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OPTIMAL SYNTHESIS OF PLANAR LINKAGES

Abstract. This paper investigates the optimal synthesis of planar linkages. The main idea of this paper is to find the initial approximations based on the use of Burmester points for function generator linkages, path generator linkages, motion generator linkages. The results of the numerical synthesis of the linkages depend on the choice of the initial approximations. A more flexible method to the search for initial approximations is the method based on the use of Burmester points. This method allows the determination of the initial approximations analytically for three, four or five by established initial data of synthesis. In this case, the problem is reduced to determining the solutions of polynomials, respectively the second, third and fourth degree. The method consists in that the synthesized linkage is conditionally divided into initial kinematic chains and closing kinematic chains, and Burmester points are determined for each chain. After the choice of initial approximations, an objective function is formed according to the output criteria, depending on the synthesis parameters, using the Chebyshevsky (best) or quadratic approximation problems. The synthesis parameters of planar linkages are determined from objective function minimum. According to this method, a program for the synthesis of planar linkages has been developed. An example is included to demonstrate the method.

Keywords: synthesis, optimal, planar linkages, initial approximations, Burmester points.

Introduction. Synthesis of planar multiple bar linkages has been extensively studied in the last ten years. Dimensional synthesis is one of the most important stages in the design of the linkages, since at this stage the basic kinematic properties necessary for the mechanism are formed to perform the functions assigned to it. The dimensional synthesis of linkages is divided into three types [1]:

1) It is required to realize the given function of the position of the output link of the mechanism - synthesis of transmission mechanisms ("function generation");

2) It is required to reproduce the trajectory of the working point in the plane - the synthesis of the guide mechanisms ("path generation");

3) It is necessary to reproduce the given motion of the solid body in the plane - the synthesis of the motion mechanisms ("motion generation").

When exact realization of the given motion is required, the problem arises of exact synthesis. However, the number of output object positions that can be reproduced accurately is generally limited. On the other hand, any movement in practice cannot be reproduced with perfect accuracy due to inaccuracies in the manufacture of elements (links, kinematic pairs, etc.) of the mechanism.

Therefore, the methods of approximate synthesis of the linkages have developed greatly. The problems of the kinematic synthesis of linkages reduce to the problem of approximation of a function. This formulation of the problem of linkages synthesis was proposed in the work of P.L. Chebyshev [2]. By way of compiling synthesis equations which follow from the constraint equations can be divided into geometric and algebraic methods [3-6]. The geometric synthesis methods are compiled on the basis of the equation of the projected closed kinematic chain. The algebraic constraint equations used methods that are imposed on the output link of moving mechanism. By the method of solving the synthesis equations, the existing methods for the synthesis of linkages can be divided into two groups: 1) analytical methods; 2) optimization methods.

In analytical synthesis, part of the constant parameters of the mechanism is calculated directly by analytical formulas. These formulas are obtained as a result of solving the synthesis equations in an explicit form [7, 8]. Upon optimal synthesis of linkages, additional synthesis conditions, such as the optimal transmission angle, the minimum value of the generalized force at the input, etc. can be taken into account. In connection with the advent of modern high-speed computers, optimal synthesis of linkages have been created, which were considered in [9-16]. The advantages of optimization methods for the synthesis of linkages are particularly evident in cases where the "classical" methods of kinematic synthesis based on kinematic geometry or various methods of approximation, are inapplicable or ineffective.

Initial approximations for plane linkages. The success of the search for the optimal linkage largely depends on the choice of the initial approximation, determined by classical methods, while the linkage designed by classical methods often requires optimization taking into account additional synthesis conditions. The results of the numerical synthesis of linkages depend on the choice of the initial approximations. The choice of initial approximations can be made using the metric parameters of the mechanism analog.In this case, it is possible to obtain only one mechanism, which reproduces an approximately desired trajectory. The initial approximations can be found using random search methods, for example the LP_τ sequence generator. In this case, the initial approximations are distributed in a given multidimensional space using the LP_τ sequence [17, 18]. The method makes it possible to obtain the most complete picture of optima distribution of considered functional; however, large dimension parameters of the synthesis can greatly increase the computational volumes. A more flexible method to the search for initial approximations is the method based on the use of Burmester points. This method allows determining the initial approximations analytically for three, four or five by established initial data of synthesis. In this case, the problem is reduced to finding solutions of polynomials, respectively the second, third and fourth degree. The principle of method lies in the fact that the synthesized mechanism is conditionally divided into initial kinematic chains (IKC) and closing kinematic chains (CKC), and for each chain Burmester points are determined [19, 20]. For example, to synthesize a path-generator four-bar linkage, this mechanism is divided into the IKC, which is a dyad O_1AC, and CKC, which is a bar O_2B (figure 1). Consider the method of finding initial approximations, based on the use of Burmester points for function generator linkages, path generator linkages, motion generator linkages.

