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NATURAL FILTRATION EQUATIONS. FIASCO “OF DARCY'S LAW”

Abstract. The theory of natural filtration equations is given. The naturalness of the new filtration equations is that they are the exact consequences of the fundamental laws of physics, directly take into account the density and porosity of the soil, the viscosity and density of the filtration fluid, drainage, the influence of gravity, etc.the falsity of the traditional continuity equation in the filtration theory is Established. New filtration equations are derived from the equation of continuum dynamics in stresses, including the density and viscosity of the liquid and the porosity of the soil.Inadequacy of the modeling filter equations with the friction law of Newton. The efficiency of simulation of filtration by Jakupova equations based on the power laws of friction with odd exponents is numerically confirmed, with the use of which the calculations of filtration in the well, drainage under the influence of gravity, displacement of oil by water from the underground area through two symmetrically located pits are carried out.
Keywords: filtration, pressure, velocity, acceleration, equations.

Falsifications and contradictions of Darcy's law equations»

$$k\mathbf{v} = -gradp + \rho\mathbf{F},\ div\mathbf{v} = 0$$

with the fundamental laws of physics detailed in [1] and [13]. There is also considered the non-representativeness of the application of "Darcy's law" in the theory of spatial filtration. It shows the contradictions of equations "Darcy's law" law of friction and the second law of Newton. It is found that the spatial equations of "Darcy's law" correspond to potential flows, which contradicts the theory of viscous fluid. The equations of the "Darcy's law" do not comply with the law of conservation of energy. Based on the fact that the equations of the "Darcy's law" are composed of derivatives of the 1st order, contradictory problems of setting boundary conditions are revealed.

The equations of the Forchheimer model also contradict the laws of physics:

$$\rho_f \frac{\partial \mathbf{v}}{\partial t} = -\varphi[grad(p+\rho_f gz) + \rho_f \frac{\nu}{K}\mathbf{v} + \rho_f c_f K^{-\frac{1}{2}}|\mathbf{v}|\mathbf{v}],\ div\mathbf{v} = 0,$$

φ -coefficient of porosity of the medium, K – coefficient of permeability, ρ_f -is the fluid density, c_f -is the dimensionless coefficient of friction of Forchheimer. The viscosity of the liquid is also included ν.

The Forchheimer dynamics equation contains only local acceleration, but there is no transfer of medium particles $(\mathbf{v}\cdot\nabla)\mathbf{v}$, which is a *gross error*.

NumeroffC.H. as early as 1968г.paid attention to necessity of account acceleration (forces of inertia) in basic equalizations of theory of filtration and offered in [10] to use the next system of equalizations:

$$\frac{1}{g\sigma}\frac{\partial \mathbf{v}}{\partial t}+\frac{1}{(g\sigma)^2}(\mathbf{v}\cdot\nabla)\mathbf{v}+gradh+f(\mathbf{v})\mathbf{v}=0,\ div\mathbf{v}=0$$

The same Numeroff first specified on impermissibility of breaches of the second Newton's law, that says of : "mass on an acceleration equal to force". If an acceleration is equal to the zero, then on the first Newton's law a body accomplishes rectilineal motion with permanent speed or reposes. In the model of Нумерова, as well as in the model ofForchheimerthe first derivatives enter from the sought after functions, consequently, there is a problem of raising of regional terms.

1. Falseness of traditional equalization of indissolubility with the coefficient of porosity in the theory of filtration

In [3-5] and other in equalization of indissolubility is plug porosity

$$\frac{\partial s\rho}{\partial t}+div\rho\mathbf{v}=0$$

(In Wikipedia given in a form $\frac{\partial m\rho}{\partial t}+div\rho\mathbf{v}=0$). We will prove falsity of this equalization of indissolubility.

We will proceed from the fact that the elementary volume of the continuous medium is represented by the sum of the individual volumes of soil $\delta\tau_g$ and to the leakliquid $\delta\tau_f$: $\delta\tau=\delta\tau_g+\delta\tau_f$. Mass $\delta\tau$ is equal to the sum of the masses of soil and liquid $\delta m=\delta m_g+\delta m_f, \delta m_g=\rho_g\delta\tau_g, \delta m_f=\rho_f\delta\tau_f$. .

Complete porosity is entered by attitude of volume $\delta\tau_f$ of pores toward a volume $\delta\tau$:

$$\mathrm{m}_0=\delta\tau_f/\delta\tau=\delta\tau_f/(\delta\tau_g+\delta\tau_f)$$

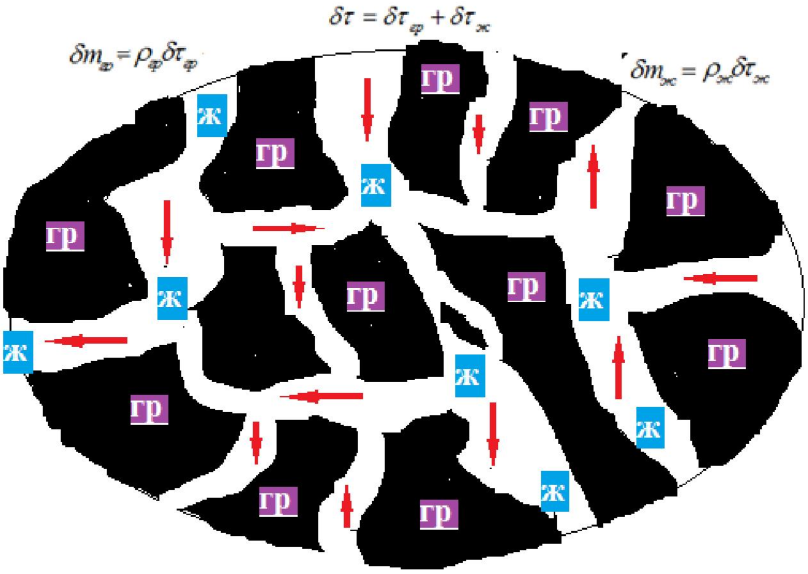
Effective porosity [3-5] there is a relation $s=\delta\tau_f/\delta\tau_g$.

1^0. Sources, flowsare contained in the volume of liquid $\delta\tau_f$.

Law of maintenance of mass taking into account by volume intensity J of flows and sources of liquid $\frac{d\delta m}{dt}=\delta m_f J$ we will present through the masses of soil and liquid (on a picture the elements of soil are drawn by a black):

$$\frac{d(\delta m_g+\delta m_f)}{dt}=\delta m_f J \tag{1}$$

Soil, a liquid flows through the pores of that, is immobile and mass of him is permanent $\delta m_g=\rho_g\delta\tau_g=const$. Therefore $\frac{d\delta m_g}{dt}=0$.



Consequently, from equalization (1) for a liquid it will be

$$\frac{d\delta m_f}{dt} = \delta m_f J, \ \frac{d\rho_f \delta\tau_f}{dt} = \rho_f \delta\tau_f J, \ \rho_f \frac{d\delta\tau_f}{dt} + \delta\tau_f \frac{d\rho_f}{dt} = \rho_f \delta\tau_f J$$

For locomotive $\mathbf{v}$ a formula takes place at a speed of liquid

$$\frac{d\delta\tau_f}{dt} = \delta\tau_f div\mathbf{v}. \text{ Turns out } \rho_f \delta\tau_f div\mathbf{v} + \delta\tau_f \frac{d\rho_f}{dt} = \rho_f \delta\tau_f J.$$

From where equalization of indissolubility flows out with the intensity of sources and flows, being in a liquid

$$\rho_f div\mathbf{v} + \frac{d\rho_f}{dt} = \rho_f J$$

This equalization of indissolubility does not contain the coefficient of porosity, consequently, **does not coincide withequalizations of type**

$$\frac{\partial s\rho}{\partial t} + div\rho\mathbf{v} = 0$$

2^0. Sources and flows are contained in soil.

