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ABSOLUTE STABILITY OF A PROGRAM MANIFOLD OF NON-AUTONOMOUS BASIC CONTROL SYSTEMS

Abstract. In this paper the inverse dynamics problemisstudied: for a given manifold restore a force field, which lies in the tangent subspace to manifold. One of the general inverse problems of dynamics is solved: the corresponding system of differential equations is but as well as the stability is considered. This inverse problem is very important for a variety of mathematical models mechanics.Absolutestability of a program manifold of non-autonomous basic control systems with stationary nonlinearity is investigated.Theproblem of stability of the basic control systems is considered in the neighborhood of a program manifold. Nonlinearitysatisfies to conditions of local quadratic relations. The sufficient conditions of the absolute stability of the program manifold have been obtained relatively to a given vector-function by means of construction of Lyapunov function, in the form "quadratic form plus an integral from nonlinearity". The obtained results are used to solve the problem of the synthesis of high-speed regulators.

Key words. Absolute stability, basic control systems, program manifold, Lyapunov function, local quadratic relation. high-speed regulators.

Introduction. The problem of constructing for systems of ordinary differential equations on a given integral curve was formulated by Yerugin in [1] and there was proposed a method for its solving. Later, this problem was developed by Galiullin, Mukhametzyanov, Mukharlyamov and others [2-19] to the problem of the construction of systems of differential equations by a given integral manifold, to solving of various inverse problems of dynamics, and to constructing of systems of program motion.The integral manifold is defined as the intersection of hypersurfaces.It should be noted that the construction of stable systems developed into an independent theory. A detailed survey of these works can be found in [2, 7, 16]. The works [2-5] are devoted to the construction of automatic control systems on the basis of a given manifold. In these works, control systems were constructed for a scalar nonlinear function $\varphi(\sigma)$, and sufficient conditions for absolute stability were established. The problem of the construction of automatic control systems for a vector nonlinear function with locally quadratic relations was solved in [6, 7].In [10, 12, 13], inverse problems of dynamics are considered in the presence of random perturbations, namely in the class of stochastic differential Ito equations. In [20 - 22], conditions for reducibility to a canonical form and conditions for the stability of a Cauchy problem were established, and the problem of the existence of periodic solutions of equations unresolved with respect to the higher derivative was investigated. Sufficient conditions for the asymptotic stability of the program manifold of degenerate automatic control systems were obtained in[8]. The problem of the exponential stability of the trivial solution was investigated in [23]. In [24, 25], exponential-stability conditions were established for automatic control systems of a certain class. The problem of the synthesis of asymptotically stable systems possessing a given property was posed in[26], where a method for the synthesis of feedback laws was also given. Questions of the stability of the trivial solution of systems with variable coefficients were considered in the works [26, 27].

In the present paper,we investigate the stability of a program manifold with respect to the given vector-function of non-autonomous basic control systems with stationary nonlinearity.

*Statement of the problem.*Note,what the general statement of the problem is as follows:

To construct a material system describing by ordinary differential equations

$$\dot{x} = f(t,x,u), \; t \in I = [0,\infty[, \tag{1}$$

where $x \in R^n$ is the state vector of the object; $u \in R^r$ - control vector; $f \in R^n$ is a continuous vector-function, on a given $(n-s)$-dimensional program manifold $\Omega(t) \equiv \omega(t,x) = 0, \omega \in R^s$

The method of solving this problem consists in finding a set of right parts of the desired systems satisfying the equality on the manifold.

Definition 1.*A set $\Omega(t)$ is called an integral manifold of equation (1) if, from that $\omega(t_0,x_0) \in \Omega(t_0)$ follows $\omega(t,x(t,t_0,x_0)) \in \Omega(t)$ for all $t \geq t_0$.*

The concepts of an integral manifold and a program manifold have the same meaning, for convenience, we will use the term program manifold.

Taking into account the necessary and sufficient conditions for the manifold $\Omega(t)$ to be integral for the system (1), we get:

$$\dot{\omega} = \frac{\partial \omega}{\partial t} + \frac{\partial \omega}{\partial x} f(t,x,u) = F(t,\omega,u). \tag{2}$$

Here $F \in R^S$ is the Erugin vector-function [1] satisfying the condition $F(t,0,u) \equiv 0$.

We will introduce for consideration a class Ξ of continuously-differentiable at times t and bounded on a norm matrices.

