure 3 - Displacement round the vertical axis (left graph) and velocity along the vertical axis at the mass of the car of M = 450 kg

Now we change the mass of the car and in the command line we write:

```
clc
global M K B V0 H L
M=958;
K=35000;
B=7300;
```

```
z10=0;
z20=0;
accel=zeros(1000);
i=1;
V0=20;
H=0.05;
L=0.8;
% L/V0 - the time of motion on the roughness
tmax=2*L/V0;
z0=[z10; z20];
dt=[0 tmax];
dt=0:tmax/500:tmax;
ii=0;
aa=zeros(1000,1);
>> opt=odeset('RelTol', 1e-8);
[t,z]=ode45(@polizei, dt,z0, opt);
subplot(2,2,1); plot(t,z(:,1))
title('displacement')
grid on
subplot(2,2,2); plot(t,z(:,2))
title('velocity')
grid on
```

The result is presented in the fig.4

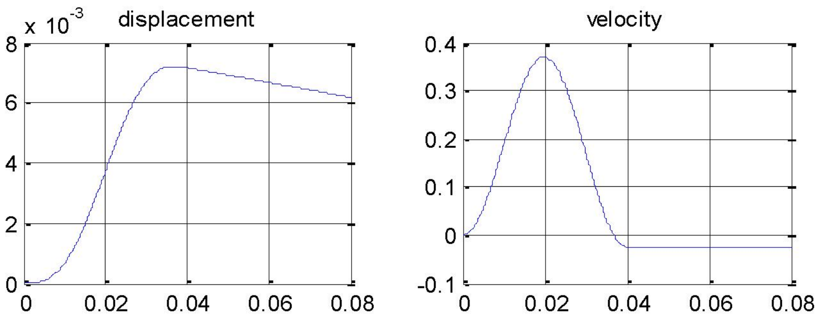

Figure 4 - Displacement round the vertical axis (left graph) and velocity along the vertical axis at the mass of the car of M = 958 kg

At the decrease of the spring rigidity till K = 20 000 the graph of the displacement round the vertical axis (left graph) and velocity along the vertical axis at the mass of the car of M = 658 kg doesn’t change (compare the fig.4 and 5). Here we give only the results in fig.5.

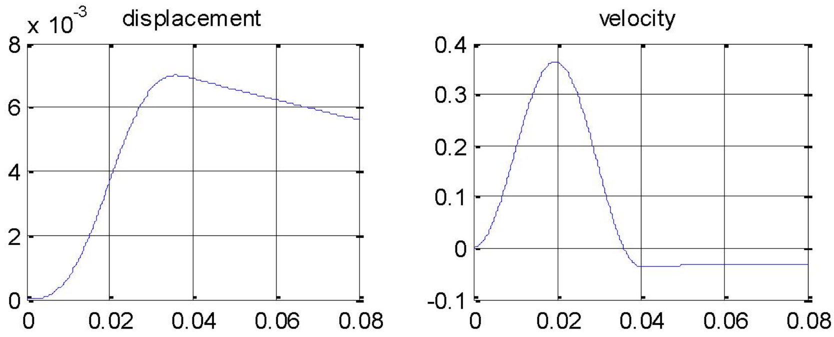

Figure 5 - Displacement round the vertical axis (left graph) and velocity along the vertical axis at the mass of the car of M = 658 kg

Then we increase the damping coefficient up to K = 20 000 (the mass of the car is 958 kg)

```
clc
global M K B V0 H L
M=958;
K=20000;
B=21900;
z10=0;
z20=0;
accel=zeros(1000);
i=1;
V0=20;
H=0.05;
L=0.8;
% L/V0 - the time of motion on the roughness
tmax=2*L/V0;
z0=[z10; z20];
dt=[0 tmax];
dt=0:tmax/500:tmax;
ii=0;
aa=zeros(1000,1);
opt=odeset('RelTol', 1e-8);
[t,z]=ode45(@polizei, dt,z0, opt);
subplot(2,2,1); plot(t,z(:,1))
title('displacement')
grid on
subplot(2,2,2); plot(t,z(:,2))
title('velocity')
grid on
```

The result is presented in the fig.6.