We have in this case

$$\frac{d\delta m}{dt} = \delta m_g J, \frac{d(\delta m_g + \delta m_g)}{dt} = \delta m_g J, \ \frac{d\delta m_g}{dt} = 0,$$

$$\frac{d\delta m_f}{dt} = \delta m_g J, \delta\tau_f (\rho_f div\mathbf{v} + \frac{d\rho_f}{dt}) = \rho_g \delta\tau_g J$$

Attributing both parts to the volume $\delta\tau_{gr}$ and using the effective coefficient of porosity, we get common equalization of indissolubility

$$\frac{d\rho_f}{dt} + \rho_f div\mathbf{v} = \frac{\rho_g J}{s}$$

For an incompressible liquid $\rho_f div\mathbf{v} = \frac{\rho_g J}{s}$, $div\mathbf{v} = \frac{\rho_g J}{s\rho_f}$.

Further passing to denotation $\rho_f \equiv \rho$, therefore $div\mathbf{v} = \frac{\rho_g J}{s\rho}$.

2.Natural equation of filtration in the soil

We will appeal to equalization of dynamics of individual volume of continuous environment in tensions [13]:

$$\frac{d}{dt}(\mathbf{v}\rho\delta\tau) = \mathbf{F}\rho\delta\tau + (\frac{\partial \mathbf{p}_x}{\partial x} + \frac{\partial \mathbf{p}_y}{\partial y} + \frac{\partial \mathbf{p}_z}{\partial z})\delta\tau$$

In a volume $\delta\tau = \delta\tau_g + \delta\tau_f$ soil is immobile, mass force only on motion ofliquid $\mathbf{F}\rho\delta\tau \Rightarrow \mathbf{F}\rho\delta\tau_f$,turns out therefore

$$\frac{d}{dt}[\mathbf{v}\rho(\delta\tau_g + \delta\tau_f)] = \mathbf{F}\rho\delta\tau_f + (\frac{\partial \mathbf{p}_x}{\partial x} + \frac{\partial \mathbf{p}_y}{\partial y} + \frac{\partial \mathbf{p}_z}{\partial z})\delta\tau$$

Soil is immobile $\frac{d}{dt}[\mathbf{v}\rho\delta\tau_g] = 0$. Equalization assumes an air

$$\rho\delta\tau_f \frac{d\mathbf{v}}{dt} + \mathbf{v}\frac{d\rho\delta\tau_f}{dt} = \mathbf{F}\rho\delta\tau_f + (\frac{\partial \mathbf{p}_x}{\partial x} + \frac{\partial \mathbf{p}_y}{\partial y} + \frac{\partial \mathbf{p}_z}{\partial z})\delta\tau,$$

$$\rho\delta\tau_f \frac{d\mathbf{v}}{dt} + \mathbf{v}(\delta\tau_f\frac{d\rho}{dt} + \rho\frac{d\delta\tau_f}{dt}) = \mathbf{F}\rho\delta\tau_f + (\frac{\partial \mathbf{p}_x}{\partial x} + \frac{\partial \mathbf{p}_y}{\partial y} + \frac{\partial \mathbf{p}_z}{\partial z})\delta\tau,$$

where $\frac{d\delta\tau_f}{dt} = \delta\tau_f \mathrm{div}\mathbf{v}$. Taking to $\delta\tau = \delta\tau_g + \delta\tau_f$,we get equalization of dynamics of liquid (filtrations) in soil in a general view

$$\frac{\delta\tau_f}{\delta\tau}[\rho\frac{d\mathbf{v}}{dt} + \mathbf{v}(\frac{d\rho}{dt} + \rho\mathrm{div}\mathbf{v})] = \mathbf{F}\rho\frac{\delta\tau_f}{\delta\tau} + (\frac{\partial \mathbf{p}_x}{\partial x} + \frac{\partial \mathbf{p}_y}{\partial y} + \frac{\partial \mathbf{p}_z}{\partial z}),$$

In equalization complete porosity is included in **natural way** " m_0 ":

$$\mathrm{m}_0 = \delta\tau_f / \delta\tau = \delta\tau_f / (\delta\tau_g + \delta\tau_f),\ 0 \le \mathrm{m}_0 \le 1,$$

$$\mathrm{m}_0[\rho\frac{d\mathbf{v}}{dt} + \mathbf{v}(\frac{d\rho}{dt} + \rho\mathrm{div}\mathbf{v})] = \mathrm{m}_0\rho\mathbf{F} + \frac{\partial \mathbf{p}_x}{\partial x} + \frac{\partial \mathbf{p}_y}{\partial y} + \frac{\partial \mathbf{p}_z}{\partial z}$$

The particles of liquid at motion in soil test braking from contiguity with the particulate matters of soil, that increases force of friction considerably $\mathbf{F}_{mpi} = -\frac{k}{s}\mathbf{v}_i$, $s = \frac{\delta\tau_f}{\delta\tau_g}$ – coefficient of porosity. (Blowing air or liquid through a millimeter-diameter tube requires a lot of effort compared to a centimeter-diameter tube.)

The theorem on the asymmetry of the stress tensor of a continuous medium, proved in [2-13], makes it possible to construct a wide spectrum of new rheological laws, from which it is possible to choose suitable models according to the flow velocity and the physical properties of the medium.

Let $u > 0$ and consider the frictional force proportional to the degree of velocity: $\mathbf{F}_{mp} = -\frac{k_u}{s}u^{m_u}\mathbf{i} - \frac{k_v}{s}v^{m_v}\mathbf{j} - \frac{k_w}{s}w^{m_u}\mathbf{k}$,whose projections on the x axis are equal to: $\mathbf{F}_1 = -\frac{k_u}{s}u_1^{m_u}\mathbf{i}$ on the plane y_1 , and $\mathbf{F}_2 = -\frac{k_u}{s}u_2^{m_u}\mathbf{i}$ on a plane y_2 .

Theincreasesofforceandspeedappearbetweenlayers:

$$\delta\mathbf{F} = \mathbf{F}_2 - \mathbf{F}_1,\ \delta\mathbf{F} = -\frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i},\ \delta u^{m_u} = u_2^{m_u} - u_1^{m_u} > 0,$$

thus $\delta\mathbf{F} \uparrow\downarrow \mathbf{i}$. The linear closeness of force is entered $\delta\mathbf{F}$ through a relation $\mathbf{f} = \frac{\delta\mathbf{F}}{\delta y}, \delta\mathbf{F} = \delta y\mathbf{f}$. There is a vector on determination $\mathbf{p}_{yxcp} = \frac{\delta\mathbf{F}}{\delta x\delta z}$ tangent tension parallel and identically directed with forces of friction

$$\mathbf{p}_{yxcp} \uparrow\uparrow \delta\mathbf{F}, \mathbf{p}_{yxcp} \uparrow\uparrow \mathbf{f}.$$

Through the coefficient of proportion we have equalities $\mathbf{f} = k'\mathbf{p}_{yxcp}$, $k' > 0$, $\mathbf{p}_{yxcp} \uparrow\downarrow \mathbf{i}$,

$$k'\mathbf{p}_{yxcp}\delta y = -\frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}.$$

This expression is multiplied by a thirl $\mathbf{i}$:

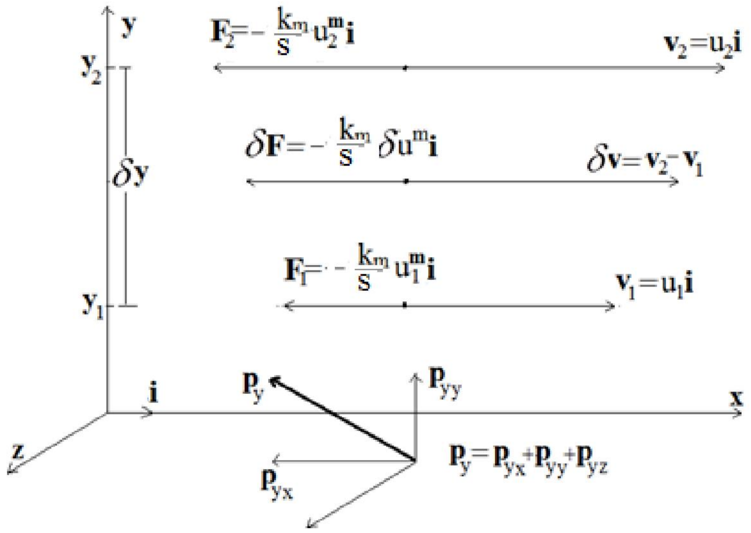
$$k'\delta y\mathbf{p}_{yxcp}\cdot\mathbf{i} = -\frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}\cdot\mathbf{i}.$$

