N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
SERIES OF GEOLOGY AND TECHNICAL SCIENCES
ISSN 2224-5278
Volume 4, Number 442 (2020), 110 – 119 **https://doi.org/10.32014/2020.2518-170X.91**

UDC 621.91.01

A. B. Akhmedov[1], D. T. Khodjibergenov[2], U. D. Khojibergenova[2], Z. A. Ibragimova[2], B. N. Absadykov[3]

[1] M. Ulugbek National University of Uzbekistan, Tashkent, Uzbekistan;
[2] M. Auezov South Kazakhstan State University, Shymkent, Kazakhstan;
[3] A. B. Bekturov Institute of Chemical Sciences, Almaty, Kazakhstan.
E-mail: ahmedov-1956@mail.ru, had_ji@mail.ru, hadji_umida@mail.ru, Zaure_1983_as@mail.ru,b_absadykov@mail.ru

THERMODYNAMIC BASES OF METAL MECHANICAL PROCESSING BY CUTTING METHOD

Abstract. Fundamental thermal mechanical phenomena arising in the process of metal cutting are difficult to give a systematic and interconnected theoretical and mathematical description. The phenomena various in essence and nature considered in the s tudies are so closely intertwined with each other that it is very difficult to describe their interaction. Based on the results of our research, it was revealed that eleven relatively independent theories that have not yet come to a holistic unity are focused on the top of the cutting edge. In such studies, it is necessary to systematize and relate the theory of chip formation, the mechanics of metal cutting, friction theories and wear during metalworking, the thermodynamics of cutting, and the theory of durability and reliability of cutting tools.

In this paper, the approaches of mathematical modeling of thermal mechanical processes of metal cutting, is based on the non-linear theory of elasticity of V.V. Novozhilov at that, the final deformation of the elementary volume of the considered three-dimensional body is accompanied by finite rotations and displacements but with small relative changes in geometric dimensions.

Key words: thermodynamics, thermal conductivity, cutting, deformation, friction, solids, stress, cutting tool, shavings, roughness, speed, shear.

Introduction. The mechanical processing of metals by cutting method is accompanied by thermodynamic processes. The source of heat when cutting metals is the work spent on the final deformation in the sheared layer and in the layers adjacent to the machined surface and the cutting surface as well as to overcome friction on the rear surfaces of the cutter. During the final deformation, the material points of the sample move relative to each other which are a source of additional heat generation [1-5].

It is known that the process of thermal mechanical cutting of solids is difficult to mathematically model due to the complexity of the process occurring there [3-5]. The various phenomena considered here are so closely intertwined with each other and their interaction is so complicated that eleven relatively independent theories that have not yet come to a holistic unity are focused on the cutter tip. These are the theory of chip formation, mechanics of metal cutting, the theory of friction and wear during metal working, the thermodynamics of cutting and the theory of resistance and reliability of cutting tools.

Here we propose a thermal mechanical theory of metal cutting based on the fundamental mechanisms of thermodynamics and nonlinear mechanics of a deformable solid. In this process, along the cutting line due to the occurrence of critical values of internal stresses at the tip of the cutting tool metal chips are detached from the base. In this case, after detaching, the chip in the form of a metal strip makes final movements [7-10].

Main part. The stress state during nonlinear deformation of an arbitrary three-dimensional elastic body under the action of the bulk system - X_i, and surface forces - S_i in the current coordinate system $O\xi_1\xi_2\xi_3$, is described by the Euler motion equation [7-10].

According to the theory of V.V. Novozhilov, nonlinear deformation of the elementary volume of the considered three-dimensional body is accompanied by finite rotations and displacements but with small relative changes in geometric dimensions. In these conditions, the actual (after deformation) curvilinear triple-orthogonal coordinate system $O\xi_1\xi_2\xi_3$ practically remains unchanged, i.e. the condition $\xi_i \approx x_i$ is fulfilled for them. Here $Ox_1x_2x_3$ is the initial coordinate system (before deformation). Then the second Piola-Kirchhoff tensor P_{ij} through the symmetric Cauchy tensor σ_{kj} in the initial coordinate system is expressed as follows [7]

$$P_{ij} = (\delta_{ik} + U_{i,k})\sigma_{kj}, \tag{1}$$

where δ_{ij} are the Kronecker symbols.

Thus, the nonlinear deformation of a three-dimensional body, relative to the initial coordinate system $Ox_1x_2x_3$, is described by the following system of equations of Lagrangian motion

$$\left[(\delta_{ik} + U_{i,k})\sigma_{kj}\right]_{,j} + X_i - \rho\ddot{U}_i = 0 \tag{2}$$

under the initial conditions

$$U_i\big|_{t=0} = U_i^0, \qquad \dot{U}_i\big|_{t=0} = V_i^0, \tag{3}$$

and boundary conditions

$$U_i\big|_{\Sigma_1} = U_i^{\Sigma}, \quad (\delta_{ik} + U_{i,k})\sigma_{kj}n_j\big|_{\Sigma_2} = S_i, \tag{4}$$

where U_i^{Σ} are the displacements $\Sigma=\Sigma_1+\Sigma_2$given on the part of the boundary, U_i^o, $V_i^o$are the coordinate functions characterizing the initial state of the body, n_j is the external normal and ρ is the density, δ_{ij} are the Kronecker symbols. Points above functions denote time derivatives. Derivatives by coordinate are denoted by a comma, i.e. tensor analysis operations take place. Here i, j, k∈(1,2,3).

For isotropic materials, according to the thermodynamic form of writing the determining Duhamel – Neumann relations [11-23] we have

$$\sigma_{ij} = \lambda(e_{kk} - \beta\vartheta)\delta_{ij} + 2\mu e_{ij}, \quad \theta = T - T_0, \tag{5}$$

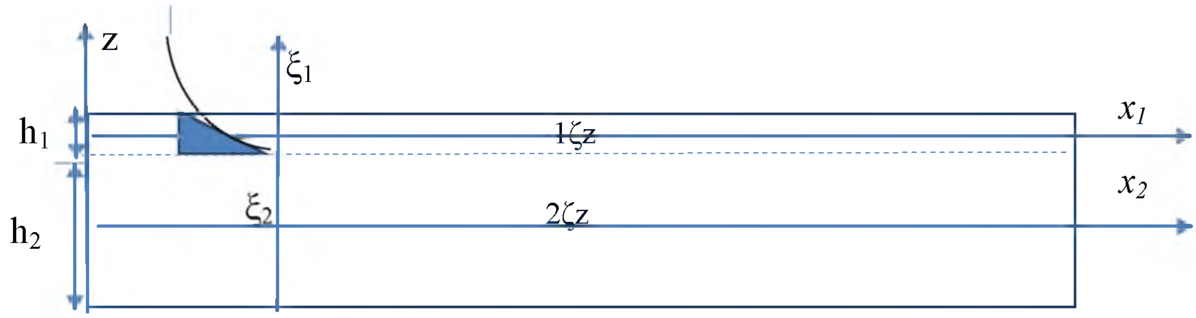
where $\lambda = \dfrac{E\nu}{(1-2\nu)(1+\nu)}$, $\mu = \dfrac{E}{2(1+\nu)}$, Lame coefficients, E - modulus of elasticity, ν - Poisson's ratio; T, T_0 - temperature of the current and initial state, β- characterize the influence of temperature stresses.

