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O. O. Dovzhenko[1], V. V. Pohribnyi[1], Ye. V. Klymenko[2], M. Orešković[3]

[1]National University "Yuri Kondratyuk Poltava Polytechnic", Poltava, Ukraine;
[2]Odessa State Academy of Civil Engineering and Architecture, Odessa, Ukraine;
[3]University North, Varaždin, Croatia.
E-mail: o.o.dovzhenko@gmail.com, v.v.pogrebnoy1960@gmail.com,
klimenkoew57@gmail.com, moreskovic@unin.hr

USE OF EXTREME PROPERTIES OF DEFORMATION FOR ESTIMATION OF STRENGTH OF CONSTRUCTIVE CONCRETE AND REINFORCED CONCRETE

Abstract. To estimate the resistance of structural concrete and reinforced concrete to destruction, as a criterion for reaching the ultimate state, extreme properties of the energy (power) of compressed concrete deformation are used.

Normal and inclined to the longitudinal axis of the bending elements cross sections are considered in the most stressed zones. The limitation of the "stress-strain" diagram used in calculations for concrete is justified by the level of the beginning of macro-destructurization, which makes it possible to exclude a re-evaluation of strength. The value of the ultimate strain is determined on the descending branch of the diagram at the point corresponding to the maximum deformation energy. Super limiting strain is accompanied by a sharp decrease of stresses and structural rupture of the material. The dependence of the ultimate strain value on the parameter of the elastic-plastic characteristics of concrete is given.

The values of the ultimate strains for concrete of different classes are obtained. The strength problem of a compressed inclined element near the supports as a component of the truss analogy is solved. The result is obtained on the basis of a variational method in the theory of plasticity. The functional of virtual velocities principle is used. Concrete is considered as a rigid-plastic body. The shear failure occurs within the boundaries of the inclined element. Intense deformation is considered localized in a thin layer on the failure surface. To determine the value of the ultimate load the upper estimate is applied. The minimum power of plastic deformation is used as a criterion. The area of implementation of the truss analogy method is specified.

Key words: concrete, reinforced concrete, extreme deformation properties, ultimate deformation, shear, truss analogy.

Introduction. To estimate the strength of concrete and reinforced concrete structures, a non-linear deformation model has recently been widely used, which forms the basis of a number of author and normative techniques [1-6]. It examines the distribution of the strain in the sections normal to the longitudinal axis of the elements and allows use the stress diagrams in a compressed zone concrete corresponding to it and experimentally confirmed. This shows the actual work in the ultimate state. The most proven relationship between stresses σ_c and strains ε_c of concrete is the fractionally rational function [5,6]. Meanwhile, it should be noted that the value of the ultimate strain of concrete, which bounds the "$\sigma_c - \varepsilon_c$" diagram on the descending branch, is not uniquely determined. At the moment, the experimentally determined values of the ultimate strain of concrete vary over a fairly wide range [2,3,7] and require a clarification.

The tasks of estimating the strength of elements in inclined sections are solved on the basis of the method of truss analogy with the introduction of empirical coefficients [8-10]. At the same time, the shear form of failure is experimentally confirmed, both within the boundaries of the compressed strut (strip) and under the dangerous inclined crack [9-12]. At the same time, the empirical approach does not allow the establishment of clear boundaries for its implementation.

In [13], the cyclic strength of asphalt concrete was determined depending on the level of deformation. In [14], the safety margins of the truss structure were established using the criterion of the minimum perceived mobile load.

In accordance with the abovementioned, the solution of the problems under consideration on a general theoretical basis is relevant. The application of the extreme principles of mechanics of a solid deformed body seems promising.

The purpose of this study is to improve the methods for estimating the strength of structural concrete and reinforced concrete based on extreme deformation properties and the theory of plasticity.