As a result $k'\mathbf{p}_{yxcp}\cdot\mathbf{i}\delta y = k'|\mathbf{p}_{yxcp}||\mathbf{i}|\delta y\cos 180^o = k'p_{yxcp}\cdot 1\cdot\delta y\cdot(-1) = -k'p_{yxcp}\delta y,$

$$-\frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}\cdot\mathbf{i} = -\frac{k_{m_u}}{s}\delta u^{m_u}|\mathbf{i}||\mathbf{i}|\cdot\cos 0^o = -\frac{k_{m_u}}{s}\delta u^{m_u}\cdot 1\cdot 1\cdot 1 = -\frac{k_{m_u}}{s}\delta u^{m_u}$$

Equalities $-k'\delta yp_{yxcp} = -\frac{k_{m_u}}{s}\delta u^{m_u}$, $p_{yxcp} = \frac{k_{m_u}}{sk'}\frac{\delta u^{m_u}}{\delta y}$,в in a limit give tangent tension

$$p_{yx} = \lim_{\delta y\to 0}p_{yxcp},\ p_{yx} = \lim_{\delta y\to 0}\frac{k_{m_u}}{sk'}\frac{\delta u^{m_u}}{\delta y} = \frac{\mu_{m_u}}{s}\frac{\partial u^{m_u}}{\partial y},\ \mu_{m_u} = \frac{k_{m_u}}{k'}$$

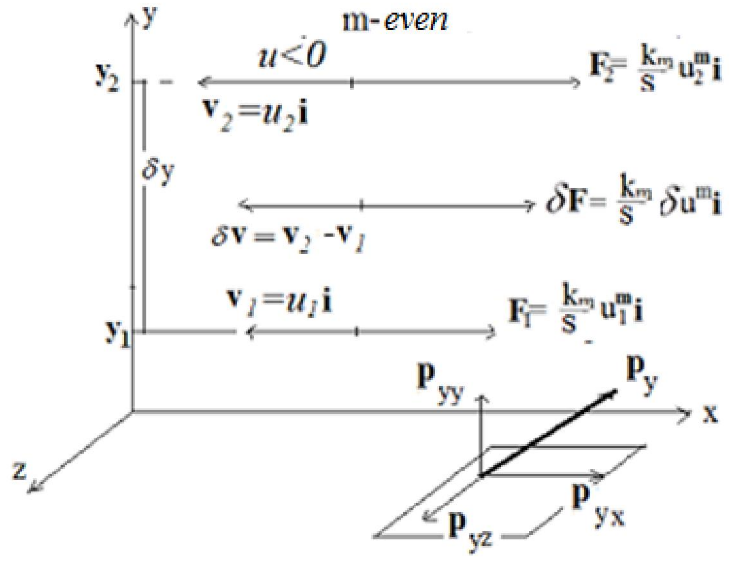


Generalizations of the got formula by transpositions of lower indexes give corresponding tangent tensions

$$p_{xy}=\frac{\mu_{m_v}}{s}\frac{\partial v^{m_v}}{\partial x},p_{xz}=\frac{\mu_{m_w}}{s}\frac{\partial w^{m_w}}{\partial x},p_{zx}=\frac{\mu_{m_u}}{s}\frac{\partial u^{m_u}}{\partial z},p_{yz}=\frac{\mu_{m_w}}{s}\frac{\partial w^{m_w}}{\partial y},p_{zy}=\frac{\mu_{m_v}}{s}\frac{\partial v^{m_v}}{\partial z}$$

Formulas are shown out, for the sake of simplicity, for $u>0, v>0, w>0$

and odd number indexes of degree $m = 1;3;5;7;9...$. The same result turns out for an odd number m и $u \le 0, v \le 0, w \le 0$.



If *m- even* **and** $u<0, v<0, w<0$ **, force of friction is equal**

$\mathbf{F}_{mp}=\frac{k_u}{s}u^{m_u}\mathbf{i}+\frac{k_v}{s}v^{m_v}\mathbf{j}+\frac{k_w}{s}w^{m_u}\mathbf{k}$ (look a picture), projections of that on an axis$x$equal: $\mathbf{F}_1=\frac{k_u}{s}u_1^{m_u}\mathbf{i}$ on a plane y_1 and $\mathbf{F}_2=\frac{k_u}{s}u_2^{m_u}\mathbf{i}$ on a plane y_2 .

The increases of force and speed appear between layers:

$$\delta\mathbf{F}=\mathbf{F}_2-\mathbf{F}_1,\ \delta\mathbf{F}=\frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i},\ \delta u^{m_u}=u_2^{m_u}-u_1^{m_u}>0,$$ thus

$\delta\mathbf{F}\uparrow\uparrow\mathbf{i}$. The linear closeness of force is entered $\delta\mathbf{F}$ as a relation

$\mathbf{f}=\dfrac{\delta\mathbf{F}}{\delta y},\delta\mathbf{F}=\delta y\mathbf{f}$. There is a vector of tangent tension on determination

$\mathbf{p}_{yxcp}=\dfrac{\delta\mathbf{F}}{\delta x\delta z}$ parallel and identically directed with forces of friction

$$\mathbf{p}_{yxcp}\uparrow\uparrow\delta\mathbf{F},\mathbf{p}_{yxcp}\uparrow\uparrow\mathbf{f}.$$

The coefficient of proportion gives equalities $\mathbf{f}=k'\mathbf{p}_{yxcp}$, $k'>0$, $\mathbf{p}_{yxcp}\uparrow\uparrow\mathbf{i}$, $k'\mathbf{p}_{yxcp}\delta y=\dfrac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}$.

This expression is multiplied by a thirl $\mathbf{i}$: $k'\delta y\mathbf{p}_{yxcp}\cdot\mathbf{i}=\dfrac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}\cdot\mathbf{i}$.

As a result $k'\mathbf{p}_{yxcp}\cdot\mathbf{i}\delta y=k'\left|\mathbf{p}_{yxcp}\right|\left|\mathbf{i}\right|\delta y\cos 0^o=k'p_{yxcp}\cdot 1\cdot\delta y\cdot(1)=k'p_{yxcp}\delta y,$

$$\frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}\cdot\mathbf{i}=\frac{k_{m_u}}{s}\delta u^{m_u}\left|\mathbf{i}\right|\left|\mathbf{i}\right|\cdot\cos 0^o=\frac{k_{m_u}}{s}\delta u^{m_u}\cdot 1\cdot 1\cdot 1=\frac{k_{m_u}}{s}\delta u^{m_u}$$

Equalities $k'\delta y p_{yxcp}=\dfrac{k_{m_u}}{s}\delta u^{m_u},\ p_{yxcp}=\dfrac{k_{m_u}}{sk'}\dfrac{\delta u^{m_u}}{\delta y}$,

in a limit give tangent tension

$$p_{yx}=\lim_{\delta y\to 0}\frac{k_{m_u}}{k'}\frac{\delta u^{m_u}}{\delta y}=\mu_{m_u}\frac{\partial u^{m_u}}{\partial y},\ \mu_{m_u}=\frac{k_{m_u}}{k'}$$

Id est the same result turns out.