Based on the thermodynamic theory, the Fourier thermal conductivity has the following form [11]:

$$c_\varepsilon\dot{\vartheta} = \lambda_\varepsilon\Delta\vartheta - T\beta\dot{\varepsilon}_{kk}, \tag{6}$$

wherec_ε,λ_ε- coefficients of heat capacity and thermal conductivity with constant deformation.

Thus, we have the general task of the nonlinear thermodynamic theory of elasticity for thermal mechanical processing of metal parts by cutting method solving which we can study the stress-strain state of the objects under consideration, including metal strips and formed chips. Due to the difficulties encountered in solving nonlinear three-dimensional problems (1)-(9) in applied problems the mechanics of a deformable solid usually turn to two-dimensional problems.



The pattern of chips formation from the processed surface

In the problem under consideration, deformation of plate – 2 does not go beyond the framework of the linear theory of elasticity. The same is true for a metal strip - 1 before exposure to the cutting tool. Thus, we have linear physical and kinematic relations. After the impact of the cutter, at the time of chip formation, the metal strip undergoes final displacements, and the strip bending is accompanied by a predominance of deflection compared to the others, i.e. the condition $U_z >> U_i$ is satisfied.

Therefore, in expressions (1)-(6), when the plates are bent by nonlinear terms along the U_i displacement vector axial displacements they are usually neglected. At that, the nonlinear components of the derivative U_z with respect to the normal coordinate z are also taken to be negligible.

Then, for the components of the second Piola – Kirchhoff tensor and the Lagrange – Green strain tensor, we hae:

$$P_{ij} = \sigma_{ij}, P_{3j} = \left(\delta_{3k} + U_{3,k}\right)\sigma_{kj}, \varepsilon_{33} = U_{3,3},$$
$$\varepsilon_{ij} = \frac{1}{2}\left(U_{i,j} + U_{j,i} + U_{3,i}U_{3,j}\right),\ \varepsilon_{i3} = \frac{1}{2}(U_{i,3} + U_{3,i}),\quad \varepsilon_{33} = U_{3,3}, \tag{7}$$

Hereinafter, the indices i, j, k, l take the values (1), (2). Let the considered three-dimensional deformable body be a metal parallelepiped with a constant thickness $h=h1+h2$ on which the cutter affects (figure). At that, strip-1 with a thickness h_1 is located above the cutting tool, and strip-2 with a thickness h_2 below it. Cartesian coordinate systems $Ox_1x_2\xi_I, I=1,2$ are located in the middle plane of the considered bands.

These bands, until the moment of exposure $x_i > v_i t$ of the cutter, are one whole composition, therefore, along the line of external influence $\xi_2 = h_2/2$ or $\xi_1 = -h_1/2$, the continuity conditions for the components of the displacement vector, stress tensor, and temperature are satisfied.

$$U_i^{(1)} = U_i^{(2)},\ U_z^{(1)} = U_z^{(2)},\quad \sigma_{iz}^{(1)} = \sigma_{iz}^{(2)},\quad \sigma_{zz}^{(1)} = \sigma_{zz}^{(2)},\ \vartheta^{(1)} = \vartheta^{(2)},\ \left(\vartheta^{(1)}\right)'_{\xi} = \left(\vartheta^{(2)}\right)'_{\xi}, \tag{8}$$

After exposure to the cutter: $x_i \le v_i t$ on the upper plane of the lower strip $\xi_2 = h_2/2$, the following boundary conditions

$$\sigma_{iz}^{(2)} = P_{\tau}\left[H(v_1t - x_1)H(v_2t - x_2)\right]_{,i},\quad \left(\vartheta^{(2)}\right)'_{\xi} = \gamma\left[1 + \vartheta_r \frac{P_n v_i}{\lambda c} g(x_i, t)\right]\vartheta^{(2)},$$
$$U_z^{(2)} = 0,\quad g(x_\alpha, t) = H(v_\alpha t - x_\alpha)H(x_\alpha + a_\alpha - v_\alpha t).\ \alpha = 1,2 \tag{9}$$

where ϑr is the coefficient characterizing the influence of the surface roughness of the cutting tool and the processed sample during friction on the resulting temperature, which, depending on the materials used, is determined experimentally; γ is the coefficient characterizing the heat transfer of the sample material and the environment; c is the speed of sound propagation in the considered sample; $H(x)$ is the Heaviside function. v_i - the displacement speed of the cutter along the coordinate axes. Under these conditions, at the lower boundary $\xi_1 = -h_1/2$ of the upper band, the following boundary conditions hold

$$U_z^{(1)} = v_i t\, tg\eta_i + \alpha\vartheta^{(1)},\quad \sigma_{iz}^{(1)} = P_{\tau}\left[H(v_1t - x_1)H(v_2t - x_2)\right]_{,i},\quad \left(\vartheta^{(1)}\right)'_{\xi} = \gamma\vartheta^{(1)}. \tag{10}$$

In a rigidly clamped boundary near the $\xi_2 = -\dfrac{h_2}{2}$ lower strip, a heat exchange process takes place

$$U_i^{(2)} = 0, \; U_z^{(2)} = 0, \quad \left(\vartheta^{(2)}\right)'_{\xi} = \gamma\vartheta^{(2)}, \tag{11}$$

The upper plane $\xi_2 = h_2/2$ of the upper band is free from loads and there is a heat exchange process

$$\sigma_{iz}^{(1)} = 0, \quad \sigma_{zz}^{(1)} = 0, \quad \left(\vartheta^{(1)}\right)'_{\xi} = \gamma\vartheta^{(1)}, \tag{12}$$