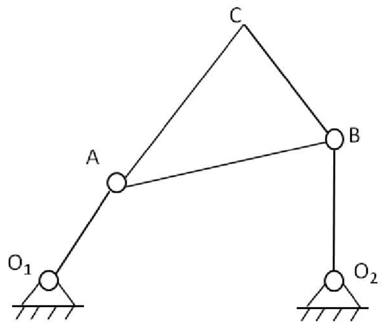

Figure 1 – Path generator four-bar linkage

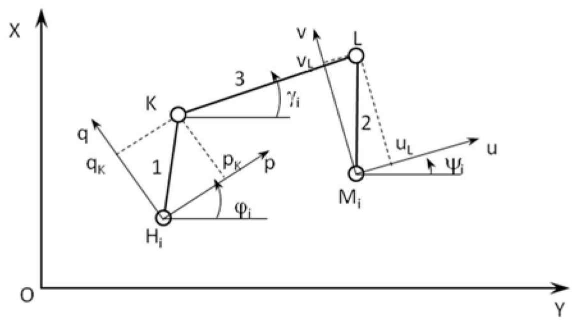

Figure 2 – Initial kinematic chain

For function generator linkages. Suppose that N positions of the two movable planes 1 and 2 are given, for the initial kinematic chain. Movable planes are determined by the coordinates x_{H_i}, y_{H_i}, x_{M_i}, y_{M_i} of the points H and M, and the rotation angles ϕ_i, ψ_i around these points (i = 1,2, ..., N), (figure 2). It is necessary to determine the Burmester points K and L in the corresponding movable planes, lying on arcs of circles with centers at the points H and M.

We will compose the algebraic equation of closure of vector contours, relative to the coordinate system

$$\left.\begin{aligned} x_{L_i} &= x_{H_i} + p_K\cos\varphi_i - q_K\sin\varphi_i + l_3\cos\gamma_i = x_{M_i} + u_L\cos\psi_i - v_L\sin\psi_i,\\ y_{L_i} &= y_{H_i} + p_K\sin\varphi_i + q_K\cos\varphi_i + l_3\sin\gamma_i = y_{M_i} + u_L\sin\psi_i + v_L\cos\psi_i,\\ i &= 1,2,\ \ldots,\ N \end{aligned}\right\} \quad (1)$$

Excluding an unknown angle γ_i , the system of Eq. (1) reduced to the form

$$A_j + B_j p_K + C_j q_K + D_j u_L + E_j v_L + F_j(p_K u_L + q_K v_L) + G_j(p_K v_L - q_K u_L) = 0 \quad (2)$$

where (3)

$$\left.\begin{aligned}
A_j &= \left[\left(x_{M_i} - x_{H_i}\right)^2 + \left(y_{M_i} - y_{H_i}\right)^2 - \left(x_{M_{i+1}} - x_{H_{i+1}}\right)^2 - \left(y_{M_{i+1}} - y_{H_{i+1}}\right)^2\right]/2 \\
B_j &= -\left(x_{M_i} - x_{H_i}\right)\cos\phi_i - \left(y_{M_i} - y_{H_i}\right)\sin\phi_i + \\
&\quad + \left(x_{M_{i+1}} - x_{H_{i+1}}\right)\cos\phi_{i+1} + \left(y_{M_{i+1}} - y_{H_{i+1}}\right)\sin\phi_{i+1} \\
C_j &= \left(x_{M_i} - x_{H_i}\right)\sin\phi_i - \left(y_{M_i} - y_{H_i}\right)\cos\phi_i - \\
&\quad - \left(x_{M_{i+1}} - x_{H_{i+1}}\right)\sin\phi_{i+1} + \left(y_{M_{i+1}} - y_{H_{i+1}}\right)\cos\phi_{i+1} \\
D_j &= \left(x_{M_i} - x_{H_i}\right)\cos\psi_i + \left(y_{M_i} - y_{H_i}\right)\sin\psi_i - \\
&\quad - \left(x_{M_{i+1}} - x_{H_{i+1}}\right)\cos\psi_{i+1} - \left(y_{M_{i+1}} - y_{H_{i+1}}\right)\sin\psi_{i+1} \\
E_j &= -\left(x_{M_i} - x_{H_i}\right)\sin\psi_i + \left(y_{M_i} - y_{H_i}\right)\cos\psi_i + \\
&\quad + \left(x_{M_{i+1}} - x_{H_{i+1}}\right)\sin\psi_{i+1} - \left(y_{M_{i+1}} - y_{H_{i+1}}\right)\cos\psi_{i+1} \\
F_j &= -\cos(\phi_i - \psi_i) + \cos(\phi_{i+1} - \psi_{i+1}) \\
G_j &= -\sin(\phi_i - \psi_i) + \sin(\phi_{i+1} - \psi_{i+1}),\ j = 1, 2, ..., N-1
\end{aligned}\right\} \quad (3)$$