Research methods. To achieve this purpose, the methods of mechanics of a solid deformed body are used. As criteria for solving problems of strength, the extreme properties of the energy (power) of concrete deformation are considered. The concept of rigid plastic body is implemented. The variational method is used in the theory of plasticity, the principle of virtual velocities, the upper estimate of the ultimate load, discontinuous solutions. The functional principle of the virtual velocities is investigated on a stationary state. As a condition of plasticity at a certain range of stresses, strength condition [15] is used, which generalize classical theories of Mohr and Mises – Henki for fragile materials. The values of the ultimate deformation of the concrete, beyond which comes the macrofailure of its structure, meets the criterion of the maximum potential deformation energy. To estimate the strength of the elements under the shear, kinematically possible schemes of its failure are considered, and that one is taken at which the plastic deformation power is minimal.

Results. The task of evaluating strength in normal sections of reinforced concrete elements under bending in [3,4] is proposed to be solved by determining the values of the moments corresponding to the maximum in the “moment – curvature” or “moment – deformation” diagrams. In this case, the condition of not exceeding the ultimate value ε_{cR} by the strain rate of the extreme fiber of the compressed zone should be observed.

The stress-strain relationship for concrete (figure 1) is fairly accurately described by a rational function or a polynomial of the 5th degree, which is harmonized according to research data [3,16]. Meanwhile, to clarify the parameters of these functions and check the condition $\varepsilon_c \le \varepsilon_{cR}$, it is necessary to establish the value ε_{cR} for different classes of concrete. ε_{cR} limits the part of the descending branch of the “$\sigma_c - \varepsilon_c$” diagram used in the calculations, where the potential deformation energy increases. With an increase in the level of deformation above the beyond, destructurization and destruction of concrete occur.

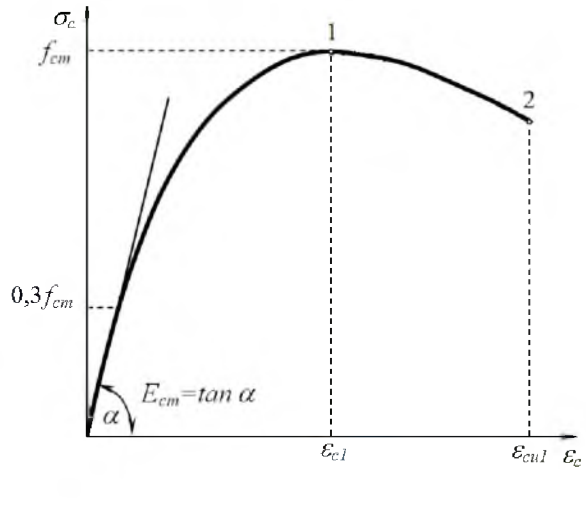

Figure 1 – "Stress – strain" diagram of compressed concrete:
f_{cm} and E_{cm} – average values, respectively, of the compressive strength of the concrete and the initial modulus of elasticity

The relations for fractional rational function and the 5th degree polynomial are written respectively in the form

$$\sigma_c / f_{cd} = (k\eta - \eta^2) / \left[1 + (k-2)/\eta\right], \tag{1}$$

$$\sigma_c / f_{cd} = a_1\eta + a_2\eta^2 + a_3\eta^3 + a_4\eta^4 + a_5\eta^5, \tag{2}$$

where f_{cd} – design value of concrete compressive strength; $k = 1{,}05E_{cd} / E_{c1,cd}$ – characteristic of the elastic-plastic properties of concrete; E_{cd} – design modulus of concrete elasticity; $E_{c1,cd}$ – secant module at the top of the diagram (fig. 1); $\eta = \varepsilon_c / \varepsilon_{c1,cd}$ – level of strain; $\varepsilon_{c1,cd}$ – strain at maximum stress; a_1, a_2, a_3, a_4, a_5 – polynomial coefficients.