Based on the nature of the problems under consideration, for the purpose of simplification, we proceed to the consideration of the corresponding two-dimensional ones, according to the non-classical theory of deformation of a nonlinear elastic parallelepiped [15,22]. The required components of the displacement vector are decomposed in a power series by the normal coordinate ζ:

$$\begin{cases} U_i^{(I)} = A_i^{(I)} + \xi B_i^{(I)} + \xi^2 C_i^{(I)} + \xi^3 D_i^{(I)} + \ldots, \\ U_z^{(I)} = a^{(I)} + \xi b^{(I)} + \xi^2 \theta^{(I)} + \ldots, \\ \vartheta^{(I)} = \vartheta_0^{(I)} + \xi\vartheta_1^{(I)} + \xi^2\vartheta_2^{(I)} + \ldots, \end{cases} \tag{13}$$

where $\Phi_1^{(I)}(\xi) = \dfrac{h_I^2}{12}\left[1 - 12\left(\dfrac{\xi}{h_I}\right)^2\right], \Phi_2^{(I)}(\xi) = \dfrac{1}{4}\left[1 - \dfrac{20}{3}\left(\dfrac{\xi}{h_I}\right)^2\right]\xi.$

$$\bar{\vartheta}^{(I)} = \frac{1}{h_I}\int_{-h_I/2}^{h_I/2} \vartheta^{(I)} d\xi, \quad \tilde{\vartheta}^{(I)} = \frac{1}{h_I}\int_{-h_I/2}^{h_I/2} \vartheta^{(I)}\xi d\xi,$$

$$u_i^{(I)} = \frac{1}{h_I}\int_{-h_I/2}^{h_I/2} U_i^{(I)} d\xi, \quad \psi_i^{(I)} = \frac{1}{h_I}\int_{-h_I/2}^{h_I/2} U_i^{(I)}\xi d\xi, \quad w^{(I)} = \frac{1}{h_I}\int_{-h_I/2}^{h_I/2} U_z^{(I)} d\xi, \quad V^{(I)} = \frac{1}{h_I}\int_{-h_I/2}^{h_I/2} U_z^{(I)}\xi d\xi,$$

where $C_i^{(I)}, D_i^{(I)}, \ldots, a^{(I)}, b^{(I)}, \theta^{(I)}$ are unknown functions of the coordinates x_1, x_2 and time t.

Taking into account (5)-(7), (13) for elastic isotropic plates the nonlinear components of the longitudinal stress tensor will have the following form:

$$\begin{aligned} \sigma_{ij} = {} & \lambda\Big\{u_{k,k} + \xi\psi_{k,k} - \frac{1}{2}\Phi_1(\xi)C_{k,k} - \frac{3h^2}{5}\Phi_2(\xi)D_{k,k} - \beta\left(\bar{\vartheta}^{(I)} + \xi\tilde{\vartheta}^{(I)} - \Phi_1^{(I)}(\xi)\vartheta_2^{(I)}\right) + \\ & + (V + 2\theta\xi) + \frac{1}{2}\left(w + V\xi - \Phi_1(\xi)\theta\right)_{,k}\left(w + V\xi - \Phi_1(\xi)\theta\right)_{,k}\Big\}\delta_{ij} + \\ & + \mu\Big\{u_{i,j} + u_{j,i} + \xi\left(\psi_{i,j} + \psi_{j,i}\right) - \frac{1}{2}\Phi_1(\xi)(C_{i,j} + C_{j,i}) - \\ & - \frac{3h^2}{5}\Phi_2(\xi)(D_{i,j} + D_{j,i}) + \left(w + V\xi - \Phi_1(\xi)\theta\right)_{,i}\left(w + V\xi - \Phi_1(\xi)\theta\right)_{,j}\Big\} \end{aligned} \tag{14}$$

$$\begin{aligned} \sigma_{zz} = {} & \lambda\Big\{u_{k,k} + \xi\psi_{k,k} - \frac{1}{2}\Phi_1(\xi)C_{k,k} - \frac{3h^2}{5}\Phi_2(\xi)D_{k,k} - \beta\left(\bar{\vartheta}^{(I)} + \xi\tilde{\vartheta}^{(I)} - \Phi_1^{(I)}(\xi)\vartheta_2^{(I)}\right) \\ & + \frac{1}{2}\left(w + V\xi - \Phi_1(\xi)\theta\right)_{,k}\left(w + V\xi - \Phi_1(\xi)\theta\right)_{,k}\Big\}\delta_{ij} + (\lambda + 2\mu)(V + 2\theta\xi) \end{aligned} \tag{15}$$

$$\sigma_{iz} = \mu\left\{\psi_i + \xi C_i - \frac{3h^2}{20}\left(1 - 20\left(\frac{\xi}{h}\right)^2\right)D_i + \left(w + V\xi - \Phi_1(\xi)\theta\right)_{,i}\right\} \tag{16}$$

It should be noted here, the line of impact of the cutter $x_i = v_i t$, $\left(\xi_2 = \frac{h_2}{2}\right.$ или $\left.\xi_1 = -\frac{h_1}{2}\right)$ it has a significant feature, because along this line, due to the infinity of some components of the stress tensor, the separation and formation of chips begins. For chips, the deformations become finite, and in the other parts there is a linear stress-strain state. To take these features into account, the area under consideration is divided into 4 sections, separate equations of motion are solved in each area, and the continuity conditions for the components of the movement vector will be used along the lines of the section. For regions 1 and 2, unknown integral constants are determined from boundary conditions (8),(10),(12):

$$\begin{cases} h_1\vartheta_2^{(1)} = \dfrac{1}{1-\gamma\dfrac{h_1}{6}}\left(\gamma\bar{\vartheta}^{(1)} - \left(1-\dfrac{\gamma h_1}{2}\right)\tilde{\vartheta}^{(1)}\right), \quad h_2\vartheta_2^{(2)} = -\dfrac{1}{1+\gamma\dfrac{h_2}{6}}\left(\gamma\bar{\vartheta}^{(2)} - \left(1+\dfrac{\gamma h_2}{2}\right)\tilde{\vartheta}^{(2)}\right), \\ h_1\theta^{(1)} = -V^{(1)} - \dfrac{\nu}{1-\nu}\left[u_{k,k}^{(1)} + h_1\psi_{k,k}^{(1)} + \dfrac{h_1^3}{10}D_{k,k}^{(1)} - \dfrac{\beta}{\gamma}\left(\tilde{\vartheta}^{(1)} + h_1\vartheta_2^{(1)}\right)\right], \\ h_2\theta^{(2)} = -6\left(\dfrac{w^{(2)}}{h_2} - \dfrac{1}{2}V^{(2)}\right), \quad V^{(1)} = V^{(2)} + 3\left(\dfrac{w^{(1)}}{h_1} + \dfrac{w^{(2)}}{h_2}\right), \\ \tilde{\vartheta}^{(1)} = b_{11}\bar{\vartheta}^{(1)} + b_{12}\bar{\vartheta}^{(2)}, \quad \tilde{\vartheta}^{(2)} = b_{21}\bar{\vartheta}^{(1)} + b_{22}\bar{\vartheta}^{(2)}, \end{cases} \tag{17}$$