If the hinges H and M are taken as the frames, we get a function generator four-bar linkage (figure 3) and coefficients of the system Eq. (2) take the following form:

$$\left.\begin{aligned}
A_j &= 0 \\
B_j &= -\left(x_{M_0} - x_{H_0}\right)\left(\cos\phi_i - \cos\phi_{i+1}\right) - \left(y_{M_0} - y_{H_0}\right)\left(\sin\phi_i - \sin\phi_{i+1}\right) \\
C_j &= \left(x_{M_0} - x_{H_0}\right)\left(\sin\phi_i - \sin\phi_{i+1}\right) - \left(y_{M_0} - y_{H_0}\right)\left(\cos\phi_i - \cos\phi_{i+1}\right) \\
D_j &= \left(x_{M_0} - x_{H_0}\right)\left(\cos\psi_i - \cos\psi_{i+1}\right) + \left(y_{M_0} - y_{H_0}\right)\left(\sin\psi_i - \sin\psi_{i+1}\right) \\
E_j &= \left(x_{M_0} - x_{H_0}\right)\left(\sin\psi_i - \sin\psi_{i+1}\right) + \left(y_{M_0} - y_{H_0}\right)\left(\cos\psi_i - \cos\psi_{i+1}\right) \\
F_j &= -\cos(\phi_i - \psi_i) + \cos(\phi_{i+1} - \psi_{i+1}) \\
G_j &= -\sin(\phi_i - \psi_i) + \sin(\phi_{i+1} - \psi_{i+1}),\ j = 1, 2, \ldots, N-1
\end{aligned}\right\} \quad (4)$$

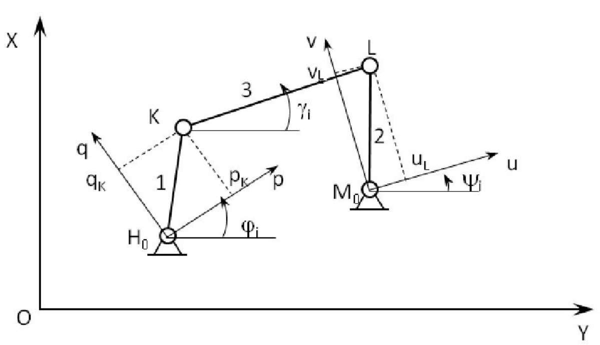

Figure 3 – Transfer four-bar

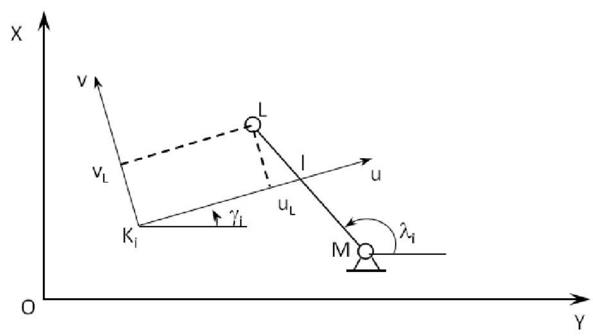

Figure 4 – The binary link

For path generator linkages. Let N be given the positions of the movable plane is determined by the coordinates X_{K_i}, Y_{K_i} of the point K by the angle of rotation around this point (figure 4). It is necessary to determine the coordinates of the point M in the fixed plane and the point L in the movable plane is lying on the arc of the circle with center at the point M.

