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# RIEMANNIAN METRIC FOR TEXTURE RECOGNITION

**Abstract.** The article discusses the recognition of textures on digital images by methods of computational topology and Riemannian geometry. Topological properties of patterns are represented by segments (barcodes) obtained by filtering by the level of photometric measure. Beginning of barcode encodes level at which topological property appears (connected component and/or “hole”), and its end - level at which the property disappears. Barcodes are conveniently parameterized by coordinates of their ends in rectangular coordinate system “birth” and “death” of topological property. Such representation in form of a cloud of points on plane is called a persistence diagram (PD). In the article show that texture class recognition results are significantly better compared to other vectorization methods of PD.

**Keywords:** Riemannian metric, persistence diagram, probability density function, persistent image (PI).

To describe the patterns of digital images, we use TDA - Topological Data Analysis [1,2]. TDA does not require any a priori assumptions about nature of data source and allows to extract new knowledge from changingshape of neighborhoods of points in space of features.

The approach is associated with persistent images [3], using Riemannian metric to calculate distances between persistence diagrams (PD) is based on analogy, which originates in quantum mechanics (Figure 1).

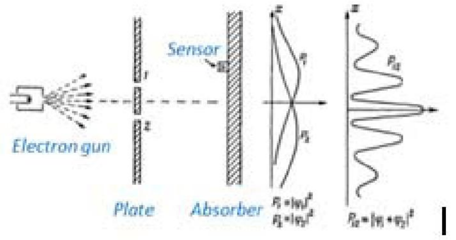



Figure 1 - Experience with two slots[4]

Electronsfromgunpass through plate with two slots.Incasewithoutabsorber, wave interference is described by joint distribution $P_{12}$. Presence of detector leads to double-humped distribution $P_1 + P_2 \neq P_{12}$.

In quantum mechanics introduces a probability amplitude $P_i = |\varphi_i|^2$ .Assuming that amplitudes from two slits add up, we obtain:

$$P_{12} = |\varphi_1 + \varphi_2|^2 = |\varphi_1|^2 + |\varphi_2|^2 + 2|\varphi_1\varphi_2|^2 = P_1 + P_2 + 2|\varphi_1\varphi_2|^2 \qquad (1)$$

Probability density defined on persistence diagrams does not form vector space. But if we introduce additive probability amplitudes, then we can transfer them to Hilbert unit sphere. The distance on such sphere does not depend on the choice of beginning of coordinates and number of points in compared diagrams.

The approach that realizes this idea is based on positive definite multiscalekernel [5,6].It is reliedon the vector representation of persistence diagram in the form ofpersistent image (PI).Since each PD consists of set of points in 2D, we start by creating a two-dimensional probability density function (pdf) using Gaussian kernel with zero mean and variance $\sigma^2$.For each probability density function, we calculate the representation in the form of square root $\phi(x) = \sqrt{pdf}$. In this case, persistence diagram, as element of geodesic $\gamma$ between the compared diagrams $X$ and $Y$, can be written as:

$$\gamma(s) = (1-s)x + s\phi(x), \tag{2}$$

where $x$ - point on diagram $X$, $\phi(x)$ is corresponding point of diagram $Y$, and $s \in [0,1]$ parametrizes the geodesic.

Without loss of generality we assume that all probability density functions lie in $[0,1]^2$. The analyzed space of probability density functions is:

$$\mathrm{P} = \{p : [0,1] \times [0,1] \to \Re\} \ \forall x, y \mid p(x,y) \geq 0, \int_0^1 \int_0^1 p(x,y)dxdy = 1\} \tag{3}$$

Transition from pdf to probability amplitudes is closely related to so-called Fisher-Rao information metric.For discrete probability space Fisher metric can be considered simply as Euclidean metric bounded by positive "quadrant" of unit sphere after corresponding change of variables.Consider Euclidean space $y = (y_0, \ldots, y_n) \in R^{N+1}$. The metric will be defined by quadratic form:

$$h = \sum_{i=0}^{N} dy_i dy_i \ , \tag{4}$$

where $dy_i$ is 1-forms, which form basis in co-tangent space.

Denote by $\frac{\partial}{\partial y_j}$ basis vectors in the tangent space, so that:

$$dy_j \left( \frac{\partial}{\partial y_k} \right) = \delta_{jk} \, . \tag{5}$$

Define $N$ -dimensional unit sphere embedded in the $(N+1)$ -dimensional Euclidean space as:

$$\sum_{i=0}^{N} y_i^2 = 1 \tag{7}$$

This embedding induces metric on sphere, which follows directly from Euclidean metric of surrounding space.Introducevariable change $p_i = y_i^2$.

