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## ON THE NONSTATIONARY PARAMETER OF STATE FOR DARK MATTER

**Abstract:** The purpose of work is a conclusion non-stationary the equations of a condition of WIMP-gas for their various models - ideal gas (Mendeleyev-Clapeyron equation), non-ideal gas (equations type of Van der Waals and Dieterichi). It is shown that their general dependence on time has the negative power. Therefore, the closer to the Planck time, and also to the time of WIMP particles birth, the more state of non-baryonic substance differ from the dust-like one.

**Keywords:** dark matter (DM), WIMP-gas, nonstationary equation of a state, early epochs of the Universe, Mendeleyev-Clapeyron equation, Van der Waals equations, equations of Dieterichi, fluctuation of temperature of relic radiation.

### Introduction

It is known that the Universe (approximately) for 73% consists of dark energy, for 23% – of dark matter and about 4% of baryonic substance – primary plasma and radiation. [1, 2]. We will note that there is a classification of dark matter - "hot", "warm" and "cold". The last is most preferable from the cosmological point of view. The class of WIMPs is among candidates for a role of particles of cold DM. The peculiarity of WIMP is that their concentration gives not only the necessary contribution to the total Universe energy balance, but also effectively describes the galaxies halos of dark matter.

At the same time, it should be noted that the most of previous articles were based on idea of halos' static character. But later the idea that density of dark matter in halos may depend on time also began to develop and, therefore, dark matter possesses by dynamic properties [3 - 10].

For searching the distribution of relic radiation in of WIMP-particles gas, we will remind that small periodic indignations in the continuous environment represent sound waves. If these changes adiabatic, the speed of a sound wave is described by expression  $v = \sqrt{(\partial P / \partial \rho)_s}$ . Since the entropy of the Universe is highly constant, later expression can be used for study the substance of non-baryonic matter (in our case - gas of WIMP-particles). So, first we will consider gas of WIMP-particles as ideal gas, and then – as real gas in which parameter of state is nonstationary.

So, due to the thermodynamic balance of WIMP-particles with particles of baryonic plasma the approximate condition takes place  $T_{BM} \propto T_{DM}$  [11]. Therefore Mendeleyev-Clapeyron equation takes on the form.

$$P_{DM} V = \frac{m_{DM}}{\mu} R T_{BM}. \quad (1)$$

It's convenient to rewrite (1) as

$$P_{DM} = \rho_{DM} \frac{R}{\mu} T_{BM} \quad (2)$$

with explicit expression of gas density at a given temperature. In our case, therefore, we are talking about baryonic matter in the form of ultrarelativistic plasma. For the standard cosmological Friedman model filled with relativistic gas (ultrarelativistic plasma), there is an approximate relation connecting the temperature of primary relativistic gas with the age of Universe. It is:

$$T_{BM} \sim t^{-1/2}. \quad (3)$$

From (2) and (3) it follows that equation of state of ideal gas, generally speaking, has the form  $P_{DM} = \bar{\omega}_{DM}(t) \cdot \rho_{DM} = \rho_{DM} \frac{R}{\mu} T_{BM}(t)$ . So, taking into account the dependence (3), its state parameter depends on time in the same way, i.e.

$$\bar{\omega}_{DM}(t) = \frac{R}{\mu} T_{BM}(t) \sim t^{-1/2}. \quad (4)$$

(ii) Van der Waals gas. Consider now the case of filling the Universe by real gas consisting of molecules and described by the Van der Waals equation of state. (Another version of the description of non-ideal dark matter is given in articles [12-15].) If the temperature is measured in degrees, then, according to [16], it takes on the form

$$\left( P_{DM} + N^2 \frac{\tilde{a}}{V^2} \right) (V - N\tilde{b}) = NkT_{BM}, \quad (5)$$

in which  $\tilde{a}$  and  $\tilde{b}$  are the constant values that describe gas of WIMP - particle,  $k$  is Boltzmann constant. Recall that the physical meaning of parameter  $\tilde{a}$  that it describes the interaction of substance molecules, parameter  $\tilde{b}$  is responsible for accounting their sizes.