We will compose the algebraic equation of closure of vector contours

$$\left.\begin{aligned} &x_{L_i} = x_{K_i} + u_L \cos\gamma_i - v_L \sin\gamma_i = x_M + l\cos\lambda_i\ ,\\ &y_{L_i} = y_{K_i} + u_L \sin\gamma_i + v_L \cos\gamma_i = y_M + l\sin\lambda_i\ ,\\ &i = 1, 2,\ \ldots,\ N \end{aligned}\right\} \tag{5}$$

Eqs. (5) can be reduced to the form

$$A_j + B_j u_L + C_j v_L + D_j x_m + E_j y_m + F_j(u_L x_M + v_L y_m) + G_j(u_L y_M - v_L x_M) = 0 \tag{6}$$

Where

$$\left.\begin{aligned} &A_j = \left[x_{K_i}{}^2 + y_{K_i}{}^2 - x_{K_{i+1}}{}^2 - y_{K_{i+1}}{}^2\right]/2\\ &B_j = x_{K_i}\cos\gamma_i + y_{K_i}\sin\gamma_i - x_{K_{i+1}}\cos\gamma_{i+1} - y_{K_{i+1}}\sin\gamma_{i+1}\\ &C_j = -x_{K_i}\sin\gamma_i + y_{K_i}\cos\gamma_i + x_{K_{i+1}}\sin\gamma_{i+1} - y_{K_{i+1}}\cos\gamma_{i+1}\\ &D_j = -x_{K_i} + x_{K_{i+1}}, E_j = -y_{K_i} + y_{K_{i+1}}, F_j = -\cos\gamma_i + \cos\gamma_{i+1}\\ &G_j = -\sin\gamma_i + \sin\gamma_{i+1}, j = 1, 2, \ldots, N-1 \end{aligned}\right\} \tag{7}$$

For motion generator linkages. Let N positions of the point L are given, and is set to define the coordinates X_{L_i}, Y_{L_i} and the rotation angle φ_i of the movable plane with respect to an unknown fixed point H_0 (figure 5). It is necessary to determine the coordinates of the point H_0 of the fixed plane and the point K in the movable plane is lying on the arc of the circle with center at the point H_0.

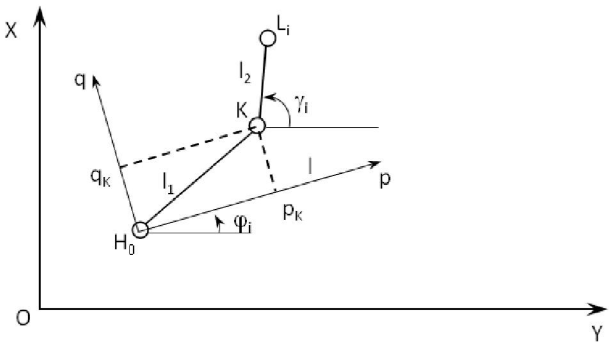

Figure 5 – The Dyad

The equation of closure of vector contours for the considered chain has the form

$$\left.\begin{aligned} &x_{L_i} = x_{H_0} + p_K \cos\phi_i - q_K \sin\phi_i + l_2 \cos\gamma_i\ ,\\ &y_{L_i} = y_{H_0} + p_K \sin\phi_i + q_K \cos\phi_i + l_2 \sin\gamma_i\ ,\\ &i = 1, 2,\ \ldots,\ N \end{aligned}\right\} \tag{8}$$

Eqs. (8) can be reduced to the form

$$A_j + B_j\ x_{H_0} + C_j y_{H_0} + D_j p_K + E_j q_K + F_j\left(x_{H_0} p_K + y_{H_0} q_k\right) + \\ + G_j\left(x_{H_0} q_K - y_{H_0} p_K\right) = 0 \quad (9)$$

where

$$\left.\begin{aligned} A_j &= \left[x_{L_i}{}^2 + y_{L_i}{}^2 - x_{L_{i+1}}{}^2 - y_{L_{i+1}}{}^2\right]/2 \\ B_j &= -x_{L_i} + x_{L_{i+1}}, C_j = -y_{L_i} + y_{L_{i+1}} \\ B_j &= -x_{L_i}\cos\phi_i - y_{L_i}\sin\phi_i + x_{L_{i+1}}\cos\phi_{i+1} + y_{L_{i+1}}\sin\phi_{i+1} \\ E_j &= x_{L_i}\sin\phi_i - y_{L_i}\cos\phi_i - x_{L_{i+1}}\sin\phi_{i+1} + y_{L_{i+1}}\cos\phi_{i+1} \\ F_j &= -\cos\phi_i + \cos\phi_{i+1}, G_j = -\sin\phi_i + \sin\phi_{i+1}, j = 1,2,...,N-1 \end{aligned}\right\} \quad (10)$$