Suppose that the right-hand side of the system (1) can be represented in the form

$$\dot{x} = f(t,x) - B(t)\xi, \quad \xi = \varphi(\sigma), \quad \sigma = P^T(t)\omega, \quad t \in I = [0,\infty), \tag{3}$$

where $x \in R^n$ is a state vector of the object, $f \in R^n$ is a vector-function, satisfying to conditions of existence of a solution $x(t) = 0$, $B \in \Xi^{n\times r}$, $P \in \Xi^{s\times r}$ are matrices, $\omega \in R^s \, (s \leq n)$ is a vector, $\xi \in R^r$ is the control vector-function of the deviation from the given program, satisfying to conditions of local quadratic connection

$$\varphi(0) = 0 \wedge \varphi^T(\sigma)\theta(t)(\sigma - K^{-1}(t)\varphi(\sigma)) > 0, \quad \forall \sigma \neq 0, \tag{4}$$

$$K_1(t) \leq \frac{\partial \varphi(\sigma)}{\partial \sigma} \leq K_2(t),$$

$$[\theta = diag\|\theta_1,\ldots,\theta_r\|] \in \Xi^{r\times r}, \quad K(t) = diag\|k_1(t),\ldots,k_r(t)\|, \quad [K(t) = K^T(t) > 0] \in \Xi^{r\times r},$$

$$[K_i(t) = K_i^T(t) > 0 \quad i = 1,2] \in \Xi^{r\times r}.$$

Note that the following estimate

$$\frac{\beta_1}{\nu_2}\|\omega\|^2 \leq \|\varphi\|^2 \leq \frac{\beta_2}{\nu_1}\|\omega\|^2 \tag{5}$$

can be obtained from the condition (4), where $\beta_1,\nu_1;\beta_2,\nu_2$ are the smallest, largest eigenvalues of matrices $P\theta P^T$, θK^{-1}.

The given program $\Omega(t)$ is exactly realized only if the initial values of the state vector satisfy the condition $\omega(t_0, x_0) = 0$. However, this condition cannot be exactly satisfied, because of always there exist initial and permanent acting perturbations. Therefore, the conditions of the stability of the program manifold $\Omega(t)$ with respect to the vector function ω should be additionally required in the construction of systems of program motion.

On the basis of relation (2) and our assumption, choosing the Yerugin function as $F = -A(t)\omega,\ A \in \Xi^{s\times s}$ we obtain the following system with respect to vector-function ω [2, 3]:

$$\dot{\omega} = -A(t)\omega - H(t)B(t)\xi, \quad \xi = \varphi(\sigma), \quad \sigma = P^T(t)\omega, \quad t \in I = [0, \infty), \tag{6}$$

where $H = \dfrac{\partial \omega}{\partial x}$ is the Jacobi matrix, nonlinearity $\varphi(\sigma)$ satisfies also to generalized conditions (4), (5).

Definition 2. *A program manifold $\Omega(t)$ is called absolutely stable with respect to vector-function ω if it is asymptotically stable in whole for solution of equations (6) for all $\omega(t_0, x_0)$ and the function $\varphi(\sigma)$ satisfying conditions (4), (5).*

Statement of the problem. *To get the condition of absolute stability of a program manifold $\Omega(t)$ of the non-autonomous basic control systems in relation to the given vector-function ω.*

Sufficient conditions of the program manifold's absolute stability. First, we consider a linear system of differential equations with respect to a vector function ω:

$$\dot{\omega} = -A(t)\omega, \quad t \in I = [0, \infty). \tag{7}$$

For this system, we have

Theorem1 [27]. *Suppose that there exists $L(t) = L^T(t) > 0$ and $-\dot{V}|_{(7)} = W$.*

Then for the asymptotic stability of a program manifold $\Omega(t)$, it is necessary and sufficient that the following relations hold

$$V = \omega^T L(t)\omega > 0,$$

$$W = \omega^T G(t)\omega > 0,$$

where V, W have the following properties

$$l_1\|\omega\|^2 \le V \le l_2\|\omega\|^2, \tag{8}$$

$$g_1\|\omega\|^2 \le W \le g_2\|\omega\|^2, \tag{9}$$

where l_1, l_2, g_1, g_2 are positive constants.

In the space, X_n we choose the region $G(R)$ as follows

$$G(R) = (t, x): t \ge 0 \wedge \|\omega(t, x)\| < R < \infty. \tag{10}$$

Basic theorem. *If there exists a real, continuous and differentiable function $V(t, \omega)$ in region (10) that is definitely positive and admits a higher limit as a whole, such that*

$$-\frac{dV}{dt}|_{(6)} = W(t, \omega)$$

will be a definite-positive function for all values of ω, then the program manifold $\Omega(t)$ is absolutely stable.