Let us rewrite (5) in the form that convenient for study of state parameter, -

$$P_{DM} \left( 1 + \nu^2 \frac{\tilde{a}}{P_{DM} V^2} \right) \left( 1 - \nu \frac{\tilde{b}}{V} \right) = \rho_{DM} \frac{R}{\mu} T_{BM}. \quad (6)$$

Here -  $\mu$  is the molar mass of substance,  $R$  - universal gas constant. Now our task is to combine (6) with (3) and find an explicit dependence of the real gas state parameter on time. For further explanation, we assume that  $\tilde{b}/V \ll 1$ . This condition describes the real property of the gas from the WIMP particles, in which its current size is essentially larger than size of all molecules themselves. In addition, it makes sense to assume that the interaction of molecules is not too large. After these simplifications, the state parameter of real gas can be express as a function of temperature as follows

$$\omega_{DM}(T_{BM}) = \frac{R}{\mu} \left( 1 - \nu^2 P_{DM} \tilde{a} \left/ \frac{m^2}{\mu^2} R^2 T_{BM}^2 \right. \right) T_{BM} \quad (7)$$

or as the function of time of two components

$$\omega_{DM}(t) = \bar{\omega}_{DM}(t) \propto t^{-1/2} > 0 \\ \omega'_{DM}(t) \propto t^{-3/2} < 0. \quad (8)$$

iii) Real gas Dieterichi links the main thermodynamic quantities in the gas. It has the form,

$$P = R \frac{T}{(V - \beta)} \cdot \exp\left(-\frac{\alpha}{RTV}\right), \quad (9)$$

were  $P$  - pressure,  $V$  - molar volume,  $T$  - absolute temperature, respectively. In addition, there is a mass of gas, and its molar mass. Finally,  $\alpha$  parameter characterizes the interaction of gas molecules, and  $\beta$  parameter describes the size of molecules. This equation can be rewritten as

$$P = \rho \left(\frac{k}{\mu}\right) \cdot \frac{T}{\left(1 - \frac{b}{V}\right)} \cdot \exp\left(-\frac{\alpha}{\frac{m}{\mu}kVT}\right). \quad (10)$$

For it examining introduce some additional conditions. First, we assume that  $\frac{b}{V} \ll 1$  as before.

Beside in exponent we'll use the equation of state of an ideal gas -  $PV = \frac{m}{\mu}kT$ . As a result, after a series of calculations for dusty matter and ultra relativistic matter, we obtain, respectively

$$\omega_D(t) + \left(\frac{k}{\mu}\right) \cdot t^{-1/2} \cdot \exp\left(-\frac{\bar{\alpha}}{\omega_D(t) \cdot t^2}\right) \approx 0, \quad (11)$$

$$\omega_D(t) + \left(\frac{k}{\mu}\right) \cdot t^{-1/2} \cdot \exp\left(-\frac{\tilde{\alpha}}{\omega_D(t) \cdot t}\right) \approx 0. \quad (12)$$

Here are the  $\bar{\alpha}$  and  $\tilde{\alpha}$  coefficients that proportional to  $k/\mu$ . For further calculation introduce new constants  $\bar{\alpha} = k/\mu, \tilde{\alpha} = k/\mu$ . In addition assume that  $-\frac{\bar{\alpha}}{\omega(t) \cdot t^2} \ll 1$  and  $-\frac{\tilde{\alpha}}{\omega(t) \cdot t} \ll 1$ . After decomposing exponential expressions into series according to the above specified parameters, we obtain the needed state parameters

$$\hat{\omega}_D = \frac{A}{2\sqrt{t}} \pm \frac{1}{2} \sqrt{\frac{A^2}{t} - \frac{4B}{t^{5/2}}}, \quad (13)$$

$$\check{\omega}_D = \frac{A}{2\sqrt{t}} \pm \frac{1}{2} \sqrt{\frac{A^2}{t} - \frac{4B}{t^{3/2}}}, \quad (14)$$