$$\begin{cases} h_2C_i^{(2)} = \dfrac{4}{9}\left[5\left(\psi_i^{(1)} + \dfrac{5}{2}\psi_i^{(2)}\right) + 3h_1V_{,i}^{(2)} + \dfrac{9}{2}h_1\left(\dfrac{w^{(1)}}{h_1} + \dfrac{w^{(2)}}{h_2}\right)_{,i} - 6\left(2\dfrac{u_i^{(1)}}{h_1} + \dfrac{u_i^{(2)}}{h_2}\right)\right], \\ h_1C_i^{(1)} = \dfrac{5}{9}\left[\left(\psi_i^{(1)} + 25\psi_i^{(2)}\right) + 6h_1V_{,i}^{(2)} + 9h_1\left(\dfrac{w^{(1)}}{h_1} + \dfrac{w^{(2)}}{h_2}\right)_{,i} - 24\dfrac{u_i^{(1)}}{h_1} + \dfrac{48}{5}\dfrac{u_i^{(2)}}{h_2}\right], \\ h_2^2D_i^{(2)} = \dfrac{20}{27}\left[10\psi_i^{(1)} + 23\psi_i^{(2)} + 6h_1V_{,i}^{(2)} + 9h_1\left(\dfrac{w^{(1)}}{h_1} + \dfrac{w^{(2)}}{h_2}\right)_{,i} - 6\left(4\dfrac{u_i^{(1)}}{h_1} + \dfrac{u_i^{(2)}}{h_2}\right)\right], \\ h_1^2D_i^{(1)} = \dfrac{10}{27}\left(\left[-18\psi_i^{(1)} + 25\psi_i^{(2)} + 6h_1V_{,i}^{(2)} + 9h_1\left(\dfrac{w^{(1)}}{h_1} + \dfrac{w^{(2)}}{h_2}\right)_{,i} - 24\dfrac{u_i^{(1)}}{h_1} + 6\dfrac{42}{5}\dfrac{u_i^{(2)}}{h_2}\right]\right), \end{cases} \tag{18}$$

$$b_{11} = \frac{\dfrac{3\gamma h_1}{6-\gamma h_1}a_{22} + \dfrac{6}{6-\gamma h_1}a_{12}}{a_{11}a_{22} - a_{21}a_{12}}, \quad b_{12} = -\frac{\dfrac{3\gamma h_2}{6+\gamma h_2}a_{22} + \dfrac{6}{6+\gamma h_2}a_{12}}{a_{11}a_{22} - a_{21}a_{12}}, \quad a_{11} = \frac{2(3-\gamma h_1)h_1}{6-\gamma h_1}, \quad a_{12} = -\frac{2(3+\gamma h_2)h_2}{6+\gamma h_2},$$

$$b_{21} = -\frac{\dfrac{6_1}{6-\gamma h_1}a_{11} + \dfrac{3\gamma h_1}{6-\gamma h_1}a_{21}}{a_{11}a_{22} - a_{21}a_{12}}, \quad b_{22} = \frac{\dfrac{3\gamma h_2}{6+\gamma h_2}a_{21} + \dfrac{6}{6+\gamma h_2}a_{11}}{a_{11}a_{22} - a_{21}a_{12}}, \quad a_{21} = -\frac{4-\gamma h_1}{6-\gamma h_1}h_1, \quad a_{22} = \frac{4+\gamma h_2}{6+\gamma h_2}h_2.$$

Here, to determine $V^{(2)}$, we will have the following partial differential equation with respect to the unknown

$$\begin{gathered} h_1^2\Delta V^{(2)} + \frac{3}{2}h_1^2\Delta\left(\frac{w^{(1)}}{h_1} + \frac{w^{(2)}}{h_2}\right) + \frac{7}{18}h_1\psi_{i,i}^{(1)} + \frac{25}{6}h_1\psi_{i,i}^{(2)} - \frac{25}{6}\frac{u_{i,i}^{(1)}}{h_1} + \frac{41}{3}\frac{u_{i,i}^{(2)}}{h_2} \\ -\beta\left(1 + \frac{\gamma h_1}{6-\gamma h_1}(1+2b_{11})\right)\bar{\vartheta}^{(1)} - \frac{2h_1}{6-\gamma h_1}\beta b_{12}\bar{\vartheta}^{(1)} + 10\frac{1-\nu}{\nu}V^{(2)} - \frac{9}{2}\left(\frac{w^{(1)}}{h_1} - \frac{w^{(2)}}{h_2}\right) = 0, \end{gathered} \tag{19}$$

After exposure to the cutting tool, to determine unknown integral constants $C_i^{(3)}, D_i^{(3)}, \tilde{\vartheta}^{(3)}, \vartheta_2^{(3)}, V^{(3)}, \theta^{(3)}$ using boundary conditions (10),(12), as a result, we will have:

$$
\begin{cases}
h_1 C_i^{(1)} = -P_\tau \left[H(v_1 t - x_1) H(v_2 t - x_2)\right]_{,i} - h_1 V^{(1)}{}_{,i} \\
h_1^2 D_i^{(1)} = \dfrac{5P_\tau}{6\mu}\left[H(v_1 t - x_1) H(v_2 t - x_2)\right]_{,i} - \dfrac{5\alpha}{6\gamma}\left(\tilde{\vartheta}^{(1)} - h_1 \vartheta_2^{(1)}\right)_{,i} - \dfrac{5}{3}\psi_i^{(1)} \\
\tilde{\vartheta}^{(1)} = \dfrac{12\gamma}{12 - \gamma^2 h_1^2}\bar{\vartheta}^{(1)}, \quad \vartheta_2^{(1)} = \dfrac{6\gamma^2}{12 - \gamma^2 h_1^2}\bar{\vartheta}^{(1)} \\
h_1 \theta^{(1)} = 6\left[-\dfrac{w^{(1)}}{h_1} + \dfrac{1}{2}V^{(1)} + v_i t\, tg\eta_i + \dfrac{\alpha}{\gamma h_1}\left(\tilde{\vartheta}^{(1)} - h_1 \vartheta_2^{(1)}\right)\right],
\end{cases}
\tag{20}
$$

Using the obtained expressions, from the boundary conditions (12) for the normal stress on the upper boundary of the formed chips, for the determination $V^{(3)}$ of neglecting the terms of the high order of smallness, we will have the following nonlinear partial differential:

$$
\begin{aligned}
& u_{k,k}^{(3)} + \frac{1}{2}W^{(3)}{}_{,k} W^{(3)}{}_{,k} + \frac{5h_1}{12}\psi_{k,k}^{(3)} - \frac{23}{24}\frac{P_\tau}{\mu}h_1 \Delta\left[H(v_1 t - x_1) H(v_2 t - x_2)\right] - h_1^2 \Delta V^{(3)} - \\
& \frac{6\gamma}{12 - \gamma^2 h_1^2}\left[2\left(\frac{1-\nu}{\nu}\frac{\alpha}{\gamma h_1} - \frac{\beta}{\gamma}\right) - \gamma h_1\left(\frac{1-\nu}{\nu}\frac{\alpha}{\gamma h_1} + \frac{\beta}{\gamma}\right)\right]\left(\frac{h_1}{24}\frac{\alpha}{\gamma}\Delta\bar{\vartheta}^{(3)} - \bar{\vartheta}^{(3)}\right) + \\
& \frac{1-\nu}{\nu}\left(4V^{(3)} - 6\frac{w^{(3)}}{h_1} + 6\frac{v_i t}{h_1}tg\eta_i\right) = 0,
\end{aligned}
\tag{21}
$$

