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MODELING OF HEREDITARY MATERIALS RELAXATION BY ABEL KERNEL

Abstract. The present work is devoted to mathematical modeling of the process of the hereditary materials relaxation. Nonlinear integral equation of a hereditary type is proposed. The Abel kernel with two unknown parameters is adopted as the kernel of the integral equation: $\alpha \in (0,1), \delta > 0$. Two new characteristics were introduced: 1) experimental rheological parameter of relaxation; 2) calculated (model) rheological parameter of relaxation. Using the least squares method, expressions are obtained to determine unknown parameters of the Abel kernel. A mathematical expression is given to approximate the process of the hereditary materials relaxation.

Using examples of rheonomic materials different in structure (polyurethane matrix, propellant, polyoxymethylene, fiberglass), it is shown that the proposed methods allow to determine Abel kernel parameters with a high accuracy and to model the process of relaxation of rheonomic materials different in structure during a long period of time: from 10^2 to $1.8 \cdot 10^6$ seconds (500 hours).

Key words: Abel kernel, relaxation, polyurethane matrix, propellant, polyoxymethylene, fiberglass.

1. Introduction. Almost all natural and artificial materials are viscoelastic ones: they deform in time under the action of load. And the reaction of many materials to external actions depends on the history of loading, i.e. these materials are hereditary [1-4]. Creep and relaxation are important characteristics of hereditary materials. The creep has been studied for an asphalt concrete and other materials in the works [5-13]. The present work is devoted to the modeling of the process for relaxation of a number of hereditary materials.

2. Methods and materials. The methods of the theory of viscoelasticity have been used for a mathematical description of the process for the rheonomic materials relaxation: the Boltzmann-Volterra integral equation with the Abel kernel; the expressions based on the least squares method are obtained to determine the parameters of the Abel kernel.

A new method for description of the process of the rheonomic materials relaxation is proposed with the introduction of new characteristics - the so-called experimental and calculated (model) rheological parameters of relaxation.

For the mathematical description of the relaxation process, the following materials were selected by the proposed method: 1) a polyurethane matrix containing salt crystals and aluminum powder (polyurethane matrix) [1]; 2) solid fuel of rocket engines (propellant) [14]; 3) polyoxymethylene [15] and 4) fiberglass TC 8/3-250 (fiberglass) [16].

3. Theoretical bases

3.1. Integral equation and relaxation kernel. The equation for the nonlinear hereditary type of a rheonomic material we will write in the following integral form

$$\sigma(t,T) = f[\varepsilon(t),T] - \int_0^t R(t-\tau) f[\varepsilon(\tau),T] d\tau, \qquad (1)$$

where T is a temperature, $T = const$; $f[\varepsilon(t),T]$ is a nonlinear function of time at $T = const$; $\varepsilon(t), \varepsilon(\tau)$ is a deformation at time moments t and τ; $R(t-\tau)$ is a relaxation kernel.

At $\varepsilon(t) = const$ from the equation (1) we have:

$$\sigma(t,T) = f(\varepsilon,T) \cdot \left[1 - \int_0^t R(t-\tau)\, d\tau\right], \qquad (2)$$

where $f(\varepsilon,T) = \sigma_0(\varepsilon,T)$ is an instantaneous deformation curve.

We take relaxation kernel in the form of the following functions:

$$R(t-\tau) = \delta(t-\tau)^{-\alpha}, \alpha \in (0,1);\ \delta > 0. \qquad (3)$$

where α, δ are parameters of the relaxation kernel.

Inserting the function (3) into the equation (2) we will have:

$$\sigma(t,T) = \sigma_0(\varepsilon,T) \cdot \left[1 - \frac{\delta}{1-\alpha} t^{(1-\alpha)}\right]. \qquad (4)$$

In the expression (4): $\alpha = \alpha(T), \delta = \delta(T)$.

3.2. Relaxation kernel parameters. Relaxation equation (4) includes three unknown parameters: $\sigma_0(\varepsilon,T)$, α and δ. It contains a function with the parameters α and δ:

$$L(t) = \frac{\delta}{1-\alpha} t^{(1-\alpha)} \qquad (5)$$

The singular parameter takes a value in the interval (0, 1). The works [5-8] suggest the parameter α to consider as the specified one, and the remaining two other parameters to determine using the least squares method. In the work [9] the above method has been improved: the determining ratios have been changed by introducing the so-called experimental rheological parameter.