Theorem 2. *Suppose that there exist matrices*

$$L(t) = L^T(t) > 0 \in \Xi^{s\times s},\ \beta = diag(\beta_1,\dots,\beta_r) > 0$$

and non-linear function $\varphi(\sigma)$ *satisfies the conditions (4), (5). Then, for the absolute stability of the program manifold* $\Omega(t)$ *with respect to the vector function* ω *it is sufficient performing of the following conditions*

$$l_1\|\omega\|^2 \le V \le l_2\|\omega\|^2, \tag{11}$$

$$g_1\|\omega\|^2 \le -\dot V \le g_2(\|\omega\|^2, \tag{12}$$

where l_1, l_2, g_1, g_2 *are positive constants.*

*Proof.*Let there exist $L(t) = L^T(t) > 0 \in \Xi^{s\times s},\ \beta = diag(\beta_1,\dots,\beta_r) > 0,$ then for the system (6) we can construct a Lyapunov function of the form

$$V(\omega,\xi) = \omega^T L(t)\omega + \int_0^\sigma \varphi^T\beta\, d\sigma > 0. \tag{13}$$

Taking into account the property (4), making the substitution

$$\varphi(\sigma) = h\sigma \quad (0 \le [h = h^T] \le k) \quad k = \min_t K(t),$$

we obtain the estimate

$$l_1(t)\|\omega\|^2 \le V \le l_2(t)]\omega]^2, \tag{14}$$

where

$$l_1(t) = l^{(1)}(t) + \lambda_1(t),\quad l_2(t) = l^{(2)}(t) + \lambda_2(t);$$

$$\lambda_1(t)\|\omega\|^2 \le \int_0^\sigma \varphi^T(\sigma)\beta\, d\sigma \le \lambda_2(t)\|\omega\|^2.$$

Here $l^{(1)}, \lambda_1, l^{(2)}, \lambda_2$ are the smallest and largest eigenvalues of matrices $L, \Lambda,\ \Lambda = P(t)H(t)\beta P^T(t)$. The diagonal elements of the matrix Λ are divided by the number 2. On the basis of property (5) the derivative of the function (13) takes the form

$$-\dot V = \omega^T G\omega + 2\omega^T G_1\xi + \xi^T G_2\xi > 0, \tag{15}$$

where

$$G(t) = -\dot L + A^T(t)L(t) + L(t)A(t);$$

$$G_1(t) = \frac{1}{2}A^T(t)P(t)\beta + L(t)H(t)B(t) + \frac{1}{2}\beta\dot P^T(t);$$

$$G_2(t) = \beta P^T(t)H(t)B(t).$$

Due to the fact that $-\dot V > 0$ the following estimates hold

$$q_1\left(\|\omega\|^2 + \|\xi\|^2\right) \le z^T Q z \le q_2\left(\|\omega\|^2 + \|\xi\|^2\right), \tag{16}$$

where

$$z = \begin{Vmatrix} \omega \\ \xi \end{Vmatrix}, \quad Q = \begin{Vmatrix} G & G_1 \\ G_1^T & G_2 \end{Vmatrix},$$

q_1, q_2 are the smallest and largest eigenvalues of matrix Q.

Taking into account the estimates (5) from (16), we get

$$\eta_1(t)\|\omega\|^2 \le -\dot{V} \le \eta_2(t)\|\omega\|^2, \tag{17}$$

$$\eta_1 = q_1\left(1 + \frac{\beta_1}{\nu_2}\right); \quad \eta_2 = q_2\left(1 + \frac{\beta_2}{\nu_1}\right).$$

On the basis of (11), (17), the inequalities are valid

$$l_2^{-1}V_0 \exp[-\int_{t_0}^{t}\alpha_1(\tau)d\tau] \le \|\omega\|^2 \le l_1^{-1}V_0 \exp[-\int_{t_0}^{t}\alpha_2(\tau)d\tau], \tag{18}$$

where

$$\alpha_1(t) = \frac{\eta_2(t)}{l_1(t)}; \quad \alpha_2(t) = \frac{\eta_1(t)}{l_2(t)}.$$