The equation of sphere then takes the form of a condition of the probability normalization:

$$\sum_i p_i = 1, \tag{8}$$

and metric becomes:

$$h = \sum_i dy_i \, dy_i = \sum_i d\sqrt{p_i} d\sqrt{p_i} = \frac{1}{4} \sum_i \frac{dp_i dp_i}{p_i} = \frac{1}{4} \sum_i p_i d(\log p_i) d(\log p_i) \, . \tag{9}$$

The last expression represents a quarter of Fisher's information metric[7]. Probabilities are parametric functions of the manifold of variables $\theta$, thus $p_i = p_i(\theta)$. Then we obtain the induced metric on parametric manifold:

$$h = \frac{1}{4}\sum_i p_i d(\log p_i(\theta))d(\log p_i(\theta)) = \frac{1}{4}\sum_{jk}\sum_i p_i(\theta)\frac{\partial \log p_i(\theta)}{\partial\theta_j}\frac{\partial \log p_i(\theta)}{\partial\theta_k}d\theta_j d\theta_k, \quad (10)$$

or in coordinate form Fisher's information metric is determined by the tensor:

$$g_{ik}(\theta) = 4h_{jk}^{fisher} = 4h\left(\frac{\partial}{\partial\theta_j}, \frac{\partial}{\partial\theta_k}\right). \quad (11)$$

Geodesic in Fisher metric is difficult to compute. Therefore, we will use representation proposed in the paper [8]. It strongly simplifies subsequent calculations. Instead of probabilities, we will consider the space:

$$\Psi = \{\psi : [0,1]\times[0,1] \to \Re\} \mid \psi \geq 0, \text{и} \int_0^1 \int_0^1 \psi^2(x,y)dxdy = 1\} \quad (12)$$

For any two tangent vectors $v_1, v_2 \in T_\psi(\Psi)$, Fisher-Rao metric is defined asscalar product in Hilbert space:

$$\langle \upsilon_1, \upsilon_2 \rangle = \int_0^1\int_0^1 \upsilon_1(x,y)\upsilon_2(x,y)dxdy. \quad (13)$$

It implies that representation in the form of square root $\psi = \sqrt{p}$ makes space a unit Hilbert sphere with a given metric in the form ofscalar product.For two points $\psi_1, \psi_2$ in such space, geodetic distance between them is defined as:

$$d_H(\psi_1, \psi_2) = \cos^{-1}(\langle \psi_1, \psi_2 \rangle), \quad (14)$$

where in calculating the scalar product of two points $\psi_1, \psi_2$ we normalize scalar product using standard Frobenius norm. Computational complexity for such distances between persistence diagrams increasesas $O(K^2)$, for $K \times K$ discretization on $[0,1]^2$. Increasing of K leads to increasing ofaccuracy of determination of distances, but increases computational complexity.

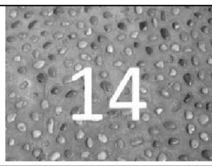
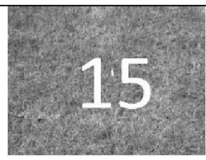
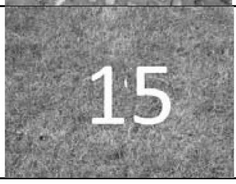
**Numerical results.**For the experiment, we chose the value of resolutionparameter of persistent images $K = 200$. Fromstandard image database [9] four texture classes were selected, two of which contain vegetation images and two - images of inanimate nature.Each class contains 40 images.

In experiments,we calculated the average value and variance of Riemannian distances of PI both within each class and pairwise for all pairs of texture classes.When calculating the distance between two classes, Riemannian distances between all possible pairs of PI textures of two compared classes are computed.

The results are shown in the table.Thus, firstly, all distances between PI of textures within one class are computed.Then, all pairwise distances between PI of textures of different classes are calculated. The average distances between the corresponding PI for Betti 0 and Betti 1 within the class are less than corresponding distances to PI from another class.

However, variances within classes are quite high. Therefore, there may occur cases when Riemannian distance between two arbitrarily taken textures from different classes may be less than the average distance inside the class.For practical use, it is usually necessary to determine the aboutness of not two separately taken textures, but belonging of the considered sample to certain class of textures: grass, stones, water, etc.In such a case, it is appropriate for classes building ofaveraged topological features.Average distances and variance within each of the classes are shown in the table.