In accordance with the work [9] we introduce the parameter determined by the formula:

$$r_e(t,T) = \frac{\sigma_e(t,T)}{\sigma_0^e(\varepsilon,T)}, \qquad (6)$$

where $\sigma_e(t,T)$ is a stress value at a time moment t at a temperature T determined experimentally; $\sigma_0^e(\varepsilon,T)$ is a stress value at a time moment t_0 at a temperature T determined also experimentally.

We shall call $r_e(t,T)$ as an experimental rheological relaxation parameter.

Similar to the parameter $r_e(t,T)$ we shall introduce the calculated (model) rheological relaxation parameter $r_m(t,T)$ determined under the formula:

$$r_m(t,T) = \frac{\sigma_m(t,T)}{\sigma_0^m(\varepsilon,T)}, \qquad (7)$$

where $\sigma_m(t,T)$ is a stress value at a time moment t at a temperature T, determined by the calculations; $\sigma_0^m(\varepsilon,T)$ is a stress value at a time moment t_0 at a temperature T, determined also by the calculations.

According to the least squares method the parameters values $\sigma_0^m(\varepsilon,T)$ and $\delta = \delta(T)$ should satisfy the following condition:

$$S[\sigma_0^m(\varepsilon,T), \delta(T)] = \sum_{i=1}^{M} \left[\sigma_0^m(\varepsilon,T) \cdot \left(1 - \frac{\delta}{1-\alpha} t_i^{(1-\alpha)} - \sigma_{ei}\right)^2\right] \to \min, \qquad (8)$$

where $S[\sigma_0^m(\varepsilon,T), \delta(T)]$ is a sum of squared deviations; σ_{ei} is a stress value at a time moment t_i determined experimentally;

M total number of the considered time moments.

From the following partial derivatives $\frac{\partial S[\sigma_0^m(\varepsilon,T),\delta(T)]}{\partial\sigma_0^m(\varepsilon,T)} = 0$ and $\frac{\partial S[\sigma_0^m(\varepsilon,T),\delta(T)]}{\partial\delta(T)} = 0$ we will obtain the expressions for the determination of the parameters $\sigma_0^m(\varepsilon, T)$ and $\delta(T)$:

$$1 = \frac{\sum_{i=1}^{M} r_{ei} \sum_{i=1}^{M} t_i^{2(1-\alpha)} - \sum_{i=1}^{M} t_i^{(1-\alpha)} \sum_{i=1}^{M} r_{ei} t_i^{(1-\alpha)}}{M \sum_{i=1}^{M} t_i^{2(1-\alpha)} - \left[\sum_{i=1}^{M} t_i^{(1-\alpha)}\right]^2}, \tag{9}$$

$$\delta(T) = \frac{\sum_{i=1}^{M} (1 - r_{ei}) t_i^{(1-\alpha)}}{\frac{1}{1-\alpha} \sum_{i=1}^{M} t_i^{2(1-\alpha)}}, \tag{10}$$

where $r_{ei} = r_e(t = t_i, T)$.

In the expression (9) it is accepted that

$$\sigma_0^m(\varepsilon, T) \approx \sigma_0^e(\varepsilon, T). \tag{11}$$

3.3. Approximating expression. After determination of the parameters α and δ the so-called coefficients of the similarity are calculated under the expression:

$$r_m(t_S, T) = 1 - \frac{\delta}{1-\alpha} t_S^{(1-\alpha)}, \tag{12}$$

where $t_S \in [0, t_M]$.

