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SOLVABILITY OF BOUNDARY VALUE PROBLEMS WITH NON-LOCAL CONDITIONS FOR MULTIDIMENSIONAL HYPERBOLIC EQUATIONS

Abstract. In this paper, we study the solvability of new nonlocal boundary value problems for hyperbolic equations in a multidimensional bounded domain. For the problem under study, the existence and uniqueness theorems of regular solutions are proved.

Keywords: hyperbolic equation, boundary value problems, regular solutions, existence, uniqueness.

1. Introduction and statement of the problem. It is well known that studies of the properties of correcting boundary value problems for differential equations are important both for applications and for mathematical and mechanical problems. The theory of boundary value problems for high-order elliptic equations attracts particular attention of mathematicians.

The aim of the work is to study the solvability in classes of regular solutions of a new boundary value problem for the equation

$$u_{tt} - \Delta u + c(x,t)u = f(x,t), (x,t) \in Q. \tag{1}$$

This equation is an ordinary second-order hyperbolic equation, and the solvability of natural boundary value and initial-boundary value problems for it is well studied [24-26].

The study of the solvability of boundary value problems for quasi-hyperbolic equations (by analogy with quasi-elliptic equations) apparently began with the works of V.N. Vragova [1,2]. A study of the solvability of problems for high orders of equations in Sobolev spaces was carried out in [1-3], a number of similar results were obtained in [4-11].

Investigations of nonlocal problems with integral conditions for linear parabolic equations, for differential equations of odd orders, and for some classes of non-stationary equations, have recently been actively studied in the works of A.I. Kozhanova [7,9,10]. In [1–6], the main attention was paid to situations related to degenerate equations of the form (1).

In [12], a criterion was obtained for the strong solvability of the mixed Cauchy problem for the Laplace equation.

It is known that the Dirichlet problem for polyharmonic equations is uniquely solvable for any right-hand side of the equation. In [13–16], a new representation of the Green function of the Dirichlet problem for the polyharmonic equation in a multidimensional ball is constructed explicitly. In [17,18], a representation of the Green function of the Neumann problem for the Poisson equation in a multidimensional unit ball was obtained.

The problems of finding solvability conditions for boundary value problems for a polyharmonic equation in a ball were investigated in [19]. In [20], for derivatives of the $2l$-th order equation with constant (and only higher) real coefficients, normal derivatives were studied under boundary conditions. For these problems, sufficient conditions for the Fredholm solvability of the problem are obtained and formulas for the index of the problem are given.

In [21-23], the behavior of solutions of the Dirichlet problem for the Poisson equations and the biharmonic equation in an unbounded domain was studied.

In this paper, we obtain a theorem on the existence and uniqueness of a regular solution of a nonlocal in time boundary value problem for hyperbolic equations in a multidimensional bounded domain by the method of a priori estimation and passage to the limit.

Let Ω - be a bounded region of space R^n with a smooth boundary Γ, Q – a cylinder $\Omega\times(0,T), 0<t<T<+\infty$, $S=\Gamma\times(0,T)$ - a side boundary Q, $f(x,t)$ – given functions defined for $x\in\overline{\Omega}$, $t\in[0,T]$

For equation (1) we study the time-nonlocal boundary value problem

$$u|_S = 0, \tag{2}$$

$$u(x,0) = \alpha u(x,T), \tag{3}$$

$$u_t(x,0) = \beta u_t(x,T), \alpha,\beta \in R. \tag{4}$$

In studies of this kind of nonlocal problems, the parameter continuation method, a priori estimation method, and passage to the limit are usually used. For a hyperbolic equation, the parameter continuation method is not applicable, since the smoothness of the right side of the equation will be lost.

We define a functional space in which the properties of uniqueness and existence of a solution to the boundary value problem (1) - (4) will be studied. Namely, we define space $W_2^{2,2}(Q)$ as the set of functions from space $L_2(Q)$, that have generalized derivatives of spatial variables up to the second order inclusive and with respect to variable t up to the second order inclusive, belonging to the same space. We define the norm in space $W_2^{2,2}(Q)$.

$$\|\upsilon\|_{W_2^{2,2}(Q)} = \left(\int_Q\left[\upsilon^2 + \sum_{i,j=1}^{n}\upsilon^2_{x_ix_j} + (\upsilon_{tt})^2\right]dxdt\right)^{1/2}$$

obviously, space $W_2^{2,2}(Q)$ with this norm will be a Banach space.