It is proposed to determine the calculated value of the ultimate strain ε_{cdR} using the extreme property of deformation – the achievement of the maximum deformation energy of concrete, from the equation

$$\sigma_c\varepsilon_{cdR} = \max(\sigma_c\varepsilon_c). \tag{3}$$

To obtain the polynomial coefficients, the characteristic points 1 and 2 are considered, as well as the area of the diagram bounded by the deformation ε_{cdR} (figure 1). The results are given in table 1.

Table 1 – 5th degree polynomial coefficients

Coef-ficient	Concrete compression class									
	C12/15	C16/20	C20/25	C25/30	C30/35	C32/40	C35/45	C40/50	C45/55	C50/60
a_1	2.9777	2.8383	2.7013	2.5758	2.4873	2,3852	2.302	2.2463	2.1595	2.0663
a_2	-3.4783	-3.1001	-2.7361	-2.3919	-2.1608	-1,8925	-1.6834	-1.5473	-1.3431	-1.1369
a_3	2.1287	1.7705	1.4358	1.1021	0.8943	0,6499	0.4727	0.3632	0.2103	0.0752
a_4	-0.7334	-0.5939	-0.4685	-0.3317	-0.2554	-0,1629	-0.1032	-0.0696	-0.0292	-0.0049
a_5	0.1053	0.0852	0.0675	0.0457	0.0346	0,0203	0.0119	0.0074	0.0025	0.0003

Functions (1) and (2), the initial modulus of elasticity E_{cd} and the secant modules at the specified points $E_{c1,cd}$, $E_{cR,cd}$ determine the stress diagram in the compressed zone of the reinforced concrete element under the condition that the strain achieves in the most compressed fiber the value $\varepsilon_{cR,cd}$.

The ultimate value of a bending moment that a reinforced concrete element can perceive

$$M_u = f_{yd}A_s d(1 - \chi\bar{\xi}\omega) = f_{cd}bd^2\bar{\xi}\omega(1 - \chi\bar{\xi}\omega) \tag{4}$$

in [4] it is recommended to determine from the condition

$$M_u = \max(\varepsilon_c), \tag{5}$$

where d – working section height; $\bar{\xi} = x/d$ – relative height of the compressed zone of concrete; ω – ratio of stress diagram completeness; χ – characteristic that determines the distance from the point of application of the resultant force in concrete to the compressed face of the element (figure 2).

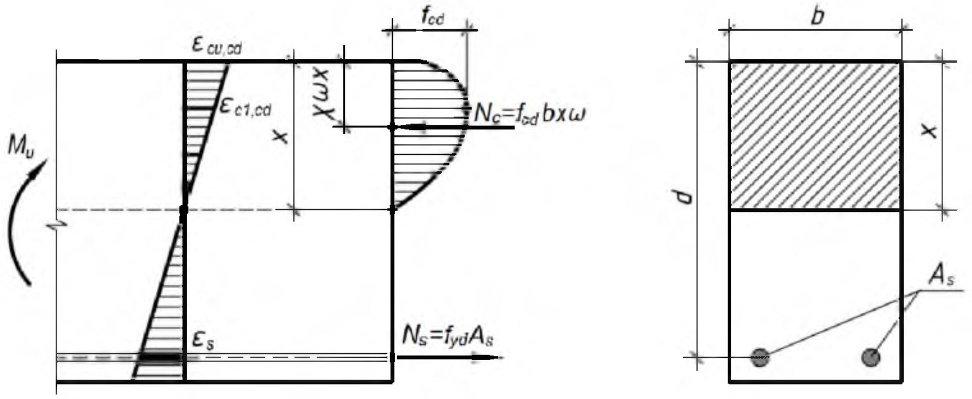

Figure 2 – The design scheme of the reinforced concrete element in the normal section: distribution of forces, stresses and strains