Taking into account (12) we will determine the value $\overline{\sigma_0}(\varepsilon, T)$:

$$\overline{\sigma}_0(\varepsilon, T) = \frac{1}{M} \sum_{S=1}^{M} \frac{\sigma_e(t_S, T)}{r_m(t_S, T)}.$$

Approximating analytical expression for the process of the rheonomic materials relaxation has the following form:

$$\sigma_m(t, T) = \overline{\sigma_0}(\varepsilon, T) \cdot \left(1 - \frac{\delta}{1-\alpha} t^{(1-\alpha)}\right). \tag{13}$$

4. Results and discussion. The relaxation function of the polyurethane matrix is determined by the pure shear experiment under dynamic loading conditions at the temperature of 26.1 °C [1]. The work [14] also determines the function of the propellant relaxation at pure shear under conditions of dynamic loading. The functions of polyoxymethylene relaxation at tension at the temperature of 20 °C are determined at two values of conditionally instantaneous deformation: 2% and 10% [15]. The relaxation functions of fiberglass cut at an angle of 45% to the fabric base at tension at the temperature of 23.5 °C are determined at three values of conditionally instantaneous deformation: 0.31%, 0.7% and 1.55% [16].

Experimental and calculated values of relaxation stresses of the materials under consideration are presented in figures 1-4. As can be seen, the degree of coincidence between the calculated and experimental stress values is high.

Thus, the obtained results have shown that the suggested methods make it possible to determine Abel kernel parameters with high accuracy and simulate the process of relaxation of rheonomic materials different in structure during a long process of time (from 100 seconds to 500 hours).

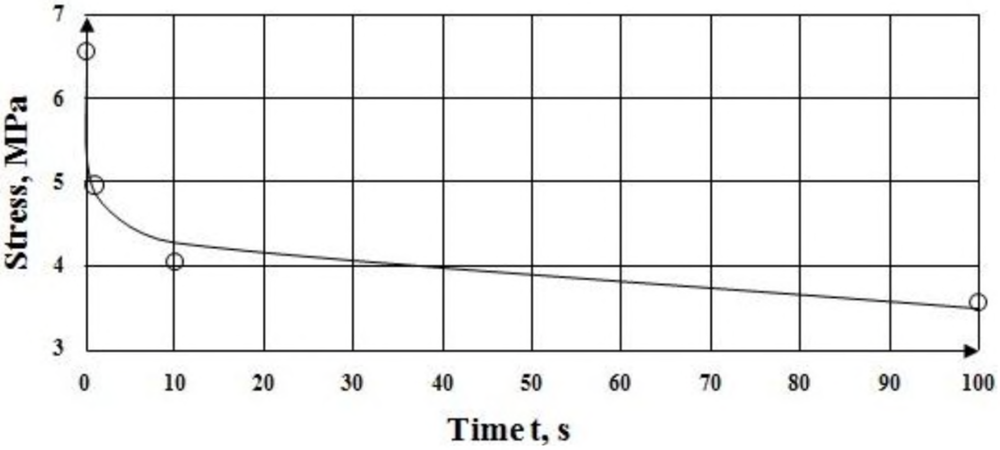

Figure 1 – Polyurethane matrix relaxation curve at the temperature of 26.1 ℃: ○ – experiment, ▬ – calculation