Assume that

$$\alpha_1 = \sup_{t\in I}[-\alpha_1(t)] \wedge \alpha_2 = \inf_{t\in I}[-\alpha_2(t)];$$

$$l_1 = \inf_{t\in I} l_1(t) \wedge l_2 = \sup_{t\in I} l_2(t).$$

from inequality (18) we obtain

$$l_2^{-1}V_0 \exp[\alpha_1(t - t_0)] \le \|\omega\|^2 \le l_1^{-1}V_0 \exp[\alpha_2(t - t_0)].$$

Whence follows the next estimates which hold on the sphere R:

$$\|\omega\|^2 \le R^2 \exp[\alpha_2(t - t_0)]. \tag{19}$$

Conditions of the synthesis of high-speed regulators. The obtained results we use to solve of the problem of the syntewsis of high-speed regulators.

Let $t = t_0^*$. Then from (19) we get

$$R^2 \exp[\alpha_2(t_0^* - t_0)] = \varepsilon^2$$

or solving with respect to $t_0^* - t_0$:

$$t_0^* - t_0 = \alpha_2^{-1} \ln\frac{\varepsilon^2}{R^2}. \tag{20}$$

The control time $t_\rho = t^* - t_0$ on the basis of (20) is defined as follows

$$t_\rho = \alpha_2^{-1} \inf_{\omega_0} \ln\frac{\varepsilon^2}{R^2}. \tag{21}$$

The solution of the problem of the synthesis of high-speed regulators follows from inequalities

$$\alpha_2^{-1} \inf_{\omega_0} \ln \frac{\varepsilon^2}{R^2} \le t_s .$$

here t_s is the specified time.

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АБСОЛЮТНАЯ УСТОЙЧИВОСТЬ ПРОГРАММНОГО МНОГООБРАЗИЯ НЕ АВТОНОМНЫХ ОСНОВНЫХ СИСТЕМ УПРАВЛЕНИЯ

Аннотация. В статье рассматривается обратная задача динамики: для заданного многобразия воостанавливается поле сил, которые расположены на перпендикулярной полуплоскости к многообразию. Решается более общая задача динамики: исследуются устойчивость систем соответствующих диффференциальных уравнений. Эти обратные задачи очень важны для различных моделей механических систем. Исследуется абсолютная устойчивость программного многообразия не автономных основныхсистем управления со стационарными нелинейностями. Условия устойчивости основных систем исследованы вокрестности заданного программного многообразия. Нелинейности удовлетворяют условиям локальной квадратичной связи. Достаточные условия абсолютной устойчивости программного многообразия, относительно заданной вектор-функции, получены с помощью построения функции Ляпунова, «кадратичная форма плюс интеграл от нелинейности». Полученные результаты использованы длярешениязадачисинтеза быстродействующихрегуляторов.

Ключевые слова. Абсолютная устойчивость, основная система управления, пророграммное многообразие, функция Ляпунова, локальная квадратичная связь, быстродействующие регуляторы.

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АВТОНОМДЫ ЕМЕС НЕГІЗГІ БАСҚАРУ ЖҮЙЕЛЕРІНІҢ БАҒДАРЛАМАЛЫҚ КӨПБЕЙНЕСІНІҢ АБСОЛЮТ ОРНЫҚТЫЛЫҒЫ

Аннотация. Мақалада динамиканың кері есебі зерттеледі: яғни, берілген көпбейне үшін, көпбейнеге перпендикуляр жазықшада жататын күш өрісі тұрғызылады. Динамиканың жалпы есебі шешіледі: яғни сәйкес дифференциалдық теңдеулер жүйесінің орнықтылығы зерттейледі. Бұл кері есеп механиканың түрлі математикалық моделдері үшін өте маңызды. Стационар бейсызықты автономды емес негізгі басқару жүйелерінің абсолют орнықтылығы зерттеледі. Негізгі басқару жүйелерінің орнықтылығы бағдарламалық көпбейненің маңайында қарастырылады. Бейсызықтықтар локалды квадраттық байланыстарды қанағаттандырады. Бағдарламалық көпбейненің берілген вектор-функция бойынша абсолют орнықтылығының жеткіліктк шарттары “квадраттық форма қосу бейсызықтын интеграл” түріндегі Ляпунов функциясын құру арқылы алынады. Алынған нәтижелер тез жылдамдықты реттегіштерді синтездеу есебіне қолданылды.

Түйін сөздер. Абсолюттік отнықтылық, негізгі басқару жүйелері, бағдарламалық көпбейне, Ляпунов функциясы, локалді квадраттық байланыс, жоғары жылдамдықты реттегіш.

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