2. The regularized nonlocal boundary value problem and the main result. We consider the following regularized problem in order to apply the continuation method with respect to the parameter

$$u_{tt} - \Delta u + c(x,t)u - \varepsilon\Delta u_t = f(x,t) \in L_2(Q) \tag{5}$$

$$u|_S = 0 \tag{6}$$

$$u(x,0) = \lambda\alpha u(x,T), \tag{7}$$

$$u_t(x,0) = \lambda\beta u_t(x,T). \tag{8}$$

For the regularized problem (5) - (8), we obtain estimates in the corresponding spaces. To this end, we multiply equation (5) by u_t integrate over Q, then we obtain.

$$\int_Q u_{tt}u_t dxdt - \int_Q \Delta u u_t dxdt + \int_Q cuu_t\, dxdt - \varepsilon\int_Q \Delta u_t\, u_t dxdt = \int_Q fu_t dxdt;$$

$$I_1 = \frac{1}{2}\int_Q \frac{\partial}{\partial t}(u_t^2)dxdt = \frac{1}{2}\int_\Omega [u_t^2(0,T) - u_t^2(x,0)]dx = \frac{(1-\lambda^2\beta^2)}{2}\int_\Omega u_t^2(x,T)dx;$$

Here we take into account the boundary condition (7) and $|\beta| < 1$, such that $\frac{1-\lambda^2\beta^2}{2} > 0$.

$$I_2=-\int_Q \Delta u u_t=\sum_{i=1}^{n}\int_Q u_{x_i}u_{x_it}dxdt=\frac{1}{2}\sum_{i=1}^{n}\int_Q \frac{\partial}{\partial t}(u_{x_i}^2)dxdt=$$
$$=\frac{1}{2}\sum_{i=1}^{n}\int_\Omega [u_{x_i}^2(x,T)-u_{x_i}^2(x,0)]dx=\frac{1-\lambda^2\alpha^2}{2}\sum_{i=1}^{n}\int_Q u_{x_i}^2(x,T)dx;$$
$$I_3=\int_Q cuu_tdxdt=\frac{1}{2}\int_Q \frac{\partial}{\partial t}(cu^2)dxdt-\frac{1}{2}\int_Q c_tu^2dx\,dt=$$
$$=\frac{1}{2}\int_\Omega [c(x,T)u^2(x,T)-c(x,0)u^2(x,0)]dx-\frac{1}{2}\int_Q c_tu^2dx\,dt=$$
$$=\frac{1}{2}\int_\Omega [c(x,T)-\lambda^2\alpha^2c(x,0)]u^2(x,T)dx-\frac{1}{2}\int_Q c_tu^2dx\,dt;$$
$$I_4=-\varepsilon\int_Q \Delta u_t\, u_tdx=-\varepsilon\sum_{i=1}^{n}\int_Q u_{x_ix_it}\,u_t\,dxdt=\varepsilon\sum_{i=1}^{n}\int_Q {u_{x_it}}^2dxdt;$$

$$\frac{1-\lambda^2\beta^2}{2}\int_\Omega u_t^2(x,T)dx+\frac{1-\lambda^2\alpha^2}{2}\sum_{i=1}^{n}\int_\Omega u_{x_i}^2(x,T)dx+$$
$$+\frac{1}{2}\int_\Omega [c(x,T)-\lambda^2\alpha^2c(x,0)]\,u^2(x,T)dx+\frac{1}{2}\int_Q \Delta u_tu_tdx\,dt+\varepsilon\sum_{i=1}^{n}\int_Q u_{x_it}^2\,dxdt=\int_Q fu_tdxdt;$$

We will require that the conditions are met

$$c(x,T)-\lambda^2\alpha^2c(x,0)\geq 0\ \ ,\ c_t(x,T)\leq 0\,.$$

Under the condition $\vartheta|_S=0$ the embedding theorem holds, i.e. Friedrichs inequality [20]:

$$\int_\Omega \vartheta(x,T)dx\leq m_0\sum_{i=1}^{n}\int_\Omega {\vartheta_{x_i}}^2(x,T)dx.\ (F)$$

We apply it to the function $\vartheta=u_t$.

$$I_f=-\int_Q fu_tdxdt\leq\frac{\delta^2}{2}\int_Q u_t^2dx\,dt+\frac{1}{2\delta^2}\int_Q f^2dxdt\leq\frac{\delta^2m_0}{2}\sum_{i=1}^{n}\int_Q u_{x_it}^2dx\,dt+\frac{1}{2\delta^2}\int_Q f^2dxdt.$$

We choose $\frac{\delta^2m_0}{2}=\frac{\varepsilon}{2}$.