We obtained a system of equations of the form for all the three cases

$$A_j + B_j\ x_1 + C_j x_2 + D_j x_3 + E_j x_4 + F_j\left(x_1 x_3 + x_2 x_4\right) + G_j\left(x_1 x_4 - x_2 x_3\right) = 0, \\ j = 1,2,\ ...,\ N-1 \quad (11)$$

If three positions of moving planes (N = 3) are given from Eq. (11) we obtain a system of two equations with four unknowns. In this case, two parameters of the mechanism, for example x_1, x_2 are given arbitrarily and the system Eq. (11) is solved with respect to the remaining two unknowns. The solution has the form

$$\left.\begin{aligned} x_3 &= d_1 / d_0, \\ x_4 &= d_2 / d_0, \end{aligned}\right\} \quad (12)$$

where

$$\begin{aligned} d_0 &= \left|(D_j + F_j x_1 - G_j x_2) \quad (E_j + G_j x_1 + F_j x_2)\ \right|, \\ d_1 &= \left|(-A_j - B_j x_1 - C_j x_2) \quad (E_j + G_j x_1 + F_j x_2)\ \right|, \\ d_2 &= \left|(D_j + F_j x_1 - G_j x_2) \quad (-A_j - B_j x_1 - C_j x_2)\ \right|, \\ j &= 1,2 \end{aligned}$$

If four positions of moving planes (N = 4) are given from Eq. (11) we obtain a system of three equations with four unknowns.

In this case, one parameter of the mechanism, for example x_1, is given arbitrarily and system Eq. (11) is solved with respect to the remaining three unknowns.

Alternately, excluding the two unknowns (for example, x_3 and x_4), we obtain a cubic equation that is solved by analytically known methods [21].

$$k_3 x_2^3 + k_2 x_2^2 + k_1 x_2 + k_0 = 0, \quad (13)$$

where

$$\begin{aligned} & k_0 = h_1 + h_2 x_1 + h_4 x_1^2 + h_6 x_1^3, k_1 = h_3 + h_8 x_1 + h_7 x_1^2, k_2 = h_5 + h_6 x_1, k_3 = h_7, \\ & h_1 = d_{24} d_{31} - d_{21} d_{34},\ h_2 = d_{14} d_{21} - d_{11} d_{24} + d_{24} d_{32} - d_{22} d_{34} - d_0 d_{31}, \\ & h_3 = d_{11} d_{34} - d_{14} d_{31} + d_{24} d_{33} - d_{23} d_{34} - d_0 d_{21}, \\ & h_4 = d_{14} d_{22} - d_{12} d_{34} + d_0 d_{11} - d_0 d_{32}, \\ & h_5 = d_{13} d_{34} - d_{14} d_{33} + d_0 d_{11} - d_0 d_{23}, \\ & h_6 = d_0 d_{12},\ h_7 = d_0 d_{13}, \\ & h_8 = d_{14} d_{23} - d_{14} d_{32} + d_{12} d_{34} - d_{13} d_{24} - d_0 d_{22} - d_0 d_{33}, \end{aligned}$$

$$d_0 = |E_j \ F_j \ G_j|, d_{11} = |-A_j \ F_j \ G_j|,$$
$$d_{12} = |-B_j \ F_j \ G_j|, d_{13} = |-C_j \ F_j \ G_j|, d_{14} = |-D_j \ F_j \ G_j|,$$
$$d_{21} = |E_j \ -A_j \ G_j|, d_{22} = |E_j \ -B_j \ G_j|, d_{23} = |E_j \ -C_j \ G_j|, d_{24} = |E_j \ -D_j \ G_j|,$$
$$d_{31} = |E_j \ F_j \ -A_j|, d_{32} = |E_j \ F_j \ -B_j|, d_{33} = |E_j \ F_j \ -C_j|, d_{34} = |E_j \ F_j \ -D_j|,$$
$$j = 1, 2, 3$$

The cubic equation Eq. (13) can be solved analytically and have one or three real roots [21] that substituting into Eq. (11) we obtain two equations in two unknowns x_3, x_4, are defined analogously to Eq. (12).