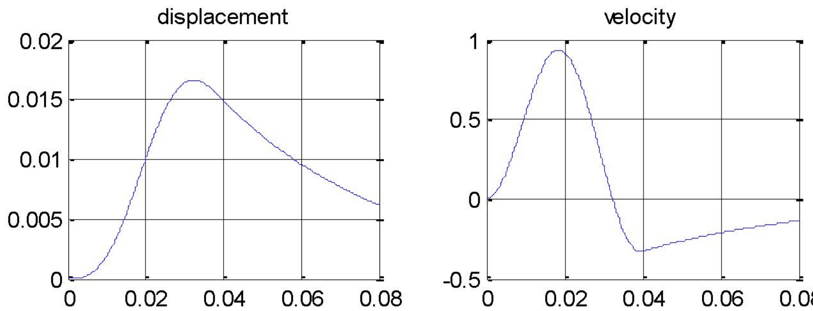

Figure 6 - Displacement round the vertical axis (left graph) and velocity along the vertical axis at the mass of the car of M = 958 kg

The figure 6 shows that the change of the damping coefficient significantly affects the displacement round the vertical axis and the velocity along this axis (compare figures 4 and 6).

Assignments for individual work:

9. Decrease the damping coefficient by 3 times till zero. How did the process change during the motion along the obstacle? After overcoming the obstacle?

10. Change the parameters of the obstacle. Consider the options of the narrow and low obstacle (the height and width are 10 cm and 40 cm, 10 cm and 100 cm, 4 cm and 100 cm)

Conclusion. The article considers the calculation and visualization of the car collision with an obstacle by using the Matlab software. It contains the formulation of the problem, car and obstacle models, modeling parameters (the mass of the car, the speed of the car, parameters of the suspension: the width, height, profile) and the mathematical model of the collision profile. There are the calculation and

visualization of the profile of the road roughness. The *m*-file titled "polizei.m" is created for calculation and visualization of the car's motion. Calculation results are presented in graphs of displacement around a vertical axis and velocity along the vertical axis at a various mass of the car, coefficients of elasticity and damping of the suspension. There is also the discussion of the results of the calculation and visualization. The program allows making experiments with a change of width and height of roughness of the road, of the mass of the car, of suspension parameters.

К.А.Қабылбеков[1], Х.К.Абдрахманова[2],
П.А.Саидахметов[1], Е.Б. Исаев[1], Б.Ш. Кеделбаев[1]

[1]М.Әуезов атындағы Оңтүстік Қазақстан мемлекеттік университеті, Шымкент, Қазақстан;
[2]Оңтүстік Қазақстан мемлекеттік педагогикалық университеті, Шымкент, Қазақстан

АВТОМОБИЛЬДІҢ КЕДЕРГІДЕН ӨТУІН ЗЕРТТЕУ

Аннотация. Matlab жүйесінде автомобилдің кедергіден өтуін есептеу мен бейнелеу ұсынылады. Проблеманы тұжырымдау, автомобил мен кедергінің көрініс моделі, моделдеудің параметрлері (автомобилдің массасы мен жылдамдығы, аспаның серпімділік және демпферлік коэффициенттері, кедергі параметрлері: ені мен биіктігі) және кедергіден өту процесінің математикалық моделі келтірілген. Жолдың тегіс еместігінің параметрлері есептеліп, бейнеленген. Автомобилдің кедергіден өту қозғалысын есептеу мен бейнелеу үшін polizei.m атты m-файл құрастырылған. Есептеу нәтижелері автомобилдің массалары, аспаның серпімділік жіне демпферлік коэффициенттері әртүрлі жағдайларындағы вертикал өс бойындағы және осы өсті айнала қозғалысының жылдамдықтарының графиктері берілген. Есептеу мен бейнелеу нәтижелері талқыланған. Бағдарлама автомобиль массасын, кедергі параметрлерін, жолдың тегіс еместікгінің биіктігін өзгертіп эксперименттер жасауға мүмкіншілік береді.