The authors proposed formulas for engineering calculation, defining the above characteristics by parameter k

$$\eta_u = \eta_R \sqrt{1.8/k}\,, \tag{6}$$

$$\omega = 0.5 + 0.24\sqrt[4]{k-1}\,, \tag{7}$$

$$\chi = 2/3 - 0.12\sqrt[4]{k-1}\,, \tag{8}$$

$$\text{at } k \neq 2 \;\; \eta_R = \frac{1}{2k-4}\left[\sqrt{\left(\frac{k^2-2k-3}{2}\right)^2 + 4k^2 - 8k} + \frac{k^2-2k-3}{2}\right], \text{ at } k = 2 \;\; \eta_R = 4/3, \tag{9}$$

where $\eta_u = \varepsilon_{cu,cd} / \varepsilon_{c1,cd}$ – strain level at $M_u = \max$; $\eta_R = \varepsilon_{cR,cd} / \varepsilon_{c1,cd}$ – ultimate strain level (table 2).

Table 2 – Ultimate strain level of concrete η_R

Concrete compression class									
C12/15	C16/20	C20/25	C25/30	C30/35	C32/40	C35/45	C40/50	C45/55	C50/60
1.8093	1.7144	1.6269	1.5705	1.5264	1.4841	1.4495	1.4272	1.3934	1.358

The condition that the value of the current concrete strain ε_c does not exceed the value $\varepsilon_{cR,cd}$ indicates that the concrete is working up to the boundary of its macro-destructuring and can be written as

$$\eta_u \leq \eta_R\,. \tag{10}$$

Estimation of the strength of reinforced concrete elements on the support areas by the inclined section using the method of truss analogy considers the shear form of failure of the compressed element (strut). In this case, the calculated dependencies include a number of empirical coefficients.

For the theoretical justification of the implementation of the truss analogy, the problem of the strength of an inclined prism loaded at the ends of the compressive and tangential components of the transverse force is solved. The scheme of a rigid-plastic body is used. The prerequisites for applying the theory of plasticity are represented in [17-20]. The solution is based on the principle of virtual velocities, whose functional in the absence of inertial and mass forces is

$$I = \int_{S_l} W_{cl}\, dS - \int f_i^* V_i \, dS, \tag{11}$$

where f_i^* – forces in the direction of velocities V_i, given on the surface of the body S; W_{cl} – power density of plastic deformation of concrete.

The functional is investigated on a stationary state $\delta I = 0$.

The simplest one is a solution in discontinuous functions of velocities.

The plastic deformation is considered localized in a thin layer on the failure surface S_l, which divides the element into two absolutely rigid disks. The jumps of the tangential ΔV_t and normal ΔV_n to the surface S_l components of the movement velocity V of one disk relative to another are expressed through of the angle of inclination of the failure surface γ and the ratio of the velocities $m = V_1 / V_2$ in the direction of the action of forces T_u and N_u. The angle of the direction of velocity V to the surface S_l is $\psi = arctg\, m - \gamma$.

The variational method is used in the theory of plasticity. Parameters m and γ vary. The power W_{cl} on the area S_l is considered as a function of deformation velocities, which takes into account the dilatancy of concrete.

The kinematic scheme of the failure of the strut is shown in figure 3.

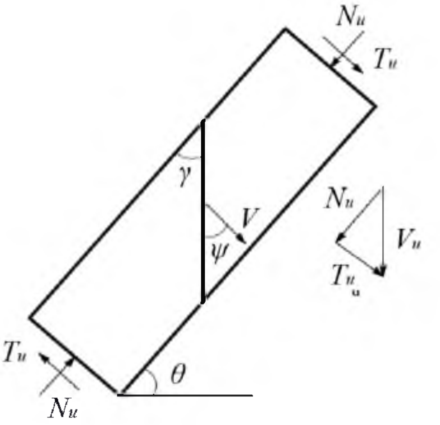

Figure 3 – The kinematic scheme of the compressed element failure

The formula for estimating the strength of a reinforced concrete inclined element has the form

$$\frac{V_u}{f_{cd} b l_c} = \frac{\sqrt{4(1-\chi+\chi^2)/3}\sqrt{(m-\tan\gamma)^2+(1+m\,\mathrm{an}\,\gamma)^2}-(1-\chi)(m-\tan\gamma)}{(\tan\theta+m)\tan\gamma}\sqrt{1+\tan^2\theta}, \qquad (12)$$

where b and l_c – cross section dimensions; $l_c = z/\sqrt{2}$, $z = 0.9d$.