Conclusion of viscid constituents of normal tensions

The analogical reasoning is set the formula of constituent $\overset{0}{\mathbf{p}}_{xx}$ normal tension $\mathbf{p}_{xx}=-p\mathbf{i}+\overset{0}{\mathbf{p}}_{xx}$.

Let forces of friction be equal: $\mathbf{F}_1=-\dfrac{k_u}{s}u_1^{m_u}\mathbf{i}$ on a plane x_1 и $\mathbf{F}_2=-\dfrac{k_u}{s}u_2^{m_u}\mathbf{i}$ on a plane

$x_2=x_1+\delta x$, $\delta\mathbf{F}=\mathbf{F}_2-\mathbf{F}_1,\ \delta\mathbf{F}=-\dfrac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i},\ \delta u^{m_u}=u_2^{m_u}-u_1^{m_u}>0$, thus $\delta\mathbf{F}\uparrow\downarrow\mathbf{i}$.

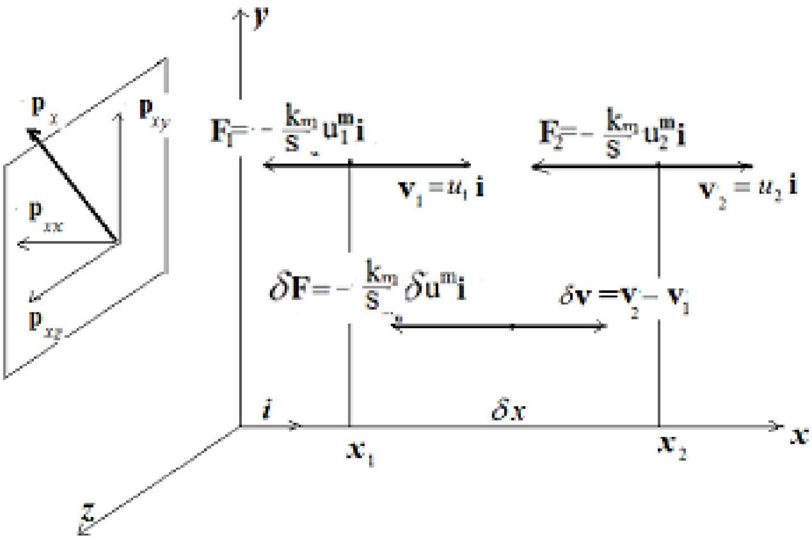
The linear closeness of force is equal: $\mathbf{f}=\dfrac{\delta\mathbf{F}}{\delta x},\delta\mathbf{F}=\delta x\mathbf{f}$.

There is a vector of normal viscid tension on determination

$\mathbf{p}^0_{xxcp} = \dfrac{\delta \mathbf{F}}{\delta y \delta z}$ parallel and identically directed with forces of friction $\mathbf{p}^0_{xxcp} \uparrow\uparrow \delta\mathbf{F},\ \mathbf{p}^0_{xxcp} \uparrow\uparrow \mathbf{f}$.

The coefficient of proportion results in equalities $\mathbf{f} = k''\mathbf{p}^0_{xxcp},\ k'' > 0$,

$$\mathbf{p}^0_{xxcp} \uparrow\downarrow \mathbf{i},\ k'\mathbf{p}_{xxcp}\delta x = -\frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}.$$



This expression is multiplied by a thirl $\mathbf{i}$: $k'\delta x\mathbf{p}_{xxcp} \cdot \mathbf{i} = -\dfrac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}\cdot\mathbf{i}$.

As a result $k'\mathbf{p}^0_{xxcp}\cdot\mathbf{i}\delta x = k'\left|\mathbf{p}^0_{xxcp}\right|\left|\mathbf{i}\right|\delta x\cos 180^o = k'p^0_{xxcp}\cdot 1\cdot\delta x\cdot(-1) = -k'p^0_{xxcp}\delta x,$

$-\dfrac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}\cdot\mathbf{i} = -\dfrac{k_{m_u}}{s}\delta u^{m_u}\left|\mathbf{i}\right|\left|\mathbf{i}\right|\cdot\cos 0^o = -\dfrac{k_{m_u}}{s}\delta u^{m_u}\cdot 1\cdot 1\cdot 1 = -\dfrac{k_{m_u}}{s}\delta u^{m_u}$ Equalities

$$-k'\delta x p^0_{xxcp} = -\frac{k_{m_u}}{s}\delta u^{m_u},\quad p^0_{xxcp} = \frac{k_{m_u}}{sk'}\frac{\delta u^{m_u}}{\delta x},$$

in a limit give viscid member of normal tension

$$p^o_{xx} = \lim_{\delta x\to 0}\frac{k_{m_u}}{sk''}\frac{\delta u^{m_u}}{\delta x} = \frac{\mu_{m_u}}{s}\frac{\partial u^{m_u}}{\partial x},\quad \mu_{m_u} = \frac{k_{m_u}}{k''} > 0$$

Generalizations of the got formula by transpositions of lower indexes give corresponding tensions

$$p^o_{yy} = \frac{\mu_{m_v}}{s}\frac{\partial v^{m_v}}{\partial y},\quad p^o_{zz} = \frac{\mu_{m_w}}{s}\frac{\partial w^{m_w}}{\partial z}$$

Obviously, complete normal tensions are the sum of hydrodynamic pressure andviscid constituents

$$p_{xx} = -p + p^o_{xx} = -p + \frac{\mu_{m_u}}{s}\frac{\partial u^{m_u}}{\partial y}, p_{yy} = -p + p^o_{yy} = -p + \frac{\mu_{m_v}}{s}\frac{\partial v^{m_v}}{\partial y},$$

$p_{zz} = -p + p_{zz}^{o} = -p + \frac{\mu_{m_w}}{s}\frac{\partial w^{m_w}}{\partial z}$ Single values of indexes of degrees

$m_u = 1, m_v = 1, m_w = 1$ correspond to the law of friction law of Newton. All exponentsmust be odd integers [2]. The filtrationequations for an incompressible fluid according to Newton's law of friction with coefficients of porosities in a gravity field have the form (x_3 =z):

$$\mathrm{m}_0[\rho(\frac{\partial v_i}{\partial t} + \sum_{j=1}^{3} v_j \frac{\partial v_i}{\partial x_j}) + v_i \rho div\mathbf{v}] + \frac{\partial}{\partial x_i}(p + \mathrm{m}_0 \rho g z) = \frac{\mu}{s}\sum_{j=1}^{3}\frac{\partial^2 v_i}{\partial x_j^2}, i = 1,2,3,$$ relating to

$\rho = const$ we find

$$\mathrm{m}_0[\frac{dv_i}{dt} + v_i \frac{\rho_g J}{s\rho}] = -\frac{\partial H}{\partial x_i} + \frac{\mu}{s\rho}\Delta v_i, i = 1,2,3, \quad H = \frac{p}{\rho} + \mathrm{m}_0 g z$$

The equations in dimensionless variables (with strokes) with the effective porosity coefficient in the continuity equation:

$$\mathrm{m}_0[\frac{dv'_i}{dt'} + v'_i \frac{\rho_g J\mathrm{L}}{s\rho\mathrm{U}}] = -\frac{\partial h'}{\partial x'_i} + \frac{\nu}{s\mathrm{UL}}\Delta v'_i, i = 1,2,3,$$

$$div\mathbf{v}' = \frac{\rho_g J\mathrm{L}}{s\rho\,\mathrm{U}} \tag{2}$$

Thus, the equations include the density ρ_g soil and porosity coefficients, which emphasizes their significant effect on filtration.

Dimensionless numbers are formed:

$$\mathrm{m}_0[\frac{dv'_i}{dt'} + v'_i\,\mathrm{Ds}] = -\frac{\partial h'}{\partial x'_i} + \frac{1}{s\mathrm{Re}}\Delta v'_i, \ i = 1,2,3, \ div\mathbf{v}' = \mathrm{Ds},$$

$\mathrm{Ds} = \frac{\rho_g J\mathrm{L}}{s\rho\ \mathrm{U}}$, $\mathrm{Re} = \frac{\mathrm{UL}}{\nu}$ - Reynoldsnumber.