Түйін сөздер. Автомобил, масса, аспа (подвеска), кедергі параметрі- ені және биіктігі, вертикаль өс бойында және оны айнала қозғалуы.

К.А.Кабылбеков[1], Х.К.Абдрахманова[2],
П.А.Саидахметов[1], Е.Б. Исаев[1], Б.Ш.Кеделбаев[1]

[1]Южно-Казахстанский государственный университет им. М.Ауэзова, Шымкент, Казахстан;
[2]Южно--Казахстанский государственный педагогический университет, Шымкент, Казахстан

ИССЛЕДОВАНИЕ НАЕЗДА АВТОМОБИЛЯ НА ПРЕПЯТСТВИЕ

Аннотация. Предлагается программа расчета и визуализации наезда автомобиля на препятствие в системе Matlab. Приведены формулировка проблемы, модели автомобиля и препятствия, параметры моделирования (масса автомобиля; скорость автомобиля, параметры подвески: коэффициент упругости и коэффициент демфпирования, параметры препятствия: ширина, высота, профиль) и математическая модель процесса наезда. Проведен расчет и визуализация профиля неровности дороги Для расчета и визуализации движения автомобиля создан m-файл под названием polizei.m. Результаты расчетов представлены в графиках перемещения вокруг вертикальной оси и скорости вдоль вертикальной оси при различных массах автомобиля и коэффициентов упругости и демпфирования подвески. Обсуждены результаты расчетов и визуализации.

Программа позволяет проводить эксперименты с изменением ширины и высоты неровности дороги, массу автомобиля, параметров подвески.

Ключевые слова. Автомобиль, масса, подвеска, профиль препятствия- ширина и высота, перемещение вокруг и вдоль вертикальной оси.

Information about authors

Kabylbekov K.A. – cand.ph-math.sc., associate professor of the chair of Physics, M.Auezov South-Kazakhstan State University, kenkab@mail.ru, https://orcid.org/0000-0001-8347-4153,;

Abdrakhmanova Kh.K.- cand.chem.sc., associate professor of the chair of Physics, South-Kazakhstan State Pedagogical University, khadi_kab@mail.ru, https://orcid.org//0000-0002-6110-970X;

Saidakhmetov P.A.- cand.ph-math.sc., associate professor of the chair of Physics, M.Auezov South-Kazakhstan State University, timpf_ukgu@mail.ru, https://orcid.org/0000-0002-9146-047X;

Kedelbaev B.Sh.- doc. eng. sciences, professor of the chair of "Microbiology", M. Auezov South-Kazakhstan State University, b.sh.kedelbaev@mail.ru, .- https://orcid.org/0000-0001-7158-1488;

Issaev E.B.- cand.techn.sciences, associate professor of the department "Biology" of M. Auezov SKSU, erzhanisaev@mail.ru, https://orcid.org/0000-0001-7536-5643

REFERENCES

[1] Porsev S. V. Computer simulation of physical processes in the package MATLAB. M.: Hot Line-Telecom, 2003.592 p.

[2] Kotkin G.A., Cherkassky V.S. Computer modeling of physical processes using MATLAB: Tutorial. / Novosibirsk University.

[3] Lurie M. S., Lurie O. M. Application of the MATLAB program in the study of course of electrical engineering. For students of all specialties and forms of educatio. Krasnoyarsk: SibSTU, 2006208 p.

[4] Potemkin V. system of engineering and scientific calculations MATLAB 5.x (in 2 volumes). Moscow: Dialog-MIFI, 1999.

[5] Averyanov G. P., Budkin, Dmitrieva V. V. Design automation. Computer workshop. Part 1. Solving problems of Electrophysics in MATLAB: tutorial. Moscow: MEPhI, 2009. 111 p.

[6] Dyakonov V. p. MATLAB. Complete tutorial. M: DMK Press, 2012. 768 p.

[7] Ryndin E. A., Lysenko I. E. Solving problems of mathematical physics in Matlab. Taganrog: TRTU. 2005. 62 p.