The results of the calculation with respect to the resistance of concrete to tension and compression $\chi = f_{ctd} / f_{cd} = 0.07$ are given in table 3.

Table 3 – Ultimate relative forces perceived by an inclined element

θ, °	$\tan\theta$	m	$\tan\gamma$	γ, °	$N_u / f_{cd} b l_c$	$T_u / f_{cd} b l_c$	$V_u / f_{cd} b l_c$
45	1	10.51	1.21	50.4	0.305	0.305	0.432
30	0.577	176	1.32	52.9	0.179	0.309	0.357
21.8	0.4	4054	1.33	53	0.124	0.309	0.333

Both characteristics of concrete strength are taken into account, in contrast to the dependence of the method [5,6]. The need to take into account the tensile strength of concrete f_{ctd} is due to the shear form of failure. It is well known that shear strength depends both on f_{ctd} and f_{cd}.

Comparative analysis of the results of evaluating the strength of reinforced concrete elements in inclined sections near the supports on the basis of a refined method of a truss analogy with experimental data indicates the convergence of theoretical and experimental values. The design scheme (figure 3) is confirmed by the failure pattern observed in experiments. Plastic strains are localized on the cut surface in thin layers, which is confirmed by systematic experimental studies [21].

If condition $1 \le \cot\theta \le 1.5$ is met failure occurs within the boundaries of the inclined compressed brace. In the case $\cot\theta > 1.5$ there is a shear of the concrete of the compressed zone of the bending element over a dangerous inclined crack, which is described by the well-known disk model [9,22,23].

Conclusion. The stress-strain diagram "$\sigma_c - \varepsilon_c$" used in the calculations on the descending branch must be limited to the starting point of the macro-destructuring of concrete. The value of the ultimate strain ε_{cdR} corresponds to the maximum potential deformation energy $\sigma_c \varepsilon_{cdR} = \max(\sigma_c \varepsilon_c)$.

When evaluating the strength of reinforced concrete elements in normal sections, the condition of not exceeding by the level of strain the most compressed fiber of concrete at the stage of failure of the level of ultimate strain $\eta_u \le \eta_R$ should be checked. When strain exceeds the ultimate value ($\eta_u > \eta_R$), a fragile avalanche-like failure occurs along the concrete of the compressed zone.

The strength problem of a compressed inclined element (strut) in the support sections as a component of the truss analogy is solved by a variational method in the plasticity theory of concrete. As a criterion for determining the ultimate force value perceived by the concrete strut under shear, the minimum of the plastic strain capacity is used that is localized in a thin layer on the failure surface. When designing reinforced concrete elements, it is recommended to take the inclination angle of the inclined element corresponding to the most efficient use of the resistances of concrete and shear reinforcement.

For a more accurate mapping in the calculations of the behavior of constructive concrete and reinforced concrete at the failure stage and improvement of the method of estimating their strength, the future development of the deformation model, the use of different methods with the specification of the areas of their implementation and the application of extreme principles of mechanics of deformable solid body are perspective.

О. А. Довженко[1], В. В. Погребной[1], Е. В. Клименко[2], М. Орешкович[3]

[1]«Юрий Кондратюк атындағы Полтава политехникасы» Ұлттық университеті, Полтава, Украина;
[2]Одесса мемлекеттік құрылыс және сәулет академиясы, Одесса, Украина;
[3]Солтүстік университеті, Вараждин, Хорватия

КОНСТРУКЦИЯЛЫҚ ЖӘНЕ ТЕМІР БЕТОННЫҢ БЕРІКТІГІН БАҒАЛАУДА ДЕФОРМАЦИЯНЫҢ ЭКСТРЕМАЛДЫ ҚАСИЕТТЕРІН ПАЙДАЛАНУ

О. А. Довженко[1], В. В. Погребной[1], Е. В. Клименко[2], М. Орешкович[3]