For sources and drains contained in liquid $\mathrm{Ds} = \frac{J\mathrm{L}}{\mathrm{U}}$. Here a new dimensionless number is given the name Darcy.

3. Modeling filtering equations with Newton's law of friction

In the well height*1000m*and wide*100m*with an impenetrable wall accumulated*200m*layer of water that displacesoil through the 800m layer of soil so that at the same time at the input upper boundary *y = 1000m* the water velocity is equal to 001m / s.The lower boundary of the soil, y = 0, is permeable to liquid.Filtration scales taken: cavity width *L = 100m,* water entry rate through the upper limit *U = 0.001 m / s,* water density $\rho_в = 1000 кг / м^3$. Oil density

$\rho_н = 880 кг / м^3$, kinematic viscosity of water $\nu_в = 0.00556 \cdot 10^{-4} м^2 / c$, oil kinematic viscosity $\nu_н = 22.6 \cdot 10^{-6} м^2 / c$, effective soil porosity $s = 10^{-4}$, total porosity $\mathrm{m}_0 = 10^{-5}$. Power drain in the ground is such that $\mathrm{Ds} = -0,05$. Generalized density

$\rho = \alpha_в \rho_в + \alpha_н \rho_н$ and kinematic viscosity of the medium $\nu = \alpha_в \nu_в + \alpha_н \nu_н$, $\alpha_в + \alpha_н = 1$ calculated by the equation of transfer of water concentration [8]: $\frac{\partial \alpha_в}{\partial t} + \mathbf{v} \cdot \nabla \alpha_в = 0$.

Dimensionless generalized density:

$$\rho' = \alpha_в + \alpha_н \rho_н / \rho_в, \quad \frac{\partial \alpha_в}{\partial t'} + \mathbf{v}' \cdot \nabla \alpha_в = 0$$

The difference grid 100x200, a dimensionless time step, satisfies the condition of counting stability according to a semi-implicit five-point scheme without "circuit diffusion".

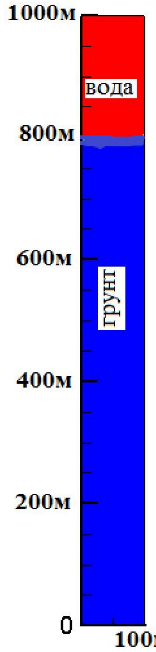
Before the ground in the area from *800m* to *1000m* the numerical calculation was carried out using the dimensionless Navier equations

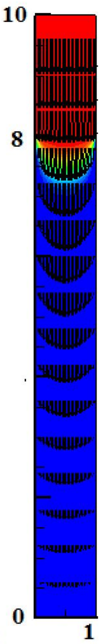
$$\rho' \frac{dv'_i}{dt'} = -\frac{\partial h'}{\partial x'_i} + \frac{1}{\mathrm{Re}_в} \Delta v'_i, i = 1,2,3, \ div\mathbf{v}' = 0$$

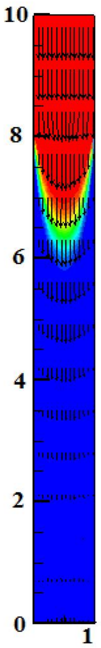
There is oil in the ground. Also used were the Navier equations, but with porosity coefficients:

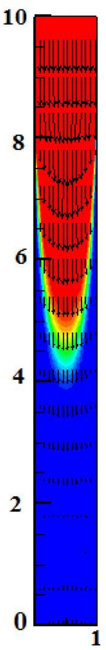
$$\rho' \mathrm{m}_0 \left(\frac{dv'_i}{dt'} + v'_i \mathrm{Ds}\right) = -\frac{\partial h'}{\partial x'_i} + \frac{1}{\mathrm{sRe}} \Delta v'_i, i = 1,2,3, \ div\mathbf{v}' = \mathrm{Ds}$$

The location of water and oil at the initial moment of time is reflected in figure 1. Sticking conditions are set at the lateral boundaries of the well $\mathbf{v}' = 0$, at the output boundary $y = 0$, the vertical derivatives of the velocity components are considered equal to 0. The water entering through the upper boundary $y = 1000m$ begins to displace oil, the diagrams of the velocity vector fields in the cavity are shown in fig.2-4. The parabolic distribution of the velocity vector across the width of the channel in the ground can be seen in fig.2-4.Separately, fig. 5 shows a parabolic vertical velocity profile in one of the soil sections. The nonphysality of this flow in the ground is evident from the pictures of the velocity vector fields, which is the content of the output: the Navier equations (the old name is Navier-Stokes) are not a filtration model









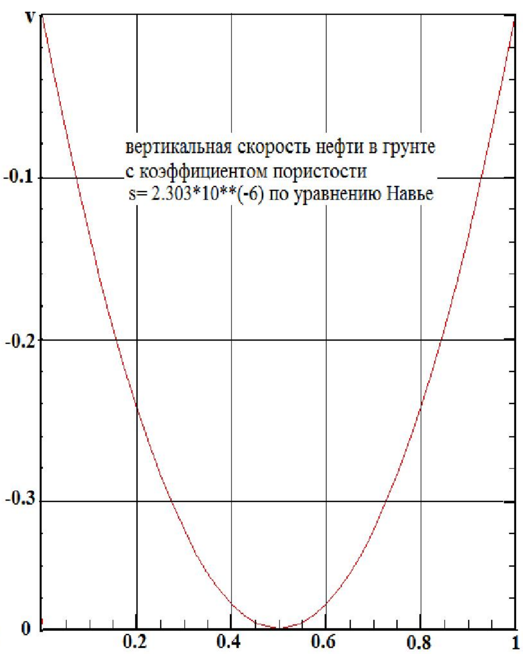


Fig.1 Fig.2 Fig.3 Fig.4 Fig..5

4.Filtration modeling by power friction equations

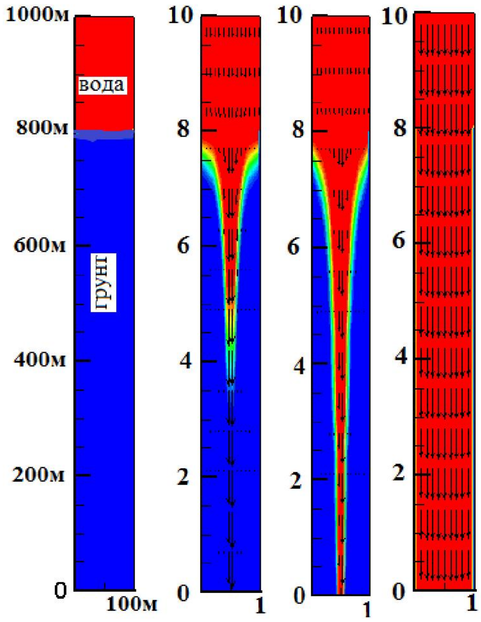


Fig.6 Fig.7 Fig.8

In this case, the dynamics of water over the ground is modeled by the equations of Jakupov's power laws of friction:

$$\rho\frac{dv_i}{dt} = -\frac{\partial p}{\partial x_i} + \sum_{j=1}^{3}\frac{\partial}{\partial x_j}\left(\frac{\alpha\mu}{m_i^{m_i-1}}\frac{\partial v_i^{m_i}}{\partial x_j}\right) + \rho F_i, i=1,2,3, div\mathbf{v}=0$$

This continuity equation corresponds to the fact that there are no sources and sinks in the flow of water. Equations in dimensionless variables have the form

$$\rho'\frac{dv'_i}{dt'} = -\frac{\partial h'}{\partial x'_i} + \sum_{j=1}^{3}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x'_j}\left[\left(\frac{\alpha U}{m_i}\right)^{m_i-1} m_i v_i'^{m_i-1}\frac{\partial v'_i}{\partial x'_j}\right], i=1,2,3, div\mathbf{v}'=0,\ \alpha=1\frac{сек}{м}$$