Table-Mean and variance of Riemannian distance between all pairs of persistent images (PIs) of textures of 4 classes

| | | | | |
|---|---|---|---|---|
| |  |  |  |  |
|  | $\langle\beta_0\rangle$=10.1<br>$\sigma(\beta_0)$= 4.4<br>$\langle\beta_1\rangle$=8.9<br>$\sigma(\beta_1)$= 4.5 | $\langle\beta_0\rangle$=11.7<br>$\sigma(\beta_0)$= 3.9<br>$\langle\beta_1\rangle$=12.9<br>$\sigma(\beta_1)$= 2.9 | $\langle\beta_0\rangle$=29.0<br>$\sigma(\beta_0)$= 4.4<br>$\langle\beta_1\rangle$=23.3<br>$\sigma(\beta_1)$= 5.3 | $\langle\beta_0\rangle$=25.3<br>$\sigma(\beta_0)$= 5.2<br>$\langle\beta_1\rangle$=13.9<br>$\sigma(\beta_1)$= 3.8 |
|  | | $\langle\beta_0\rangle$=11.7<br>$\sigma(\beta_0)$= 4.7<br>$\langle\beta_1\rangle$=7.8<br>$\sigma(\beta_1)$= 3.1 | $\langle\beta_0\rangle$=32.8<br>$\sigma(\beta_0)$= 2.9<br>$\langle\beta_1\rangle$=26.0<br>$\sigma(\beta_1)$= 3.0 | $\langle\beta_0\rangle$=29.3<br>$\sigma(\beta_0)$= 4.8<br>$\langle\beta_1\rangle$=19.0<br>$\sigma(\beta_1)$= 2.3 |
|  | | | $\langle\beta_0\rangle$=19.1<br>$\sigma(\beta_0)$= 11.7<br>$\langle\beta_1\rangle$=13.9<br>$\sigma(\beta_1)$= 6.6 | $\langle\beta_0\rangle$=38.3<br>$\sigma(\beta_0)$= 4.7<br>$\langle\beta_1\rangle$=22.6<br>$\sigma(\beta_1)$= 4.8 |
|  | | | | $\langle\beta_0\rangle$=23.6<br>$\sigma(\beta_0)$= 15.1<br>$\langle\beta_1\rangle$=16.1<br>$\sigma(\beta_1)$= 7.7 |

In the lines and columns there are 4 classes of textures. The numbers indicate the texture class number. Diagonal elements correspond to distances between PIs of textures inside the class.Off-diagonal elements correspond to pairwise distances between PIs of textures of the two correspondingtoline and columnofclasses. Mean value and variance are calculated separately for Riemannian distances of PI of Betti 0 and Betti 1.

**Conclusion.** Obtained results show that the described approach allows, bypassing large computational complexities, to classify reliably the textures even without the use of machine learning methods.

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## РИМАНОВА МЕТРИКА ДЛЯ РАСПОЗНАВАНИЯ ТЕКСТУР

**Аннотация.** В статье обсуждается распознавание текстур на цифровых изображениях методами вычислительной топологии и римановой геометрии. Топологические свойства паттернов представлены отрезками (баркодами), полученными при фильтрации по уровню фотометрической меры. Начало баркода кодирует уровень на котором появляется топологическое свойство (компонента связности и/или «дыра»), а его конец – уровень на котором свойство исчезает. Баркоды удобно параметризовать координатами их концов в прямоугольной системе координат «рождение» и «смерть» топологического свойства. Такое представление в форме облака точек на плоскости, называют диаграммой персистентности (ДП). В статье показано, что результаты распознавания классов текстур существенно лучше, по сравнению с другими способами векторизации ДП.

**Ключевые слова:** Риманова метрика, диаграмма персистентности, функция плотности вероятности, персистентное изображение (ПИ).

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## ТЕКСТУРАЛАРДЫ ТАНУ ҮШІН РИМАНМЕТРИКАСЫ

**Аннотация.** Мақалада сандық бейнелердегі текстураларды есептеу топология және Риман геометриясы әдістерімен тану талқыланады. Паттерлердің топологиялық қасиеттері фотометриялық өлшем деңгейі бойынша сүзу кезінде алынған кесінділермен (баркодтармен) берілген.Баркодтың басы топологиялық сипат (байланыс компоненті және/немесе "тесік") пайда болатын деңгейді, ал оның соңы – сипат жоғалатын деңгейді кодтайды. Баркодтарды топологиялық қасиеттің "туу" және "өлім" координаттарының тікбұрышты жүйесіндегі олардың ұштарының координаттарын параметрлеуге ыңғайлы. Жазықтықтағы нүктелердің бұлт түріндегі мұндай көрініс персистенттік диаграмма (ПД) деп аталады. Мақалада басқа ДП векторизация әдістерімен салыстырғанда, текстураның сыныптарын тану нәтижелері айтарлықтай жақсы екендігі көрсетілген.

**Түйін сөздер:** Риман метрикасы, персистенттік диаграммасы, ықтималдықтығыздығы функциясы, персистенттібейнелер (ПБ).

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