[1]Национальный университет «Полтавская политехника имени Юрия Кондратюка», Полтава, Украина;
[2]Одесская государственная академия строительства и архитектуры, Одесса, Украина;
[3]Университет Север, Вараждин, Хорватия

ИСПОЛЬЗОВАНИЕ ЭКСТРЕМАЛЬНЫХ СВОЙСТВ ДЕФОРМАЦИИ ДЛЯ ОЦЕНИВАНИЯ ПРОЧНОСТИ КОНСТРУКТИВНОГО БЕТОНА И ЖЕЛЕЗОБЕТОНА

Аннотация. Для оценивания сопротивления конструктивного бетона и железобетона разрушению в качестве критерия достижения предельного состояния используются экстремальные свойств энергии (мощности) деформирования сжатого бетона.

Рассматриваются нормальные и наклонные к продольной оси изгибаемых элементов сечения в наиболее напряженных зонах.

Обосновано ограничение применяемой в расчетах диаграммы «напряжения – деформация» бетона уровнем начала макродеструктуризации, что позволяет исключить переоценку прочности. Диаграмма описывается дробно-рациональной функцией и гармонизированным полиномом 5-ой степени. Для уточнения коэффициентов полинома рассматриваются характерные параметры диаграммы: точка в ее вершине, соответствующая максимуму напряжений, точка предельного уровня деформации, до которой бетон как материал еще способен воспринимает возрастающую нагрузку, а также площадь диаграммы, ограниченной предельной деформацией. Используются начальный модуль деформации бетона и секущие модули в вершине диаграммы и в точке, которая ее ограничивает на нисходящем участке. Приведены коэффициенты полинома для бетонов разной прочности.

Решена задача прочности балочного железобетонного элемента в нормальном сечении. Предельное значение изгибающего момента определяется из условий максимальной величины нагрузки и не превышения деформацией уровня, с достижением которого начинается макродеструктуризация. Значение предельной деформации на нисходящей ветви диаграммы установлено в точке, соответствующей максимуму энергии деформирования.

Запредельная деформация сопровождается резким снижением напряжений и нарушением структуры материала. Приведена зависимость величины предельной деформации и параметров расчетного нормального сечения от упруго-пластической характеристики бетона. Получены значения уровня предельной деформации для бетонов разных классов.

Определена прочность сжатого наклонного элемента возле опор как составляющего ферменной аналогии. Рассмотрено разрушение путем среза моделирующей сжатый элемент бетонной призмы, нагруженной по торцам нормальной сжимающей и касательной силами.

Результат получен на основе вариационного метода в теории пластичности. В качестве пластического потенциала принято условие прочности. Используется математический аппарат теории пластичности с учетом дилатансии бетона, определяющий зависимости напряжений от скоростей деформаций и функционал принципа виртуальных скоростей.

Бетон рассматривается как жестко-пластическое тело. Интенсивная деформация считается локализованной в тонком слое на поверхности разрушения. Примыкающие к поверхности разрушения области принимаются абсолютно жесткими. Для определения величины предельной нагрузки применяется верхняя оценка.

Функционал метода виртуальных скоростей исследуется на стационарное состояние. Варьируются угол наклона поверхности разрушения и соотношение скоростей в ортогональных направлениях, через которые выражены нормальные и касательные к поверхности среза скачки скоростей. В качестве критерия использован минимум мощности пластической деформации.

Предложена подтвержденная экспериментальными исследованиями кинематическая схема разрушения сжатого бетонного элемента и зависимость для оценки его прочности. Приведены результаты решения при разных углах наклона призмы к продольной оси элемента. Уточнена область реализации метода ферменной аналогии.

При проектировании железобетонных элементов рекомендуется принимать угол наклона наклонной полосы, соответствующий наиболее эффективному использованию сопротивлений бетона и поперечной арматуры.