Based on the previously obtained results, we find that the temperature fluctuations of cosmic microwave background (CMB) which corresponds to the real gas model Dieterichi. So, we find the peculiar velocity (in the notations (4))

$$v' = v'_D(t) = \bar{\omega}_D(t) \propto t^{-3/2} t_0 \quad (15)$$

and the corresponding fluctuation of the relic radiation

$$\left(\frac{\delta T}{T}\right)_D = v'_D(t) \cdot \cos \theta \propto t^{-3/2} \cdot \cos \theta. \quad (16)$$

Note that the temperature fluctuations of the relic radiation for different observed angles were previously considered in [17].

### Conclusion

In conclusion, we note the findings.

i) The nonstationary equations of state of WIMP gas for their various models - ideal gas (Mendeleev-Clapeyron equation), non-ideal gas (Van der Waals and Dieterichi equations) are derived.

(ii) It's shown that total dependence of the state parameter on time has a power form with a negative state indicator. Therefore, the closer to Planck time, as well as to the time of birth of the WIMP particles, the indicator of substance state increases. This fact shows that the state of non-baryonic matter in the earliest Universe is significantly different from dust-like matter.

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Note that by comparing (1) with the speed of sound, it is easy to see that it depends on time like (4), i.e. as  $v = \bar{v}_{MC}(t) = \bar{w}_{DM}(t) \propto t^{-1/2}$ . This velocity is the peculiar velocity of the relic radiation, which changes the rate of Hubble expansion. (Interpretation of this type of fluctuations is given in [2]). Consequently, the temperature fluctuation of the relic radiation in the epoch is described by the expression (as we usually assume  $c = 1$ )  $\left(\frac{\delta T}{T}\right)_{MC} = \bar{v}_{MC}(t) \cdot \cos \theta \propto t^{-1/2} \cdot \cos \theta$ . So the variation of the temperature of the relic radiation will depend not only on the angle of observation, but also on the time.

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### ҚАРАҢҒЫ МАТЕРИЯ ҮШІН БЕЙСТАЦИОНАР КҮЙ ПАРАМЕТРЫ

**Аннотация:** Жұмыстың мақсаты - WIMP газының стационарлық емес теңдеулерін түрлі модельдер үшін - идеал газ (Менделеев-Клапейрон теңдеуі), идеал емес газ (Van der Waals және Diterich теңдеулерінің түрі) теңдеулерін есептеу. Олардың жалпы уақыт тәуелділігі теріс екендігі көрсетілген. Сондықтан Планк уақыты мен Вимп бөлшектерінің пайда болу уақыты неғұрлым жақын болған сайын барионды емес бөлшектердің күйі шантәріздестен соғұрлым өзгешелене түседі.

**Түйін сөздер:** кара материя (DM), WIMP газы, стационарлық емес теңдеуі, Әлемнің ерте дәуірі, Менделеев-Клапейрон теңдеуі, Ван дер Ваалс теңдеулері, Дитерери теңдеуі, Фондық сәуленің температуралық ауытқуы.

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### О НЕСТАЦИОНАРНОМ ПАРАМЕТРЕ СОСТОЯНИЯ ТЕМНОЙ МАТЕРИИ

**Аннотация:** Целью работы является вывод нестационарных уравнений состояния WIMP-газа для их различных моделей - идеального газа (уравнение Менделеева-Клапейрона), неидеального газа (типа уравнений Ван-дер-Ваальса и Дитеричи). Показано, что их общая зависимость от времени характеризуется отрицательным показателем. Поэтому чем ближе к времени Планка, так и к времени рождения WIMP-частиц, тем больше состояние небарионного вещества отличается от пылеподобного.

**Ключевые слова:** темная материя (DM), WIMP-газ, нестационарное уравнение состояния, ранние эпохи Вселенной, уравнение Менделеева-Клапейрона, уравнение Ван-дер-Ваальса, уравнение Дитеричи, флуктуация температуры реликтового излучения.