If five positions of moving planes ($N = 5$) are given, from Eq. (11) one by one excluding the three unknowns (for example, x_2, x_3, x_4), we obtain a fourth-order equation of the form

$$k_4 x_1^4 + k_3 x_1^3 + k_2 x_1^2 + k_1 x_1 + k_0 = 0, \tag{14}$$

where

$$k_0 = h_3 h_3 + h_1 h_3, k_1 = h_1 h_{10} + (h_2 + h_4) h_8 + 2 h_3 h_6, k_2 = h_6 h_5 + (h_7 + h_5) h_8 + 2 h_3 h_{11} + (h_2 + h_4) h_{11},$$
$$k_3 = h_8 h_7 + (h_7 + h_5) h_{10} + 2 h_6 h_{11}, k_4 = h_{11} h_{11} + h_9 h_{10},$$
$$h_1 = d_{41}(d_{33} - d_{21}) - d_{31}(d_{43} + d_{11}), h_2 = -d_{41}(d_{22} - d_{13}) + d_{31}(d_{23} + d_{12}),$$
$$h_3 = -d_{41} d_{23} - d_{31} d_{13},$$
$$h_4 = -(d_{32} - d_{11})(d_{43} + d_{11}) + (d_{42} - d_{21})(d_{33} + d_{21}),$$
$$h_5 = -(d_{32} - d_{11})(d_{23} + d_{12}) + (d_{42} - d_{21})(d_{13} + d_{22}),$$
$$h_6 = -d_{13}(d_{32} - d_{11}) - d_{23}(d_{42} + d_{21}),$$
$$h_7 = -d_{22}(d_{33} - d_{21}) + d_{12}(d_{43} + d_{11}), h_9 = d_{22}(d_{13} - d_{22}) - d_{12}(d_{23} + d_{12}),$$
$$h_{10} = -d_{13}(d_{13} - d_{22}) - d_{23}(d_{23} + d_{12}), h_{11} = d_{12} d_{13} + d_{22} d_{23}$$
$$d_0 = |D_j \ E_j \ F_j \ G_j|$$
$$d_{11} = |-A_j \ E_j \ F_j \ G_j|, d_{12} = |-B_j \ E_j \ F_j \ G_j|, d_{13} = |-C_j \ E_j \ F_j \ G_j|,$$
$$d_{21} = |D_j \ -A_j \ F_j \ G_j|, d_{22} = |D_j \ -B_j \ F_j \ G_j|, d_{23} = |D_j \ -C_j \ F_j \ G_j|,$$
$$d_{31} = |E_j \ F_j \ -A_j \ G_j|, d_{32} = |E_j \ F_j \ -B_j \ G_j|, d_{33} = |E_j \ F_j \ -C_j \ G_j|,$$
$$d_{41} = |D_j \ E_j \ F_j \ -A_j|, d_{42} = |D_j \ E_j \ F_j \ -B_j|, d_{43} = |D_j \ E_j \ F_j \ -C_j|,$$
$$j = 1, 2, 3, 4.$$

The fourth-order equation Eq. (14) can also be solved analytically [21], it can have two or four real roots or have none. If there are two real roots and one of them was determined analytically, then the second root can be determined from the following equation

$$x_2 = (h_3 + h_6 x_1 + h_{11} x_1^2)/(h_8 + h_{10} x_1)$$

If there are four real roots, then x_3 and x_4 are determined from

$$x_3 = (d_{11} + d_{12} x_1 + d_{13} x_2)/d_0,$$
$$x_4 = (d_{21} + d_{22} x_1 + d_{23} x_2)/d_0$$

After the choice of initial approximations, an objective function is formed according to the output criteria depending on the synthesis parameters $\vec{P}$, using the Chebyshevsky (best) or quadratic approximation problems. For the Chebyshev approximation problem, the synthesis parameters are determined as a minimum of the functional [22-25]

$$S(\vec{P}) = \max_{i=1,N} \left| \Delta q_i (\vec{P}) \right| \Rightarrow \min_{\vec{P}} S(\vec{P}), \tag{15}$$

where $\Delta q_i(\vec{P})$ is the weighted difference function for the selected mechanism [22-25].

According to this method, a program for the synthesis of planar linkages has been developed.

Conclusion. The optimal synthesis of planar linkages was developed. The method of searching for initial approximations based on the use of Burmestor points for function generator linkages, path generator linkages, motion generator linkages was considered. The objective function was formed according to the output criteria, depending on the synthesis parameters of linkages, using the Chebyshevsky or quadratic approximation problems. The synthesis parameters of linkages are determined from functional minimum. The program for the synthesis of planar linkages has been developed. The program used the following optimization methods: the Niedler-Mead method (the deformable polyhedron), the kinematic inversion method, the coordinate descent method, the spiral coordinate descent method, the quadratic interpolation-extrapolation method and the sliding tolerance method. We synthesized path generator four-bar linkage at 19 preset positions of the coupler point and planar linkages of Assur of the third and fourth classes. Using the method of optimal synthesis, the gripper with desired law of motion of the bucket edges was designed. A prototype of the gripper was manufactured and tested.