Указано на необходимость и перспективность дальнейшего развития деформационной модели, использования и усовершенствования методов оценивания прочности с уточнением областей их реализации и применение экстремальных принципов механики твердого деформированного тела для более точного и обоснованного отображения в расчетах поведения конструктивного бетона и железобетона на стадии разрушения.

Ключевые слов: бетон, железобетон, экстремальные свойства деформации, предельная деформация, срез, ферменная аналогия.

Information about authors:

Dovzhenko Oksana Oleksandrivna, Professor of the Department of Reinforced Concrete and Stone Structures and Resistance of Materials, National University “Yuri Kondratyuk Poltava Polytechnic”, Candidate of Technical Sciences, Associate Professor, Poltava, Ukraine; o.o.dovzhenko@gmail.com; https://orcid.org/0000-0002-2266-2588

Pohribnyi Volodymyr Volodymyrovych, Associate Professor of the Department of Reinforced Concrete and Stone Structures and Resistance of Materials, National University “Yuri Kondratyuk Poltava Polytechnic, Candidate of Technical Sciences, Senior Researcher, Poltava, Ukraine; v.v.pogrebnoy1960@gmail.com; https://orcid.org/0000-0001-7531-2912

Klymenko Yevgenii Volodymyrovych, head of the Department of Reinforced Concrete Structures and Transport Facilities of the Odessa State Academy of Civil Engineering and Architecture, Doctor of Technical Sciences, Professor, Odessa, Ukraine; klimenkoew57@gmail.com; https://orcid.org/0000-0002-4502-8504

Matija Orešković, Associate Professor, University North, PhD, Associate Professor, Varaždin, Croatia; moreskovic@unin.hr; https://orcid.org/0000-0001-5684-0496

REFERENCES

[1] Kaminski K., Wydra M. (2015) Calculation of Reinforced Concrete Beam Cross Sections According to General Method Included in EC2, Applied Mechanics and Materials, 797: 88-95 (in Eng.).

[2] Karpenko N., Sokolov B., Radaykin O. (2013) Industrial and Civil Construction [Promyshlennoye i grazhdanskoye stroitelstvo] 1: 28-30 (in Russ.).

[3] Bambura A.N., Gurkovsky A.B. (2003) Building structures [Budivel`ni konstrukciyi] 59: 121-130 (in Russ.).

[4] Pavlikov A.M., Boiko O.V. (2012) Calculation of the strength of the normal cross sections of the beam elements by a nonlinear deformation model [Rozraxunok micznosti normal`ny`x pereriziv balkovy`x elementiv za nelinijnoyu deformacijnoyu modellyu]. Poltava. Ukraine (in Ukr.).

[5] EN 1992-1-1. Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings. European Committee for Standardisation, 2004 (in Eng.).

[6] SBC V.2.6-98:2009. Concrete and reinforced concrete structures. Basic design principles [DBN V.2.6-98:2009. Betonni ta zalizobetonni konstrukciyi. Osnovni polozhennya proektuvannya]. Kiev, Ukraine, 2011 (in Ukr.).

[7] Bezgodov E., Shvedov V. (2019) Comparison of complete stress-strain curves of concrete using test prisms and test cylinders, E3S Web of Conferences, 97. DOI: 10.1051/e3sconf/20199702013 (in Eng.).

[8] Braz D.H, Barros R., Da Silva Filho J.N. (2019) Comparative analysis among standards of the area calculation of transversal reinforcement on reinforced concrete beams of high resistance subjected by shear force, Rev. IBRACON Estrut. Mater., 12, 1. DOI: 10.1590/s1983-41952019000100011 (in Eng.).

[9] Snezhkina O.V. (2019) Engineering method for assessing the strength of reinforced concrete beams, IOP Conf. Ser.: Mater. Sci. Eng. DOI: 10.1088/1757-899X/537/2/022050 (in Eng.).