Now, for 4 plots, we will use the boundary conditions (9) and (11) to determine them, so we will have:

$$
\begin{cases}
h_2^2 D_i^{(4)} = \dfrac{6u_i^{(4)}}{5h_2} - \psi_i^{(4)} + \dfrac{2P_\tau}{5\mu}\left[H(v_1 t - x_1) H(v_2 t - x_2)\right]_{,i}, \; h_2 \theta^{(4)} = -\dfrac{6}{h_2}w^{(4)}, \\
h_2 C_i^{(4)} + = \dfrac{48}{25}\dfrac{P_\tau}{\mu}\left[H(v_1 t - x_1) H(v_2 t - x_2)\right]_{,i} - \dfrac{6u_i^{(4)}}{25h_2} - \dfrac{9}{5}\psi_i^{(4)}, \quad V^{(4)} = 0, \\
h_2 \vartheta_2^{(4)} = -\dfrac{\dfrac{2}{2+\gamma h_2} - \left(1 + \dfrac{\gamma h_2}{2+\gamma h_2}\right)\left(1 + \vartheta_r \dfrac{P_n v_i}{\lambda c} g(x_i, t)\right)}{1 + \dfrac{1}{6}\dfrac{6 + h_2\gamma}{2+\gamma h_2} - \left(1 + \dfrac{6 + h_2\gamma}{2+\gamma h_2}\right)\gamma h_2\left(1 + \vartheta_r \dfrac{P_n v_i}{\lambda c} g(x_i, t)\right)}\gamma\bar{\vartheta}^{(4)} \\
\tilde{\vartheta}^{(4)} = \dfrac{2}{2+\gamma h_2}\left[1 - \dfrac{\dfrac{2}{2+\gamma h_2} - \left(1 + \dfrac{\gamma h_2}{2+\gamma h_2}\right)\left(1 + \vartheta_r \dfrac{P_n v_i}{\lambda c} g(x_i, t)\right)}{1 + \dfrac{1}{6}\dfrac{6 + h_2\gamma}{2+\gamma h_2} - \left(1 + \dfrac{6 + h_2\gamma}{2+\gamma h_2}\right)\gamma h_2\left(1 + \vartheta_r \dfrac{P_n v_i}{\lambda c} g(x_i, t)\right)}\left(1 + \dfrac{h_2\gamma}{6}\right)\right]\gamma\bar{\vartheta}^{(4)},
\end{cases}
\tag{22}
$$

The obtained analytical expressions for the components of the displacement vector and the Cauchy stress tensor in geometrically nonlinear form exactly satisfy the boundary conditions (9)-(12).

The components of the displacement vector - (19) and the symmetric stress tensor depend on unknown integral quantities $u_i^{(I)}, \psi_i^{(I)}, W^{(I)}, \bar{\vartheta}^{(I)}$ which are functions of the coordinates x_1, x_2 and t. In order to obtain a closed system of resolving equations for these unknowns, we introduce the following integral quantities from the components of the stress tensor: $N_{ij}^{(I)} = \int_{-0.5h_I}^{0.5h_I} \sigma_{ij}^{(I)} d\xi$ - normal forces, $Q_i^{(I)} = \int_{-0.5h_I}^{0.5h_I} \sigma_{iz}^{(I)} d\xi$ - shear forces, $M_{ij}^{(I)} = \int_{-0.5h_I}^{0.5h_I} \sigma_{ij}^{(I)} \xi d\xi$ - internal bending moments. In the obtained integral expressions, substituting the components of the stress tensor, we will have

$$\begin{cases} N_{ij}^{(I)} = G^{(I)}\left\{\dfrac{2\nu}{1-2\nu}\left(u_{k,k}^{(I)} + V + \dfrac{1}{2}w_{,i}^{(I)}\, w_{,i}^{(I)}\right)\delta_{ij} + u_{i,j}^{(I)} + u_{j,i}^{(I)} + \dfrac{1}{2}w_{,i}^{(I)}\, w_{,j}^{(I)}\right\}, \\ M_{ij}^{(I)} = D^{(I)}\left\{\dfrac{2\nu}{1-2\nu}\left(\psi_{k,k}^{(I)} + 2\theta + \dfrac{1}{2}V_{,i}^{(I)}\,\theta_{,i}^{(I)}\right)\delta_{ij} + \psi_{i,j}^{(I)} + \psi_{j,i}^{(I)} + \dfrac{1}{2}V_{,i}^{(I)}\,\theta_{,j}^{(I)}\right\}, \\ Q_i^{(I)} = G^{(I)}k^2\left[\psi_i^{(I)} + \left(w^{(I)} - \dfrac{h^2}{30}\theta^{(I)}\right)_{,i}\right], \quad k^2 = \dfrac{5}{6} \end{cases} \tag{23}$$

In order to obtain the equations of elastic plate's motion, we perform the integration procedure (2) taking into account (3) -(5) and (7) over ξ in the interval *[-0.5h$_I$; 0.5h$_I$]:*

$$\begin{cases} N_{ij,j}^{(I)} + p_i^{(I)} - m\ddot{u}_i^{(I)} = 0 \\ M_{ij,j}^{(I)} - Q_i^{(I)} + m_i^{(I)} - \dfrac{h^2}{12}m\ddot{\psi}_i^{(I)} = 0 \\ Q_{i,i}^{(I)} + (N_{ij}^{(I)}w_{,i}^{(I)})_{,i} + (M_{ij}^{(I)}V_{,i}^{(I)})_{,i} + S_z^{(I)} - m\ddot{w}^{(I)} = 0, \end{cases} \tag{24}$$