[8] Kabylbekov K.A., Abdrakhmanova Kh.K., Abekova J., Abdraimov R.T., Ualikhanova B.S. Calculation and visualization of a system-an electron in a deep square potential well, with use of the software package of MATLAB. Proceeding of the III International Scientific and Practical Conference «Topical researches of the World Science» (Iune 28, 2017,Dubai, UAE). №7(23). Vol.I, July 2017, P. 7-13.

[9] Kabylbekov K., Saidullaeva N., Spabekova R., Omashova G, Tagaev N., Bitemirova A., Berdieva M. Model of a blank form for computer laboratory work on research of the speed selector. Journal of Theoretical and Applied Information Technology15th July 2017. Vol.95. No 13, P 2999-3009, c 2005 – ongoing JATIT & LLS. Indexada en Scopus.

[10] Kabylbekov K.A., Omashova G.,Spabekova R., Saidullaeva N., Saidakhmetuv P., Junusbekova S. Management and organization of computer laboratory work in physics education. Espacios. Vol. 38 (N° 45) Año 2017. Pág. 35. Indexada en Scopus.

[11] Kabylbekov K., Omashova G, Spabekova R, Saidullaeva N, Saidakhmetuv P. Junusbtkova S., Management and organization of computer laboratory work in physics education. Espacios. Vol. 38 (N° 45) Año 2017. Pág. 35. Indexada en Scopus, Google Schollar.

[12] Kabylbekov K.A., Ashirbaev Kh.A., Arysbaeva A.S., Jumagaliyeva A.M. Model of the form of the organization of computer laboratory work in the study of physical phenomena. Modern high technologies, №4, Moscow, 2015, P. 40-43.

[13] Kabylbekov K.A., Madiyarov N.K., Saidakhmetov P.A. Independent design of research tasks of computer laboratory works on thermodynamics. Proceedings of the IX International Scientific and Methodological Conference. Teaching natural sciences (biology, physics, chemistry) mathematics and computer science. Tomsk-2016, P. 93-99.

[14] Kabylbekov K.A, Omashova G. Sh, Spabekova R.S, Saidakhmetov P.A, Serikbaeva G.S, Aktureeva G. Organization of computer laboratory works on study the turn-on and turn-off current of the power supply by using MATLAB software package. News of the National Academy of Sciences of the Republic of Kazakhstan, Series of Physics and Mathematics, Almaty, Volume 3, Number 313 2017, P. 139-146.

[15] Kabylbekov K.A, Omashova G. Sh, Spabekova R.S, Saidakhmetov P.A, Serikbaeva G.S , Aktureeva G. Organization of computer labs for the study of velocity and height distribution of molecules from the Earth's surface by using MATLAB software package. Bulletin of NAS RK, Almaty, Volume 3, Number 367, 2017, P. 111-119.

[16] Kabylbekov K.A, Ashirbayev H.A, Abdrakhmanova Kh.K, Dzhumagalieva A.I., Kydyrbekova J.B. Organization of laboratory work on study of electric and magnetic fields by using MATLAB software package. Proceedings of the National Academy of Sciences of the Republic of Kazakhstan, Series of Physics and Mathematics, Almaty, Volume 3, Number 313, 2017, P. 206-213.

[17] Kabylbekov K. A., Spabekova R. S., Omashova G.Sh., Abzhapparov A.A., Polatbek A, Serkebayeva S. G. The use of the software package MATLAB for solving problems on bifurcated electrical circuits. Bulletin of NAS RK, Almaty 2017, Volume 4, Number 368, P. 101-108.

[18] Kabylbekov K. A., Ashirbaev H. A., Abdrakhmanova Kh.K., Dzhumagalieva A. I., Kadyrbekova J. B. Organization of the performance of the laboratory work "Modeling the electric field of a system consisting of a dielectric square and a long charged conductor" by using MATLAB software package. News of the National Academy of Sciences of the Republic of Kazakhstan, Series of Physics and Mathematics, Almaty 2017, Volume 4, Number 314, P. 252-259.