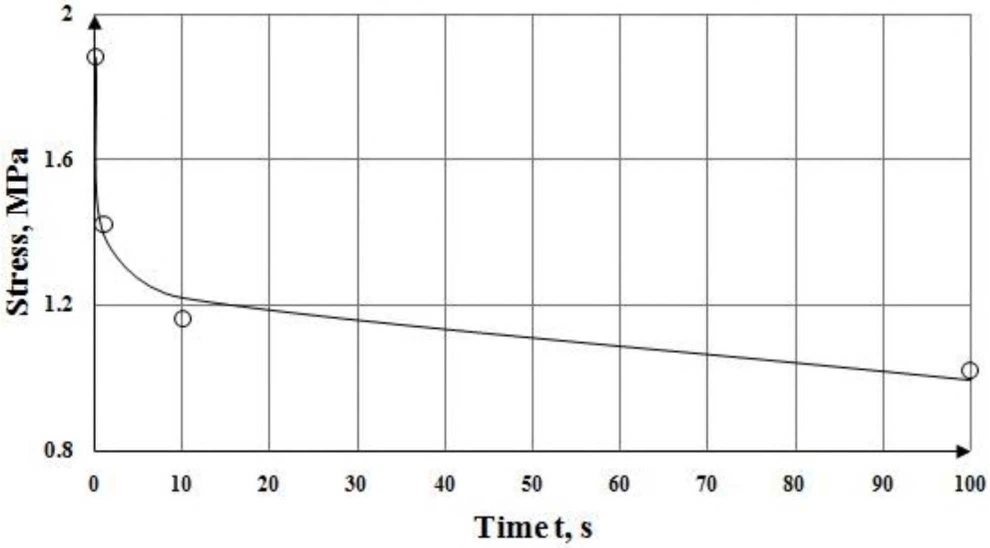

Figure 2 – Propellant relaxation curve: ○ – experiment, ▬ – calculation

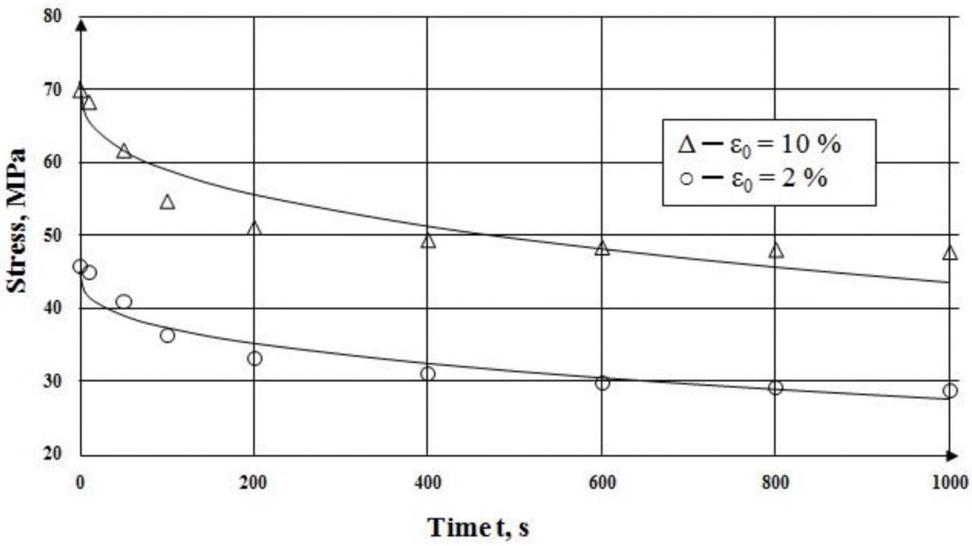

Figure 3 – Polyoxymethylene relaxation curve at the temperature of 20 ℃: ○ – experiment, ▬ – calculation

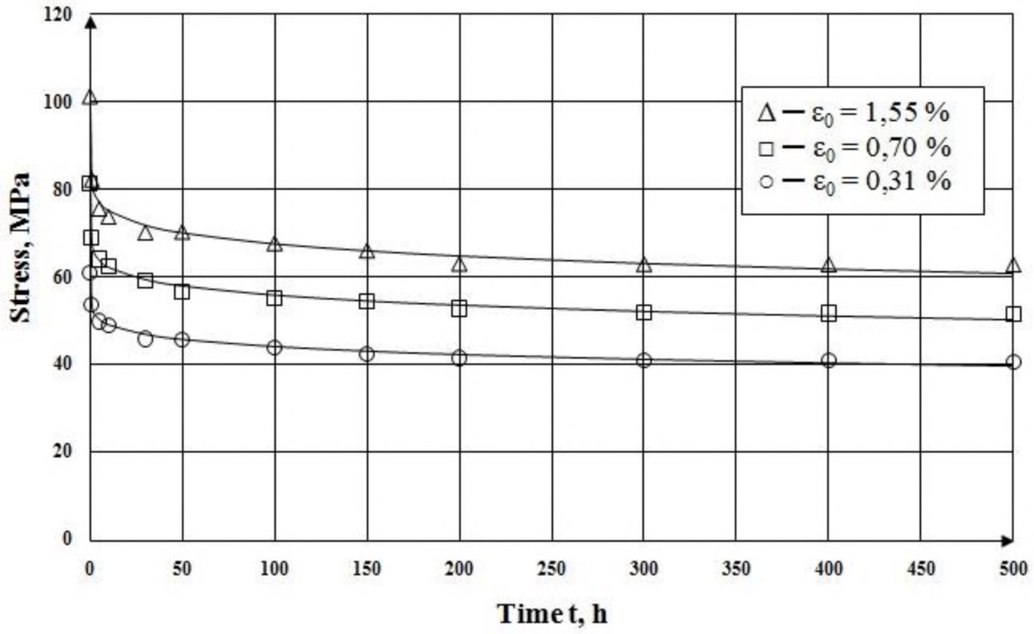

Figure 4 – Fiberglass relaxation curve at the temperature of 23.5 °C: ○ – experiment, ▬ – calculation

Conclusion. 1. The method has been proposed for the mathematical description of the process for the rheonomic materials relaxation with introduction of the new parameters: experimental and calculated (model) rheological parameters of relaxation.