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ЖАЗЫҚ ИІНТІРЕКТІ МЕХАНИЗМДЕРДІҢ ОҢТАЙЛАНДЫРЫЛҒАН СИНТЕЗІ

Аннотация. Берілген жұмыста жазық иінтіректі механизмдердің оңтайлы синтезі зерттеледі. Бұл мақаланың негізгі мақсаты бағытталған иінтіректі механизмдер, орынауыстырғыш иінтіректі механизмдер, берілісті иінтіректі механизмдер үшін Бурместер нүктелерін пайдалануға негізделген бастапқы жуықтауын анықтау. Жазық көп буынды иінтіректі механизмдерінің синтезі соңғы он жылда қарқынды дамып келеді. Геометриялық синтез – иінтіректі механизмдерін жобалаудағы ең маңызды кезеңдердің бірі, себебі дәл осы кезеңде механизм өзіне жүктелген функцияларды орындау үшін қажетті негізгі кинематикалық қасиеттер қалыптасады. Иінтіректі механизмдердің геометриялық синтезі үш түрге бөлінеді: берілісті механизмдернің синтезі («функция генераторы»); бағыттаушы механизмдердің синтезі («траектория генераторы»); қозғалмалы механизмдердің синтезі («жазықтық-параллель қозғалыс генераторы»). Берілген қозғалыстың нақты орындалуы қажет болған кезде, дәл синтездеу мәселесі туындайды. Дегенмен дәл шығаруға болатын шығыс объектісінің орналасу саны шектеулі. Екінші жағынан, іс-әрекеттегі кез келген қозғалысты механизм элементтерінің (буындар, кинематикалық жұптар және т.б.) қателіктеріне байланысты дәлдікпен көбейту мүмкін емес. Сондықтан механизмдерді синтездеу теориясында соңғы жылдары, негізінен, механизмдерді жуық синтездеу әдістері жасалды. Осылайша, кинематикалық синтездің жоғарыда аталған барлық мақсаттары функцияны жақындату мәселесіне дейін азаяды. Мұндай синтездеу мәселелерін тұжырымдау П.Л. Чебышевтің классикалық жұмыстарынан бастау алады. Байланыс теңдеулерінен шығатын синтездік теңдеулерді құрастыру әдісі мен қолданыстағы синтез әдістерін алгебралық және геометриялық деп бөлуге болады. Геометриялық синтез әдістері құрастырылған кинематикалық тізбектің тұйықтық теңдеулері негізінде құрастырылады. Алгебралық синтез әдістері механизмнің шығыс буынының қозғалысына қойылған байланыс теңдеулерін қолданады. Синтез теңдеулерін шешу тәсілдерімен байланыстыра отырып, қолданыстағы иінтіректі механизмдерді синтездеу әдістерін екі топқа бөлуге болады: 1) аналитикалық әдістер; 2) сандық-оңтайландыру әдістері. Аналитикалық синтезде механизмнің тұрақты параметрлерінің бөлігі тікелей аналитикалық формулалар арқылы есептеледі. Бұл формулалар синтез теңдеулерін нақты түрде шешу нәтижесінде алынған. Иінтіректі механизмдерінің синтезін оңтайландыру кезінде қосымша синтез жағдайларын ескеруге болады, мысалы, қозғалыс берудің оңтайлы бұрышы, жалпыланған кіріс күшінің минималды мәні және т.б. Механизмдерді синтездеудің оңтайландыру әдістерінің артықшылығы, әсіресе, кинематикалық геометрияға немесе әртүрлі жуықтау әдістеріне негізделген кинематикалық синтездің «классикалық» әдістері қолданылмайтын немесе тиімсіз болған жағдайда көрінеді. Иінтіректі механизмдердің сандық синтезінің нәтижесі бастапқы жақындаудың таңдауына байланысты. Бастапқы жақындауды іздеудің ең тиімді әдісі – Бурместер нүктелерін пайдалануға негізделген әдіс. Бұл әдіс синтездің үш, төрт және бес берілген бастапқы деректердің аналитикалық бастапқы жақындауын анықтауға көмегі тиеді. Бұндай жағдайда екінші, үшінші және төртінші дәрежедегі полиномдардың шешімдерін анықтау қажет болады. Әдісте, синтезделген иінтіректі механизм бастапқы және тұйықталған кинематикалық тізбекке бөлінеді, сонымен бірге әрбір тізбектің Бурместер нүктесі анықталады. Бастапқы жақындауды таңдағаннан соң, квадратты жақындау немесе Чебышев есептерінің көмегімен синтез параметрлеріне байланысты шығыс өлшемдері бойынша мақсатты функция құрылады. Жазық иінтіректі механизмдер синтезінің параметрлері мақсатты функцияның минимумына байланысты анықталады. Осы әдіске сәйкес, жазық иінтіректі механизмдер синтезінің бағдарламасы жасалды. Берілген әдісті демонстрациялау үшін үлгі келтірілді.