Then we have

$$\frac{1-\beta^2}{2}\int_\Omega u_t^2(x,T)dx+\frac{1-\alpha^2}{2}\sum_{i=1}^{n}\int_\Omega {u_{x_i}}^2(x,T)dx+\varepsilon\sum_{i=1}^{n}\int_Q {u_{x_it}}^2dxdt\leq\frac{1}{2\varepsilon}\int_Q f^2dxdt; \qquad (9)$$

At the beginning, we must obtain a uniform estimate for λ for a fixed ε. Equation (5) is multiplied by u_{tt} and integrated over Q, then we obtain

$$-\int_Q u_{tt}\Delta u_tdxdt+\int_Q \Delta u\,\Delta u_tdxdt-\int_Q c\,u\,\Delta u_tdxdt+\varepsilon\int_Q {\Delta u_t}^2dxdt=-\int_Q f\,\Delta u_tdxdt.$$

Then we have

$$\frac{1-\lambda^2\beta^2}{2}\sum_{i=1}^{n}{u_{x_it}}^2(x,T)dx+\frac{1-\lambda^2\alpha^2}{2}\int_\Omega [\Delta u\quad(x,T)]^2dx+\varepsilon\int_Q (\Delta u_t)^2dxdt=$$

$$= -\int_Q f\Delta u_t dxdt + \int_Q cu\Delta u_t dxdt\,;$$

Estimating the right-hand side and taking into account $C_0 = \max_Q |c|$ we have

$$\frac{1-\lambda^2\beta^2}{2}\sum_{i=1}^{n}\int_\Omega {u_{x_i t}}^2(x,T)dx + \frac{1-\lambda^2\alpha^2}{2}\int_\Omega [\Delta u\,(x,T)]^2 dx + \frac{\varepsilon}{2}\int_Q (\Delta u_t)^2 dxdt \le$$

$$\le \frac{1}{\varepsilon}\int_Q f^2 + \frac{{c_0}^2}{\varepsilon}\int_Q u^2 dxdt = I_f. \qquad (10)$$

Applying the Newton-Leibniz formula:

$$u\,(x,t) = \int_0^t u_\tau(x,\tau)d\tau + u(x,0) \qquad \text{(N-L)}$$

using the boundary condition (7) we obtain

$$u(x,T) = \frac{1}{1-\lambda\alpha}\int_0^T u_t(x,t)dt\,.$$

Using the inequality $|(a+b)^2 \le 2(a^2+b^2)|$ we obtain

$$u^2(x,t) \le 2\left(\int_0^t u_t(x,t)dt\right)^2 + 2\alpha_1\left(\int_0^T u_t(x,t)dt\right)^2 \le$$

$$\le 2\left(\int_0^t u_\tau^2\,d\tau\right)\left(\int_0^t d\tau\right) + 2\alpha_1\left(\int_0^T u_t^2\,dt\right)\left(\int_0^T dt\right) \le (2T + 2\alpha_1 T)\int_0^T u_t^2 dt;$$

Integrate over Q:

$$\int_Q u^2\,(x,t)dxdt \le 2(1+\alpha_1)T^2 \int_Q u_t^2 dxdt; \qquad (11)$$

Here $\alpha_1 = \left(\frac{\lambda\alpha}{1-\lambda\alpha}\right)^2$.

This is the required inequality. Denote by $k_0 = 2(1+\alpha_1)T^2$. Further, continuing inequality (11), we have:

$$I_f \le \frac{1}{\varepsilon}\int_Q f^2 dxdt + \frac{C_0^2 k_0}{\varepsilon}\int_Q u_t^2 dxdt \le \frac{1}{\varepsilon}\int_Q f^2 + \frac{C_0^2 k_0 m_0}{\varepsilon}\sum_{i=1}^{n}\int_Q u_{x_i t}^2 dxdt;$$

Using inequality (9), we have

$$I_f \le \frac{1}{\varepsilon}\int_Q f^2 dxdt + \frac{C_0^2 k_0 m_0}{2\varepsilon}\frac{1}{\varepsilon^2}\int_Q f^2 dxdt = \frac{1}{\varepsilon}\left(1 + \frac{C_0^2 k_0 m_0}{2\varepsilon^2}\right)\int_Q f^2 dxdt\,;$$

The Newton-Leibniz representation (N-L) using (7) gives:

$$u(x,t) = \int_0^t u_\tau(x,\tau)d\tau + \frac{\lambda\alpha}{1-\lambda\alpha}\int_0^T u_t(x,t)dt;$$