[19] Kabylbekov K. A., Abdrakhmanova Kh.K., Ermakhanov M.N., Urmashev B.A., Jaтkanbayev E.T. Calculation and visualization of a body motion in a gravitational field. NEWS of the National Academy of Sciences of the Republic of Kazakhstan. Series of geology and technical sciences.Volume 4, Number 430 (2018), P. 87-98.

[20] Kabylbekov K. A., Abdrakhmanova Kh.K., Omashova G.Sh., Lakhanova K.M., Abekova Zh.A. Organization of computer laboratory work "Calculation and visualization of small forced oscillations" N E W S of the National Academy of Sciences of the Republic of Kazakhstan, Series of geology and technical sciences. Volume 3, Number 430 (2018), P. 145-155.

[21] Kabylbekov K. A., Abdrakhmanova Kh.K., Omashova G.Sh., Kedelbaev B., Abekova Zh.A. Calculation and visualization of electric field of a space –charged sphere. N E W S of the National Academy of Sciences of the Republic of Kazakhstan. Series of geology and technical sciences. Volume 5, Number 431 (2018), P. 201–209. doi.org/10.32014/2018.2518-170X.26

[22] Kabylbekov K. A., Abdrakhmanova Kh. K., Saidakhmetov P. A., Sultanbek T. S., Kedelbaev B. Sh. Calcuiation and visualization of isotopes separation process using MATLAB program. N E W S of the National Academy of Sciences of the Republic of Kazakhstan. Series of geology and technical sciences. Volume 5, Number 431 (2018), P. 218–225. doi.org/10.32014/2018.2518-170X.28

[23] Kabylbekov K. A., Abdrakhmanova Kh.K., Saidakhmetov P.A., Musaev J.M., Issayev Ye.B., Ashirbaev Kh.A. Calculation and visualization of a body motion under the gravity force and the and the opposing drag. N E W S of the National Academy of Sciences of the Republic of Kazakhstan Series of geology and technical sciences. Volume 6, Number 432 (2018), P. 85– 95. doi.org/10.32014/2018.2518-170X.38

[24] Kabylbekov K. A., Abdrakhmanova Kh.K., Saidakhmetov P.A., Kedelbaev B.Sh., Abdraimov R. T., Ualikhanova Б.S. Calculation and visualization of the field coaxial cable carrying steady current. N E W S of the National Academy of Sciences of the Republic of Kazakhstan Series of geology and technical sciences. Volume 6, Number 432 (2018), P. 55 – 65. doi.org/10.32014/2018.2518-170X.35

[25] Kabylbekov K. A., A.D.Dasibekov, Abdrakhmanova Kh.K., Saidakhmetov P.A., Issayev E.B., Urmashev B.A. Calculation and visualization of oscillating systems. N E W S of the National Academy of Sciences of the Republic of Kazakhstan Series of geology and technical sciences. Volume 6, Number 432 (2018), P. 110 – 120. doi.org/10.32014/2018.2518-170X.41

[26] Kenzhekhan Kabylbekov, Khadisha Abdrakhmanova, Gaukhar Omashova, Pulat Saidakhmetov, Turlan Sultanbek, Nurzhamal Dausheyeva. A Laboratory on visualization of Electrostatic and Magnetic Fields. Acta Polytechnica Hungarica Vol. 15, No. 7, 2018, P49-70.

DOI: 10.12700/APH.15.7.2018.7.3

[27] Kabylbekov K. A., Dasibekov A.D., Abdrakhmanova Kh.K., Saidakhmetov P.A. Calculation and visualization of quantum-mechanical tunnel effect. News of the National Academy of Sciences of the Republic of Kazakhstan, Series of Physics and Mathematics, Volume 2, Number 324 (2019), P. 60-68 . doi.org/10.32014/2019.2518-1726.13

[28] Kabylbekov K. A., Dasibekov A.D., Abdrakhmanova Kh.K., Saidakhmetov P.A. Calculation and visualization of small oscillations of a double plane pendulum. News of the National Academy of Sciences of the Republic of Kazakhstan, Series of Physics and Mathematics, Volume 2, Number 324 (2019), P. 69-79 . doi.org/10.32014/2018.2518-1726.14