The exponents are determined in intervals[13-14:

$$|v_i| < 1/9\ \ m_i = 1;\ \ 1/9 \le |v_i| < 3/9\ \ m_i = 3; \tag{3}$$

$$3/9 \le |v_i| < 5/9\ \ m_i = 5;\ \ 5/9 \le |v_i| < 7/9\ \ m_i = 7;\ \ 7/9 \le |v_i|\ \ m_i = 9$$

The enormous resistance to movement (leakage) of a liquid in the soil is taken into account in the mass conservation equation and in the power law equations using the effective filtration coefficient s:

$$\rho' \mathrm{m}_0\left(\frac{dv'_i}{dt'} + v'_i Ds\right) = -\frac{\partial h'}{\partial x'_i} + \sum_{j=1}^{3}\frac{1}{s\mathrm{Re}}\frac{\partial}{\partial x'_j}\left[\left(\frac{\alpha U}{m_i}\right)^{m_i-1} m_i v_i'^{m_i-1}\frac{\partial v'_i}{\partial x'_j}\right], i=1,2,3,$$

$$div\mathbf{v}' = \mathrm{Ds},\quad \mathrm{Ds} = \frac{\rho_g J\mathrm{L}}{s\rho \mathrm{U}}$$

Initially, the determination of the degrees in the ground was carried out according to the distribution (3). The computed velocity fields in FIG. 6 and 7 show the unsuitability of this velocity distribution in the ground.In the ground, the equations of the vertical velocity proved to be effective as a constant exponent

$$m_i = 17, i = 1,2,3\,:$$

$$\rho' \mathrm{m}_0 \left(\frac{dv'_i}{dt'} + v'_i Ds\right) = -\frac{\partial h'}{\partial x'_i} + \sum_{j=1}^{3} \frac{1}{s\,\mathrm{Re}} \frac{\partial}{\partial x'_j} \left[\left(\frac{\alpha U}{17}\right)^{16} 17 v'^{16}_i \frac{\partial v'_i}{\partial x'_j} \right], i = 1, 2, 3,$$

$$div\,\mathbf{v}' = \mathrm{D\,s} \tag{4}$$

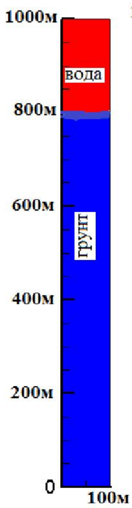


Fig.9

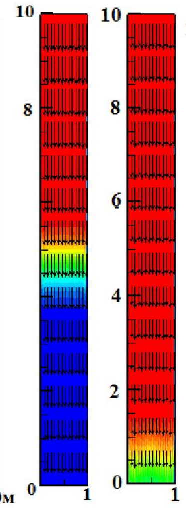


Fig.10

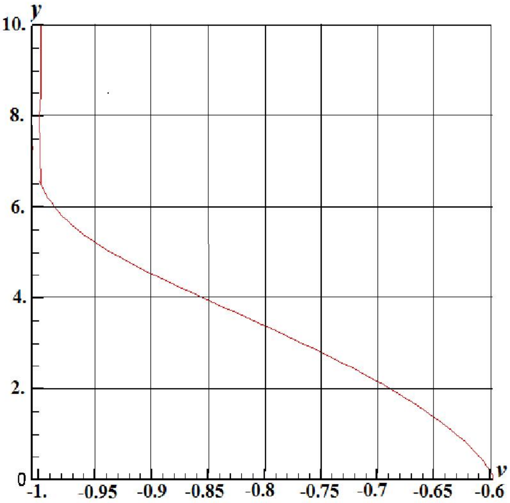


Fig.11

Fig.8 shows the field of the vector of water flow rates simply in the channel without soil, without sources-effluents, calculated according to the power law (2) and (4).

On fig. 9 and 10 show the velocity vector fields at different points in time. The difference from the parabolic velocity fields obtained by the Navier equations (-Stok`s) is simply amazing !!!Fig. 11 represents the vertical velocity distribution$v$the height of the channel in the cross section x = 50 m.

On fig. 12 shows the trapezoidal profile of the vertical velocity v in the horizontal plane of the water y = *900m,* on fig. 13 in the horizontal plane of the soil y = *400m.*

In the ground, the speed of the filtration fluid is significantly reduced, as can be seen on fig. 9, 10, 11.

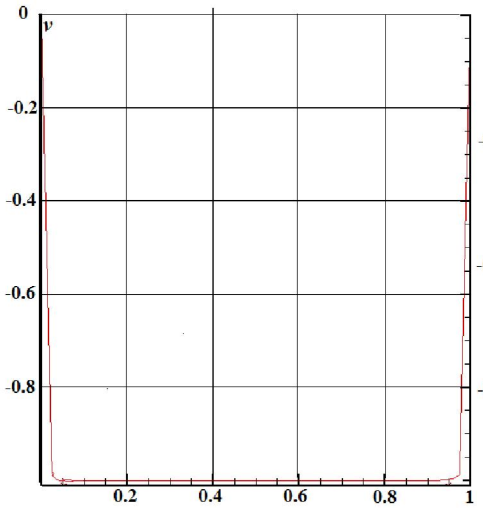


Fig.12

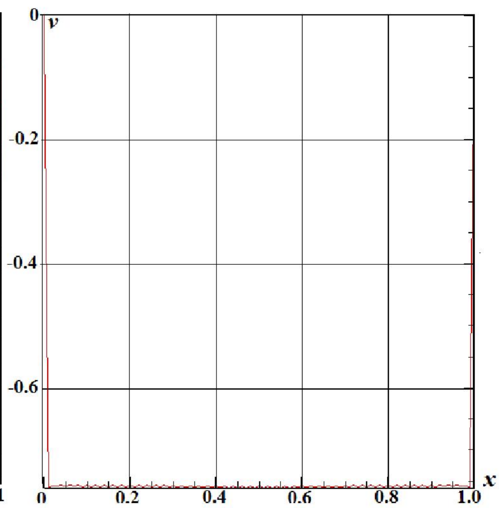


Fig.13

5. Filtration (drainage) under the action of gravity

Above are the results of calculations of forced filtration, when the upper limit of the channel *y=1000m* water is supplied at a speed of *0.001 m/s.*

In this paragraph, the numerical calculations of liquid filtration (drainage) under the influence of gravity, which acts on water from 800m to 1000m (the number of Froude is equal to $Fr=U^2/(gL)$), in the soil the effect of gravity is taken into account by the coefficient of total porosity "$m_0 = 0.00001$". In addition to the above scale, the scale of the speed adopted $U = 0.01\sqrt{gL},\ \ g=9.81м/c^2$.

Before the ground $800м \le y \le 1000м$ the equations with the Froude number are applied (for the above reasons):

$$\rho'\frac{du'}{dt'} = -\frac{\partial p'}{\partial x'_i} + \sum_{j=1}^{2}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x'_j}[(\frac{\alpha U}{m_u})^{m_u-1} m_u u'^{m_u-1}\frac{\partial u'}{\partial x'_j}],$$

$$\rho'\frac{dv'}{dt'} = -\frac{\partial p'}{\partial x'_2} + \sum_{j=1}^{2}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x'_j}[(\frac{\alpha U}{m_v})^{m_v-1} m_v v'^{m_v-1}\frac{\partial v'}{\partial x'_j}] - \frac{\rho'}{Fr}, div\mathbf{v}'=0,\ \alpha=1\frac{сек}{м}$$

In the ground, equations with a constant exponent are applied $m_i = 17, i = 1,2,3$ and with a total porosity coefficient are applied:

$$m_0\rho'(\frac{du'}{dt'} + u'Ds) = -\frac{\partial p'}{\partial x'_1} + \sum_{j=1}^{2}\frac{1}{s\mathrm{Re}}\frac{\partial}{\partial x'_j}[(\frac{\alpha U}{17})^{16}\cdot 17\cdot u'^{16}\frac{\partial u'}{\partial x'_j}],$$

$$m_0\rho'(\frac{dv'}{dt'} + v'Ds) = -\frac{\partial p'}{\partial x'_2} + \sum_{j=1}^{2}\frac{1}{s\mathrm{Re}}\frac{\partial}{\partial x'_j}[(\frac{\alpha U}{17})^{16}\cdot 17\cdot v'^{16}\frac{\partial v'}{\partial x'_j}] - \frac{\rho' m_0}{Fr}, div\mathbf{v}'=Ds$$

Fig.14 and 15 are the fields of the velocity vector at different times of oil displacement by water. Fig. 16 represents the distribution of the vertical velocity *v* along the height of the channel in the cross section x=50 m. the Difference between the distributions of the vertical velocities in Fig. 11 and Fig.16 the obvious.