Түйін сөздер: оңтайлы, жазық иінтіректі механизмдер, бастапқы жақындау, Бурместер нүктесі.

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ОПТИМИЗАЦИОННЫЙ СИНТЕЗ ПЛОСКИХ РЫЧАЖНЫХ МЕХАНИЗМОВ

Аннотация. В настоящей работе исследуется оптимальный синтез плоских рычажных механизмов. Основная идея этой статьи – в нахождении начальных приближений, основанной на использовании точек Бурместера для передаточных рычажных механизмов, направляющих рычажных механизмов, перемещающихся рычажных механизмов. Синтез плоских многозвенных рычажных механизмов интенсивно развивается в последние десять лет. Геометрический синтез является одним из наиболее ответственных этапов проектирования рычажных механизмов, поскольку именно на этом этапе формируются основные кинематические свойства, необходимые механизму для выполнения возложенных на него функций. Геометрический синтез рычажных механизмов, подразделяется на три вида: синтез передаточных механизмов («генератор функции»); синтез направляющих механизмов («генератор траектории»); синтез перемещающих механизмов («генератор плоско-параллельного движения»). При требовании точной реализации заданного движения возникает задача точного синтеза. Однако количество положений выходного объекта, которые можно воспроизвести точно, как правило, ограничено. С другой стороны, любое движение на практике невозможно воспроизвести с идеальной точностью из-за погрешностей изготовления элементов (звеньев, кинематических пар и т.д.) механизма. Поэтому в теории синтеза механизмов за последние годы развивались главным образом методы приближенного синтеза механизмов. Таким образом, все указанные выше задачи кинематического синтеза сводятся к задаче приближения функции. Такая формулировка задач синтеза восходит к классическим работам П.Л. Чебышева. По способу составления уравнений синтеза, которые вытекают из уравнений связей, существующие методы синтеза можно разделить на алгебраические и геометрические. Геометрические методы синтеза составляются на базе уравнений замкнутости проектируемой кинематической цепи. Алгебраические методы синтеза используют уравнения связей, которые налагаются на движение выходного звена механизма. По способу решения уравнений синтеза, существующие методы синтеза рычажных механизмов можно разделить на две группы: 1) аналитические методы; 2) численно-оптимизационные методы. При аналитическом синтезе часть постоянных параметров механизма вычисляется непосредственно по аналитическим формулам. Эти формулы получаются как результат решения уравнений синтеза в явном виде. При оптимизационном синтезе рычажных механизмов можно учитывать дополнительные условия синтеза, такие как оптимальный угол передачи движения, минимальное значение обобщенной силы на входе и т.д. Преимущества оптимизационных методов синтеза механизмов проявляются особенно в тех случаях, когда "классические" методы кинематического синтеза, основанные на кинематической геометрии или различных способах аппроксимации, неприменимы или малоэффективны.

Результаты численного синтеза рычажных механизмов зависят от выбора начальных приближений. Более гибким методом поиска начальных приближений является метод, основанный на использовании точек Бурместера. Этот метод позволяет определить аналитически начальные приближения по трем, четырем или пяти заданным исходным данным синтеза. В этом случае задача сводится к определению решений полиномов соответственно второй, третьей и четвертой степени. Метод заключается в том, что синтезируемый рычажный механизм условно разбивается на исходные и замыкающие кинематические цепи, и для каждой цепи определяются точки Бурместера.

После выбора начальных приближений формируется целевая функция по выходным критериям, зависящая от параметров синтеза, при помощи задач Чебышевского (наилучшего) или квадратического приближений. Параметры синтеза плоских рычажных механизмов определяются из минимума целевой функции. В соответствии с этим методом была разработана программа синтеза плоских рычажных механизмов. Приведен пример для демонстрации данного метода.

Ключевые слова: синтез, оптимальный, плоские рычажные механизмы, начальные приближения, точки Бурместера.

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