where

$$\begin{aligned} &p_i^{(1)} = -\sigma_{iz}^{(1)}(x_1,x_2,-\tfrac{h_1}{2}), \quad p_i^{(2)} = \sigma_{iz}^{(2)}(x_1,x_2,\tfrac{h_2}{2}) - \sigma_{iz}^{(2)}(x_1,x_2,-\tfrac{h_2}{2}), \\ &p_i^{(3)} = -P_\tau\left[H(v_1t-x_1)H(v_2t-x_2)\right]_{,i}, p_i^{(4)} = P_\tau\left[H(v_1t-x_1)H(v_2t-x_2)\right]_{,i} - \sigma_{iz}^{(4)}(x_1,x_2,-\tfrac{h_2}{2}), \\ &m_i^{(1)} = \frac{h_1}{2}\sigma_{iz}^{(1)}(x_1,x_2,-\tfrac{h_1}{2}), \quad m_i^{(2)} = \frac{h_2}{2}\left\{\sigma_{iz}^{(2)}(x_1,x_2,\tfrac{h_2}{2}) + \sigma_{iz}^{(2)}(x_1,x_2,-\tfrac{h_2}{2})\right\}, \\ &m_i^{(3)} = \frac{h_1}{2}P_\tau\left[H(v_1t-x_1)H(v_2t-x_2)\right]_{,i}, m_i^{(4)} = \frac{h_2}{2}\left\{P_\tau\left[H(v_1t-x_1)H(v_2t-x_2)\right]_{,i} + \sigma_{iz}^{(2)}(x_1,x_2,-\tfrac{h_2}{2})\right\}, \\ &S_z^{(1)} = -\sigma_{zz}^{(1)}(x_1,x_2,-\tfrac{h_1}{2}), \quad S_i^{(2)} = \sigma_{zz}^{(2)}(x_1,x_2,\tfrac{h_2}{2}) - \sigma_{zz}^{(2)}(x_1,x_2,-\tfrac{h_2}{2}), \\ &S_z^{(3)} = -\sigma_{zz}^{(3)}(x_1,x_2,-\tfrac{h_1}{2}), \quad S_i^{(4)} = \sigma_{zz}^{(4)}(x_1,x_2,\tfrac{h_2}{2}) - \sigma_{zz}^{(4)}(x_1,x_2,-\tfrac{h_2}{2}), \end{aligned} \tag{25}$$

Distributed external efforts and moments. $m=\rho h$ is the line armass, and the density – ρ in thick ness is considered constant. At the same time $h_3 = h_1$, $h_4 = h_2$

Similarly, integrating the thermal conductivity equation with the heat transfer process and the continuity condition at the interface of the bands under consideration, as well as with the expansion (19), we will have the thermal conductivity equation for each strip, in terms of integral quantities

$$c_\varepsilon \dot{\bar{\vartheta}}^{(I)} = \lambda_\varepsilon \Delta\bar{\vartheta}^{(I)} - T\beta\dot{u}_{k,k}^{(I)} + \tilde{\vartheta}^{(1)} + \frac{h_I}{2}\vartheta_2^{(1)} - \gamma\left(\bar{\vartheta}^{(1)} - \frac{h_1}{2}\tilde{\vartheta}^{(1)} + \frac{h_1^2}{6}\vartheta_2^{(1)}\right) \tag{26}$$

In this case, the equations of motion (24), taking into account (14)- (23) and (25) become closed with respect to the following integral quantities: $u_i^{(I)}, \psi_i^{(I)}, W^{(I)}, \bar{\vartheta}^{(I)}$.

In terms of integral quantities, the boundary conditions corresponding to various conditions are written as follows [22].

$$I.\quad \begin{cases} u_i^{(I-1)} = u_i^{(I)},\ \psi_i^{(I-1)} = \psi_i^{(I)},\ w^{(I-1)} = w^{(I)},\ V^{(I-1)} = V^{(I)},\ \vartheta^{(I-1)} = \vartheta^{(I)}, \\ N_{ij}^{(I-1)}n_j = N_{ij}^{(I)}n_j,\ M_{ij}^{(I-1)}n_j = M_{ij}^{(I)}n_j,\ Q_i^{(I-1)}n_i = Q_i^{(I)}n_i, \\ V^{(I-1)}{}_{,i}\,n_i = V^{(I)}{}_{,i}\,n_i,\ \vartheta^{(I-1)}{}_{,i}\,n_i = \vartheta^{(I)}{}_{,i}\,n_i. \end{cases}$$

$$I = 2,4 \quad x_1 = v_1 t, t > 0.$$

$$II.\quad N_{ij}^{(I)}n_j\big|_\Sigma = 0,\ M_{ij}^{(I)}n_j\big|_\Sigma = 0,\ Q_i^{(I)}n_i\big|_\Sigma = 0,\ V^{(I)}{}_{,i}\,n_i\big|_\Sigma = 0,\ \vartheta^{(I)}{}_{,i}\,n_i\big|_\Sigma = \gamma\vartheta^{(I)},$$

$$I = \overline{1,4} \quad x_1 = 0, l$$

It should be noted that the first type of boundary conditions correspond to a hard jamming of the edges of the bands under consideration, and the second type corresponds to a joint edge taking into account the thermal exchange process. Here, for prostate records, $v_2=0$ is assumed.

The initial conditions (3), taking into account (19), can be similarly written in terms of integral quantities.

Conclusions. Based on the results obtained, the **following conclusions** can be drawn:

1. The thermal mechanical formulation of the cutting objects problem on the basis of the fundamental thermodynamic nonlinear theory of elasticity is formulated;
2. The boundary conditions for the interaction of the cutting tool and the object under consideration (13) and (14) are formulated;
3. In the obtained formulas (13) of thermal transfer at the boundary, the possibility of the appearance of additional heat from the influence of the base of the cutting tool on the surface of the objects under consideration is taken into account;
4. The formulated thermal mechanical theory of cutting method allows us to solve the related problem of thermodynamics to determine the stress-strain state and the laws of temperature distribution in the formed cutting chips.

А. Б. Ахмедов[1], Д. Т. Ходжибергенов[2], У. Д. Хожибергенова[2], З. А. Ибрагимова[2], Б. Н. Абсадыков[3]

[1]М. Ұлықбек атындағы Өзбекстан ұлттық университеті, Ташкент, Өзбекстан;
[2]М. Әуезов атындағы Оңтүстік Қазақстан мемлекеттік университеті, Шымкент, Қазақстан;
[3]Ә. Б. Бектұров атындағы Химия ғылымдары институты, Алматы, Қазақстан

МЕТАЛДАРДЫ КЕСУ КЕЗІНДЕ МЕХАНИКАЛЫҚ ӨҢДЕУДІҢ ТЕРМОДИНАМИКАЛЫҚ НЕГІЗДЕРІ

Аннотация. Металдарды кесу үдерісінде пайда болатын іргелі термомеханикалық құбылыстар жүйелі және өзара байланысы теориялық және математикалық сипаттауға қиын. Зерттеулерде қарастырылатын, мәні мен сипаты жағынан әртүрлі құбылыстар бір-бірімен тығыз байланысқандықтан, олардың өзара әрекетін сипаттау күрделі. Зерттеулеріміздің нәтижелері бойынша кесетін жиектің шыңына әлі күнге дейін тұтас бірлікке келмеген он бір тәуелсіз теориялар тоғыстырылғаны анықталды. Осындай зерттеулерде жоңқа түзілу теориясын, металдарды кесу механикасын, металл өңдеу кезінде үйкеліс және тозу теориясын, кесу термодинамикасын, кесу құралдарының беріктігі мен сенімділігі теориясын жүйелеу және өзара байланыстыру қажет.