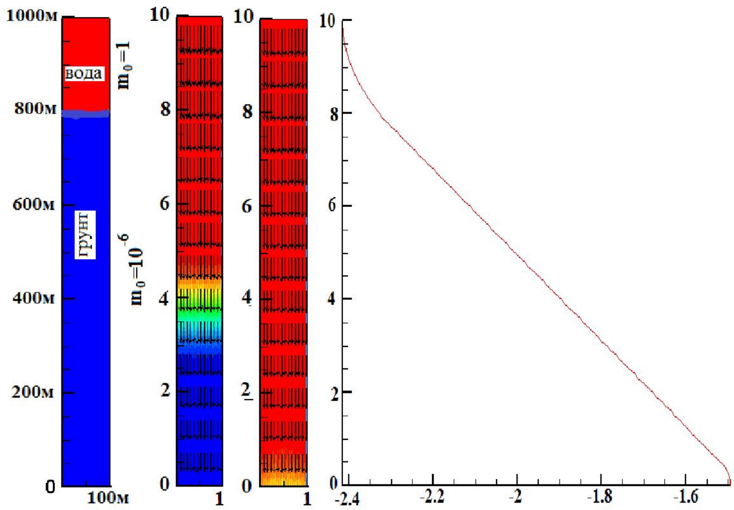


Fig.14 Fig.15 Fig..16

6. Displacement of oil by water from underground volume

Before the ground in the water are applied (for the above reasons) equations with the number of Froude:

$$\rho'\frac{du'}{dt'}=-\frac{\partial p'}{\partial x'_i}+\sum_{j=1}^{2}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x'_j}[(\frac{\alpha U}{m_u})^{m_u-1}m_u u'^{m_u-1}\frac{\partial u'}{\partial x'_j}],$$

$$\rho'\frac{dv'}{dt'}=-\frac{\partial p'}{\partial x'_2}+\sum_{j=1}^{2}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x'_j}[(\frac{\alpha U}{m_v})^{m_v-1}m_v v'^{m_v-1}\frac{\partial v'}{\partial x'_j}]-\frac{\rho'}{Fr}, div\mathbf{v}'=0,\ \alpha=1\frac{сек}{м}$$

Water is supplied to the well at a rate U=2m / s (figures.17). In linear scale taken as the diameter of the borehole *L=1 m.* the Reynolds Number is equal to $\mathrm{Re}=\alpha_в \mathrm{Re}_в+\alpha_н \mathrm{Re}_н$, where $\mathrm{Re}_в$ - Reynolds number for water, $\mathrm{Re}_н$ - Reynolds number for oil.In the ground, equations with a constant exponent are applied $m_u=11,\ m_v=11$ and with the ratio of **total porosity** $\mathrm{m}_0=0.000021$:

$$\rho'\mathrm{m}_0(\frac{du'}{dt'}+u'Ds)=-\frac{\partial p'}{\partial x'_1}+\sum_{j=1}^{2}\frac{1}{s\mathrm{Re}}\frac{\partial}{\partial x'_j}[(\frac{\alpha U}{11})^{10}\cdot 11\cdot u'^{10}\frac{\partial u'}{\partial x'_j}],$$

$$\rho'\mathrm{m}_0(\frac{dv'}{dt'}+v'Ds)=-\frac{\partial p'}{\partial x'_2}+\sum_{j=1}^{2}\frac{1}{s\mathrm{Re}}\frac{\partial}{\partial x'_j}[(\frac{\alpha U}{11})^{10}\cdot 11\cdot v'^{10}\frac{\partial v'}{\partial x'_j}]-\frac{\mathrm{m}_0\rho'}{Fr},\ div\mathbf{v}'=\mathrm{Ds}$$