2. The Abel kernel has been accepted in the integral equation in the mathematical description of the process of the rheonomic materials relaxation.

3. The expressions for determining parameters α and $\delta$of the Abel kernel have been obtained based on the least squares method.

4. Using the examples of the rheonomic materials different in structure (polyurethane matrix, propellant, polyoxymethylene, fiberglass), it is shown that the proposed methods allow determining Abel kernel parameters with high accuracy and modeling the process of the rheonomic materials relaxation different in structure during a long period of time: from 10^2 to $1.8 \cdot 10^2$ seconds (500 hours).

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АБЕЛЬ ЯДРОСЫ АРҚЫЛЫ МҰРАЛЫҚ МАТЕРИАЛДАР РЕЛАКСАЦИЯСЫН МОДЕЛЬДЕУ

Аннотация. Жұмыс мұралық материалдардың релаксация үдерісін математикалық модельдеуге арналған. Мұралық типтегі сызықтық емес интегралдық теңдеу ұсынылады. Интегралдық теңдеудің ядросы ретінде екі белгісіз: $\alpha \in (0,1), \delta > 0$ параметрлері бар Абель ядросы қабылданды. Келесідей екі жаңа сипаттама енгізілді: 1) релаксацияның эксперименттік реологиялық параметрі; 2) релаксацияның есептік (модельдік) реологиялық параметрі. Ең кіші квадраттар әдісі негізінде релаксация теңдеуінің белгісіз параметрлерін анықтауға арналған өрнектер алынды. Мұралық материалдардың релаксация үдерісін сипаттауға арналған математикалық өрнек берілді.

Құрылымы әртүрлі реономды материалдар (полиуретанды матрица, зымыран жанармайы, полиоксиметилен, шыны талшық) негізінде ұсынылған әдістер Абель ядросының параметрлерін жоғары дәлдікпен анықтауға және құрылымы әртүрлі реономды материалдардың релаксация үдерісін ұзақ уақыт бойы модельдеуге мүмкіндік беретіні көрсетілген: 10^2 секундтан $1{,}8 \cdot 10^6$ секундқа дейін (500 сағат).

Түйін сөздер: Абель ядросы, релаксация, полиуретанды матрица, зымыран жанармайы, полиоксиметилен, шыны пластик.

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МОДЕЛИРОВАНИЕ РЕЛАКСАЦИИ НАСЛЕДСТВЕННЫХ МАТЕРИАЛОВ ЯДРОМ АБЕЛЯ

Аннотация. Настоящая работа посвящена математическому моделированию процесса релаксации наследственных материалов. Предложено нелинейное интегральное уравнение наследственного типа. В качестве ядра интегрального уравнения принято ядро Абеля с двумя неизвестными параметрами: $\alpha \in (0,1), \delta > 0$. Введены две новые характеристики: 1) экспериментальный реологический параметр релаксации; 2) расчетный (модельный) реологический параметр релаксации. Используя метод наименьших квадратов, получены выражения для определения неизвестных параметров ядра Абеля. Дано математическое выражение для аппроксимации процесса релаксации наследственных материалов.

На примерах разных по структуре реономных материалов (полиуретановая матрица, ракетное топливо, полиоксиметилен, стеклопластик) показано, что предложенные методы позволяют с высокой точностью определять параметры ядра Абеля и моделировать процесс релаксации разных по структуре реономных материалов в течение длительного периода времени: от 10^2 до $1{,}8 \cdot 10^6$ секунд (500 часов).

Ключевые слова: ядро Абеля, релаксация, полиуретановая матрица, ракетное топливо, полиоксиметилен, стеклопластик.

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