Металдарды кесу арқылы механикалық өңдеу термодинамикалық үдерістермен сүйемелденеді. Металл кесу кезінде жылу көзі кесілетін қабатта және өңделген беті мен кесу бетіне іргелес қабаттарда соңғы деформацияға, сондай-ақ кескіштің артқы беті бойынша үйкелуді еңсеруге кететін жұмыс болып саналады. Соңғы деформация үдерісінде үлгінің материалдық нүктелері бір-біріне қатысты қозғалады, бұл қосымша жылу түзудің көзі болып саналады. Кесу үдерісінде бөлінетін жылу оның пайда болу орындарына шоғырланбайды, ал термодинамика заңдарына сәйкес үлгінің көлемі бойынша жоғары температуралы нүктелерден төмен температуралы нүктелерге таралады. Механикалық жұмыстың 95% - ға жуығы металдарды кесу кезінде деформация мен үйкеліс жылуға өтеді. Сондықтан металдарды кесу кезінде жоңқалар мен байланыс беттері 500÷1000° аралығында қызады, бұл ретте бөлінетін жылу, негізінен жоңқа арқылы 50÷86% жұтылады, кескіш арқылы 10÷40%, өңделетін бұйым негізінде 3÷9%, шамамен 1% жылу қоршаған кеңістікке шығарылады.

Бұл жұмыста термодинамиканың іргелі заңдары және деформацияланатын қатты дененің сызықты емес механикасы негізінде металдарды кесудің термомеханикалық теориясы ұсынылады. Бұл үдерісте кесу құралы өткір ішкі кернеулердің сыни мәнінің пайда болуына байланысты кесу желісі бойынша металл жоңқасының түбінен үзіліп кетеді. Бұл ретте жоңқасы металл жолақ түрінде үзілгеннен кейін соңғы орын ауыстырады. Бұл жұмыста металдарды кесудің термомеханикалық үрдістерін математикалық модельдеу әдістері В.В. Новожиловтың сызықты емес серпімділік теориясына негізделеді. Үшөлшемді дененің қарапайым көлемінің соңғы деформациясы соңғы айналу және қозғалумен, алайда геометриялық өлшемдердің аз салыстырмалы өзгерістерімен бірге жүреді.

Алынған нәтижелер негізінде фундаментальды термодинамикалық сызықты емес серпімділік теориясы негізінде объектілерді кесу міндетін терммеханикалық қою жағдайы тұжырымдалды; кесетін аспап пен қарастырылатын объектінің өзара әрекеттесуі үшін шекаралық жағдайлар жасалды; d алынған шекарадағы жылу алмасу формулалары кесетін құрал негізінің қарастырылатын объектілердің беті туралы әсерінен қосымша жылудың пайда болу мүмкіндігі ескерілген; тұжырымдалған термомеханикалық кесу теориясы қалыптасқан кесу жоңқаларында температураның таралу заңдылықтары мен кернеулі-деформацияланған жағдайын анықтау бойынша термодинамиканың байланысты есебін шешуге мүмкіндік береді.

Кесу үдерістерін математикалық модельдеу мәселелерімен Қазақстан Республикасы білім және ғылым Министрлігі Ғылым Комитетінің гранттық қаржыландыру бойынша «Кесу үрдістерінің имитациялық модельдерін әзірлеу және олардың негізінде жабдықтардың тиімді параметрлері мен өңдеу режимдерін болжау» жобасы бойынша ғалымдар тобы айналысады.

Түйін сөздер: термодинамика, жылу өткізгіштік, кесу, деформация, үйкеліс, қатты дене, кернеу, кесу құралы, жоңқа, кедір-бұдырлық, жылдамдық, ығысу.

А. Б. Ахмедов[1], Д. Т. Ходжибергенов[2], У. Д. Хожибергенова[2], З. А. Ибрагимова[2], Б. Н. Абсадыков[3]

[1]Узбекистанский Национальный Университет, Ташкент, Узбекистан;
[2]Южно-Казахстанский государственный университет им. М. Ауезова, Шымкент, Казахстан;
[3]Институт химических наук им. А. Бектурова, Алматы, Казахстан

ТЕРМОДИНАМИЧЕСКИЕ ОСНОВЫ МЕХАНИЧЕСКОЙ ОБРАБОТКИ МЕТАЛЛОВ РЕЗАНИЕМ

Аннотация. Фундаментальные термомеханические явления, возникающие в процессе резания металлов, трудно поддаются системному и взаимосвязанному теоретическому и математическому описанию. Различные по сути и характеру явления, рассматриваемые в исследованиях, столь тесно переплетены друг с другом, что очень сложно описать их взаимодействие. По результатам наших исследований выявлено, что на вершину режущей кромки сфокусировано одиннадцать относительно независимых теорий, не пришедших до сих пор к целостному единству. В подобных исследованиях необходимо систематизировать и взаимно увязать теорию стружкообразования, механику резания металлов, теории трения и износа при металлообработке, термодинамику резания, теории стойкости и надежности режущих инструментов.

Механическая обработка металлов резанием сопровождается термодинамическими процессами. Источником теплоты при резании металлов является работа, затрачиваемая на конечные деформации в срезаемом слое и в слоях, прилегающих к обработанной поверхности и поверхности резания, а также на преодоление трения по задним поверхностям резца. В процессе конечной деформации материальные точки образца перемещаются относительно друг друга, что является источником дополнительного теплообразования. Выделяющаяся в процессе резания теплота не сосредоточивается в местах ее образования, а согласно законам термодинамики распространяется по объему образца от точек с высшей температурой к точкам с низшей температурой. При резании металлов около 95% механической работы деформации и трения переходит в теплоту. Поэтому при резании металлов стружки и контактные поверхности нагреваются в интервале 500÷1000°, при этом выделяемая теплота, в основном поглощается стружкой – 50÷86%, резцом – 10÷40%, обрабатываемым изделием – 3÷9%, около 1% теплоты излучается в окружающее пространство.

В данной работе предлагается термомеханическая теория резания металлов на основе фундаментальных законов термодинамики и нелинейной механики деформируемого твердого тела. В данном процессе по линии резания, в связи с возникновением критических значений внутренних напряжений на острие режущего инструмента, происходит отрыв стружки металла от основания. При этом после отрыва стружка в виде металлической полосы совершает конечные перемещения. В данной работе подходы математического моделирования термомеханических процессов резания металлов основываются на нелинейной теории упругости В.В. Новожилова, причем конечное деформирование элементарного объема рассматриваемого трехмерного тела сопровождается конечными вращениями и перемещениями, но с малыми относительными изменениями геометрических размеров.

На основании полученных результатов сформулирована термомеханическая постановка задачи резания объектов на основе фундаментальной термодинамической нелинейной теории упругости; сформулированы граничные условия для взаимодействия режущего инструмента и рассматриваемого объекта; d полученных формулах теплообмена на границе учтена возможность возникновения дополнительной теплоты от воздействия основания режущего инструмента о поверхность рассматриваемых объектов; сформулированная термомеханическая теория резания позволяет решать связанную задачу термодинамики по определению напряженно-деформированного состояния и закономерностей распространения температуры в образовавшихся стружках резания.