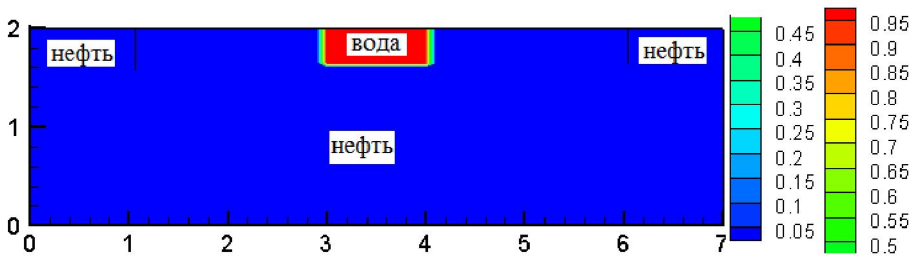


Fig.17

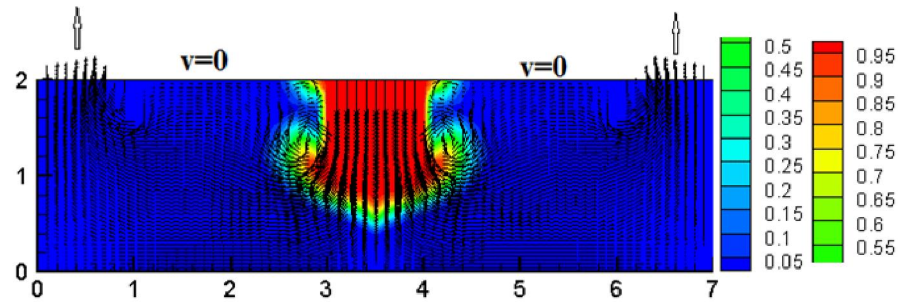


Fig.18

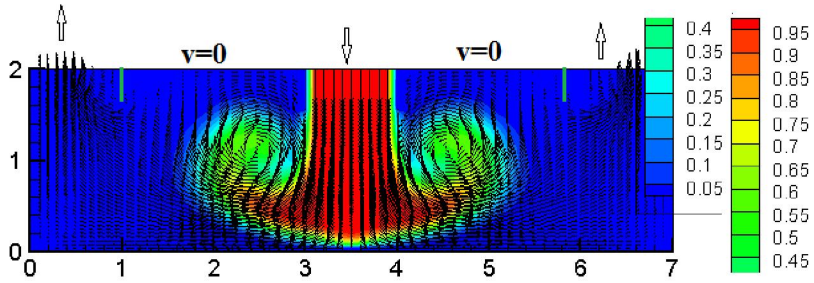


Fig.19

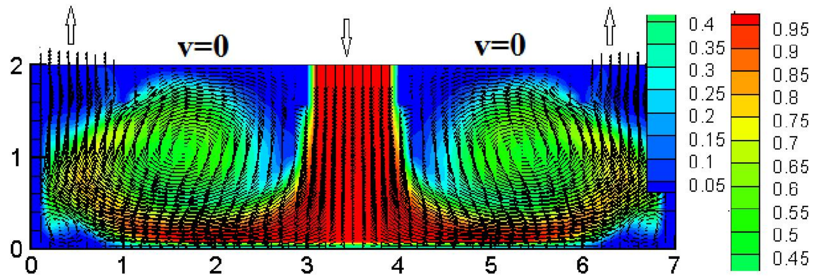


Fig.20

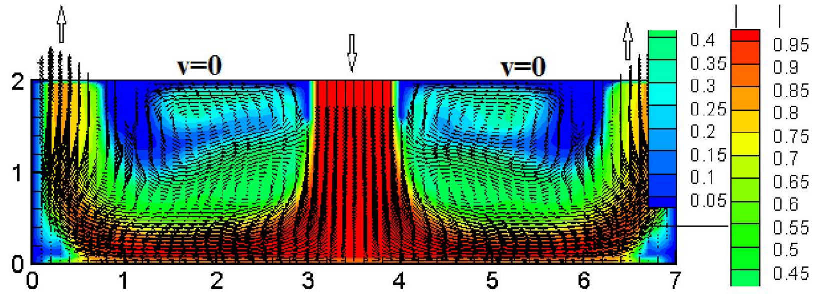


Fig.21

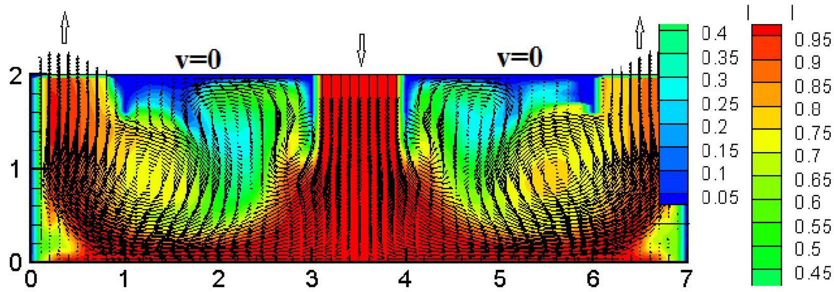


Fig.22

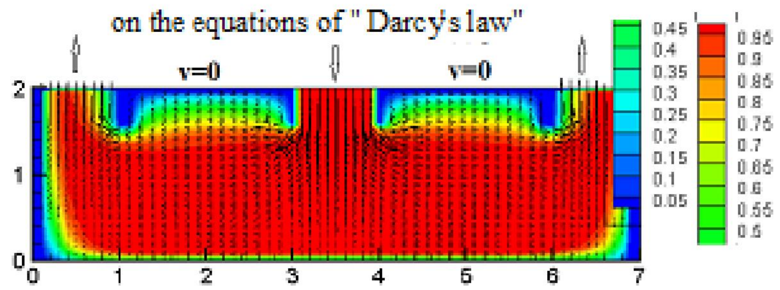


Fig.23

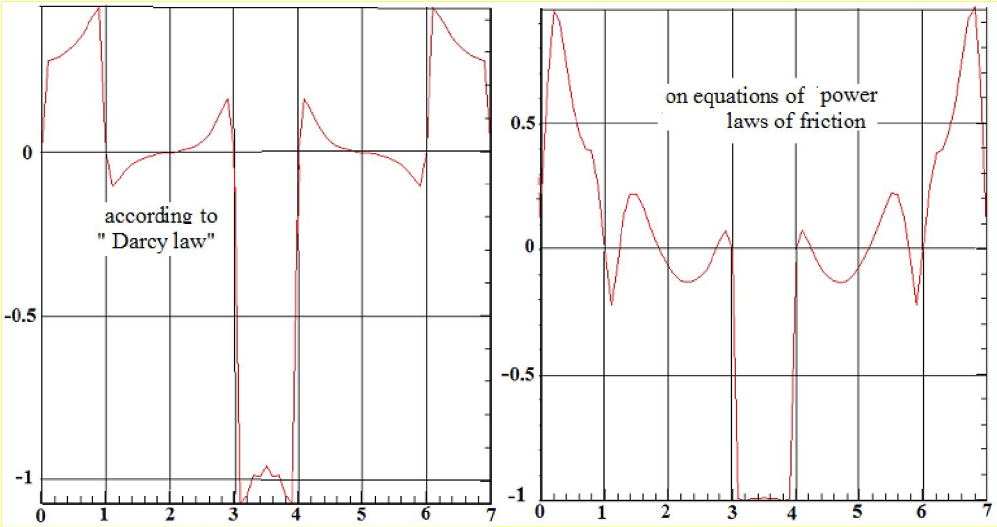


Fig.24 Fig.25

Fig.18-22 the pictures of water penetration into the soil with simultaneous oil displacement through 2 vertical pits $0<x'<1$ and 6<x'<7 are presented.Fig.23 the field of the velocity vector and the location of water at almost complete displacement of oil obtained by the equations of "Darcy's law"arepresented. Horizontal and vertical arrangement of velocity vectors $\mathbf{v}' = u'\mathbf{i} + v'\mathbf{j}$ fig.23 caused by neglect of the forces of inertia (acceleration) in the equations of "Darcy's law".The influence of the inertia forces to the filter demonstrated with paintings of fields of velocity vectors in Fig. 18-22.

Fig.24 and 25 shows a plot of the vertical velocity at the height of the wells

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СҮЗГІНІҢ ТАБИҒАТТЫҚ ТЕҢДЕУЛЕРІ.
«Дарси ЗАҢЫНЫҢ» ҚҰРЫҒАНЫ

Аннотация.Сүзгінің табиғаттық теңдеулерінің теориясы қорытлған.Жаңа теңдеу- лердің табиғаттығы мынада тұр: олар физиканың негізгі заңдарынан шыққан нақты сал- дары, грунттың тығыздығын және кеуектілігін, сүзбелі сұйықтықтың тұтқырлығы мен тығыздығын, дренажды, аурылық күштің әсерін тікелей есепке алғандығы. Сүзгі теория- сында пайдаланатын үзіліссіздік теңдеуідің жалғандығы көрсетілген. Сүзгінің жаңа тең- деулері тұтас ортаның динамикасының керенеулер арқылы жазылған теңдеулерінен шыға- рылған, олрадың құрылысына сұйықтықтың тұтқырлығы мен тығыздығы жәнегрунттың кеуектілігі еңгізілген.. Сүзгіні модельдеуге Ньютонның үйкеліс заңына сәйкес құрылған теңдеулерді пайдаланудың жәрәмсіздігі көрсетілген.Сүзгіні модельдеуге автордың дәрежелі үйкеліс заңдарына дәрежесі дақ сандарболған теңдеулерін пайдалану жәрәмді екені сандық түрде әшкереленген. Осыларды жұмсап скважинадағы сүзгі, ауырлық күш- пен болатын дренажды, жердің астындағы қоймадан мұнайды екі беттескен шурфтар арқылы сығып шығару сандық есептермен орындалған. Библ.18.

Тірек сөздер: сүзгі, қысым, жылдамдық, үдеу.

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УРАВНЕНИЯ ЕСТЕСТВЕННОЙ ФИЛЬТРАЦИИ. ФИАСКО "ЗАКОНА ДАРСИ"

Аннотация. Дана теориянатуральныхуравнений фильтраций. Натуральность новых уравнений фильтраций заключается в том, что они являются точными следствиями фунда- ментальных законовфизики, прямо учитывают плотность и пористость грунта, вязкость и плотность фильтрационной жидкости, дренаж, влияние силы тяжести и др. Установлена фальшивость традиционного уравнения неразрывности в теории фильтрации. Из уравне-ния динамики сплошной среды в напряжениях выводятся новые уравнения фильтрации, включающие плотность и вязкость жидкости и пористость грунта.Установленанеадеква-

тность моделирования фильтрации уравнениями с законом трения Ньютона.Численно подтверждена эффективность моделированияфильтрации уравнениями Джакупова, осно- ванных на степенных законах трения с нечетными показателями степеней, с применением которых проведены расчеты фильтрации в скважине, дренажа под действием силы тяжести**,**вытеснениянефти водою из подземного ареала через две симметрично расположенные шурфы.

.**Ключевые слова:** фильтрация,давление, скорость, ускорение, уравнения.

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