[10] Wei W., Gong J. (2011) Shear strength prediction of reinforced concrete flexural members with stirrups based on modified compression field theory, Journal of Building, 32: 135-141. (in Eng.).

[11] Grandić D., Šculac P., Štimac Grandić I. (2015) Shear resistance of reinforced concrete beams in dependence on concrete strength in compressive struts // *Tehnicki Vjesnik*, 22 (4): 925-934. DOI: 10.17559/TV-20140708125658 (in Eng.).

[12] Latha M.S., Revanasiddappa M., Naveen Kumar B.M. (2018) Influence of stirrup spacing on shear, resistance and deformation of reinforced concrete beam // *International Journal of Engineering & Technology*, 7 (1): 126-134, DOI: 10.14419/ijet.v7i1.9013 (in Eng.).

[13] Iskakbayev A.I., Teltayev B.B., C. Oliviero Rossi, Estayev K. (2018) A new simple damage accumulation model for predicting of an asphalt concrete cyclic strength // *News of national academy of sciences of Republic of Kazakhstan. Series of geology and technical science,* 5 (431): 38-47. DOI: 10.32014/2018.2518-170X.8 (in Eng.).

[14] Solonenko V.G., Makhmetova N.M., Musayev J.S., Bekzhanova S.E., Kvashnin M.Ya. (2019) Stresses in elements of metal railway bridges under the action of the crew // *News of national academy of sciences of Republic of Kazakhstan. Series of geology and technical science,* 2 (434): 159-165. DOI: 10.32014/2019.2518-170X.50 (in Eng.).

[15] Geniyev G.A., Kissyuk V.N., Tyupin G.A. (1974) Concrete and reinforced concrete plasticity theory [Teoriya plastichnosti betona i zhelezobetona]. M., Russia (in Russ.).

[16] Pavlikov A.M., Yurko P.A. (2010) Academic journal. Series: Industrial Machine Building, Civil Engineering [Zbirnyk naukovykh prats. Seriia: Haluzeve mashynobuduvannia, budivnytstvo] 2 (27): 23-26 (in Ukr.).

[17] Braestrup Mikael W. (2019) Concrete plasticity – a historical perspective. Proceedings of the *fib* Symposium: Concrete – Innovations in Materials, Design and Structures, Krakov, Poland. P. 29-48 (in Eng.).

[18] Mitrofanov V.P. (2006) The theory of perfect plasticity as the elementary mechanic pseudo-plastic ultimate state of concrete: bases, imitations, practical aspects, Proceedings *of the 2nd fib Congress*, Naples, Italy. P. 7-6 (in Eng.).

[19] Dovzhenko O., Pogrebnyi V., Yurko I. (2018) Shear Failure Form Realization in concrete // *News of national academy of sciences of Republic of Kazakhstan. Series of geology and technical science,* 2 (428): 212-219. (in Eng.).

[20] Pohribnyi V., Dovzhenko O., Karabash L., Usenko I. (2017) The design of concrete elements strength under local compression based on the variational method in the plasticity theory, *Web of Conferences*, 116. DOI: 10.1051/matecconf/201711602026 (in Eng.).

[21] Dovzhenko O., Pogrebnyi V., Yurko I., Shostak I. (2017) The bearing capacity experimental determination of the keyed joints models in the transport construction, *Web of conferences*, 116. DOI: 10.1051/matecconf/201711602011 (in Eng.).

[22] Gűray A. (2007) Shear strength of reinforced concrete beams with stirrups, Materials And Structures, 41 (1): 113-122. DOI: 10.1617/s11527-007-9223-3 (in Eng.).

[23] Yafei Ma, Baoyong Lu, Zhongzhao Guo, Lei Wang, Hailong Chen, and Jianren Zhang (2019) Limit equilibrium method-based shear strength prediction for corroded reinforced concrete beam with inclined bars, Materials (Basel), 12 (7). DOI: 10.3390/ma12071014 (in Eng.).