Вопросами математического моделирования процессов резания занимается группа ученых по проекту AP05132157 «Разработка имитационных моделей процессов резания и прогнозирование на их основе оптимальных параметров инструмента и режимов обработки» по грантовому финансированию Комитета науки Министерства образования и науки Республики Казахстан для прогнозирования значений возникающих силовых и температурных явлений, величину износа инструмента, которая будет сопровождаться имитацией процесса резания.

Ключевые слова: термодинамика, теплопроводность, сдвиг, деформация, трение, твердые тела, напряжение, режущий инструмент, сколы, шероховатость, скорость, сдвиг.

Information about authors:

Akhmedov Akrom Burkhonovich, Doctor of Physic and Mathematic Sciences, Head of Mechanics and Mathematic Modeling Chair, M. Ulugbek National University of Uzbekistan, Tashkent, Uzbekistan; ahmedov-1956@mail.ru; https://orcid.org/0000-0001-6897-2867

Khojibergenov Davlatbek Turganbekovich, Doctor of Technical Sciences, Director of Science and Production Department, M.Auezov South Kazakhstan State University, Shymkent, Kazakhstan; had_ji@mail.ru; https://orcid.org/0000-0003-0039-9931

Ibragimova Zaure Assilbekovna, PhD, Senior Lecturer, M.Auezov South Kazakhstan State University, Shymkent, Kazakhstan; Zaure_1983_as@mail.ru; https://orcid.org/0000-0002-7176-8186

Khojibergenova Umida Davlatbekkyzy, Student of High School "Information Technologies and Energetics" ofM. Auezov South Kazakhstan State University, Shymkent, Kazakhstan; hadji_umida@mail.ru; https://orcid.org/0000-0003-2381-8094

Absadykov Bakhyt Narikbayevich, Doctor of Technical Sciences, Deputy General Director, A.B. Bekturov Institute of Chemical Sciences, Almaty, Kazakhstan; b_absadykov@mail.ru; https://orcid.org/0000-0001-7829-0958

REFERENCES

[1] Solonenko V.G., Ryzhkin A.A. Rezaniye metallov i rezhushchiye instrument [Metal Cutting and Cutting Tools]. St. Petersburg: Higher School, 2007. 416 p.

[2] https://www.researchgate.net/publication/308166900_Metal_cutting_-_theory_and_application

[3] Krivoruchko D.V. Modelirovaniye protsessov rezaniya metodom konechnykh elementov: metodologicheskiye osnovy [Modeling of cutting processes by the finite element method: methodological basis] monografiya / D.V. Krivoruchko, V.A. Zaloga, Sumy, 2012. 496 p.

[4] Molinari A., Cheriguene R., Miguelez H., Numerical and analytical modeling of orthogonal cutting: The link between local variables and global contact characteristics, in Int. J. Mech. Sci. Vol. 53, 2011. P. 183-206.

[5] https://link.springer.com/article/10.3103%2FS0025654408030205. Numerical modeling of cutting processes for elastoplastic materials in 3D-statement.

[6] Khojibergenov D.T., Khojibergenova U.D., Akhmedov A.B., Absadykov B.N. Some aspects of the construction of a mathematical model for cutting metals // News of the National Academy of Sciences of the Republic of Kazakhstan. Series of Geology and Technical Sciences. 2019. N 2. P. 69-78. https://doi.org/10.32014/2019.2518-170X.40

[7] Novozhilov V.V. Osnovy nelineynoy teorii uprugosti [Fundamentals of the nonlinear theory of elasticity]. M. L.: Gostekhizdat, 1948. 211 p.

[8] Lur'ye A.I. Nelineynaya teoriya uprugosti [Nonlinear theory of elasticity]. M.: Nauka. Glavnaya redaktsiya fiziko-matematicheskoy literatury, 1980. 512 p.

[9] Sneddon, I.N. Klassicheskaya teoriya uprugosti [Classical theory of elasticity] / I.N. Sneddon, D.S. Berri. M.: University Book, 2008. 216 p.

[10] https://www.cambridge.org/core/books/nonlinear-elasticity/elements-of-the-theory-of-finite-elasticity/5BCD73370EEDF87E574C9D74C779ABC3.

[11] Novatskiy V. Teoriya uprugosti [Theory of elasticity]. M. Mir, 1975. 622 p.

[12] Kartashov E.M. Analiticheskaya teoriya teploprovodnosti i prikladnoy termouprugosti [Analytical theory of thermal conductivity and applied thermoelasticity] / E.M. Kartashov, V.A. Kudinov. M.: KD Librokom, 2018. 656 p.

[13] Kuz'menko A.S. Teoriya uprugosti: Osnovy pragmatichnogo podkhoda [Theory of elasticity: Fundamentals of a pragmatic approach] / A.S. Kuz'menko. M.: KD Librokom, 2013. 132 p.

[14] Lomakin V.A. Teoriya uprugosti neodnorodnykh tel [Theory of elasticity of inhomogeneous bodies] / V.A. Lomakin. M.: Lenand, 2014. 376 p.

[15] Skoptsov K.A., Sheshenin S.V. Asimptoticheskiy analiz teorii plastin Reysnera-Mindlina [Asymptotic analysis of the theory of Reisner-Mindlin plates]. M.: Publ. Moscow State University, Elasticity and Inelasticity, 2011. P. 236-240.

[16] https://link.springer.com/chapter/10.1007/978-3-662-39776-3_1. The Linear Theory of Elasticity.

[17] Akhmedov A.B. Sravnitel'nyy analiz utochnennykh teoriy izgiba tolstykh plit [Comparative analysis of refined theories of bending of thick plates] // DAN RUz. Tashkent 2007. N 2. P. 44-46.

[18] https://link.springer.com/article/10.1007/s00419-011-0513-4. A relook at Reissner's theory of plates in bending.

[19] https://link.springer.com/article/10.1007/s10409-010-0372-1. A two variable refined plate theory for the bending analysis of functionally graded plates.

[20] https://link.springer.com/article/10.1007/s40091-014-0062-5. An accurate two-dimensional theory for deformation and stress analyses of functionally graded thick plates.

[21] Timoshenko S.P. Plastinki i obolochki [Plates and shells]. M.: Gostekhizdat, 1948. 460 p.

[22] Akhmedov A.B. Neklassicheskayateoriyaizgibasloistykhkompozitov [Non-classical theory of bending of layered composites] // VestnikTGTU. Tashkent, 2015. N 2. P. 10-17.

[23] Uprugost' i neuprugost' [ElasticityandInelasticity].Materials of the International Scientific Symposium on Problems of Deformable Bodies Mechanics dedicated to the 100th anniversary of the A.A. Ilyushin birth (M., January 20-21, 2011) / Ed. prof. I.A. Kiyko, prof. G.L. Brovko, prof. R.A. Vassina. M.: Publishing House Moscow University, 2011. 483 p.