So we have

$$I_1 + I_2 + I_3 \le \left(\frac{1}{\varepsilon} + \frac{C_0^2 k_0 m_0}{2\varepsilon^3}\right) \int_Q f^2 dxdt; \tag{12}$$

If in (11) we assume $u = \Delta u$, then we obtain:

$$\int_Q (\Delta u)^2 dxdt \le k_0 \int_Q (\Delta u_t)^2 dxdt \le \frac{2k_0}{\varepsilon}\left(\frac{1}{\varepsilon} + \frac{C_0^2 k_0 m_0}{2\varepsilon^3}\right) \int_Q f^2 dxdt; \tag{13}$$

It remains to obtain an estimate for the first term, i.e. for u_{tt}. To do this, we use the following well-known inequality:

$$\left|(a_1 + \cdots + a_m)^2 \le m(a_1^2 + \cdots + a_m^2)\right|$$

$$\int_Q u_{tt}^2 dxdt = \int_Q (\Delta u - cu + \varepsilon \Delta u_t + f)^2 dxdt \le \le (4\int_Q (\Delta u)^2 + 4C_0^2 \int_Q u^2 + 4\varepsilon \int_Q (\Delta u)^2 + 4\int_Q f^2) dxdt.$$

For all terms of the right-hand side of the last inequality, estimates are obtained for fixed ε, i.e.

$$\int_Q u_{tt}^2 dxdt \le N(\varepsilon) \int_Q f^2 dxdt. \tag{14}$$

Next, let's bonus the continuation method with respect to the parameter for a fixed ε.

Now erase λ, i.e. consider the boundary value problem for $\lambda=1$.

$$\begin{cases} u(x,0) = \alpha u(x,T), \\ u_t(x,0) = \beta u_t(x,T). \end{cases}$$

Obtaining the first a priori estimate uniform in ε :

$$\frac{1-\beta^2}{2}\int_\Omega u_t^2(x,T)dx + \frac{1-\alpha^2}{2}\sum_{i=1}^n \int_\Omega u_{xi}^2(x,T)dx +$$

$$+\frac{1}{2}\int_\Omega \left[c(x,T) - a^2 c(x,0)\right]u^2(x,T)dx - \frac{1}{2}\int_\Omega c_t u^2 dxdt + \varepsilon \sum_{i=1}^n \int_Q u_{x_i t}^2 dxdt =$$

$$= \int_Q f u_t dxdt \le \frac{\delta^2}{2}\int_Q u_t^2 dxdt + \frac{1}{2\delta^2}\int_Q f dxdt. \tag{15}$$

Hence, in particular, we have

$$\frac{1-\beta^2}{2}\int_\Omega u_t^2(x,T)dx \le \frac{\delta^2}{2}\int_Q u_t^2 dxdt + \frac{1}{2\delta^2}\int_Q f^2 dxdt. \tag{16}$$

$(5)\times(A-t)u_t$, where A - is some positive number, $A > T$. For example, you can take $A = 2T$, then $A - t > A - T = T > 0$;

$$\int_Q (A-t)u_{tt}u_t dxdt - \int_Q (A-t)\Delta u u_t dxdt +$$

$$+ \int_Q (A-t)cuu_t dxdt - \varepsilon \int_Q (A-t)\Delta u_t u_t dxdt = \int_Q (A-t) f u_t dxdt;$$

Suppose that condition $\frac{\partial}{\partial t}\left[(A-t)c(x,t)\right]\le 0$, is satisfied, then we have

$$\frac{A-T}{2}\int_{\Omega}u_t^2(x,T)dx+\frac{1}{2}\int_Q u_t^2dxdt+\frac{A-T}{2}\sum_{i=1}^{n}\int_{\Omega}u_{x_i}^2(x,T)dx+\frac{1}{2}\sum_{i=1}^{n}\int_Q u_{x_i}^2dxdt$$

$$+\frac{A-T}{2}\int_{\Omega}c(x,T)u^2(x,T)dx-\frac{1}{2}\int_Q((A-T)c)_t u^2dxdt+\varepsilon\sum_{i=1}^{n}\int_Q(A-t)\,u_{x_it}^2dxdt=$$

$$=\int_Q f(A-t)\,u_tdxdt+\frac{A}{2}\int_{\Omega}u_t^2(x,0)dx+\frac{A}{2}\sum_{i=1}^{n}\int_{\Omega}u_{x_i}^2(x,0)dx+\frac{A}{2}\int_{\Omega}c(x,0)u^2(x,0)dx \quad (17)$$

We use the boundary conditions (7), (8) and (16) we have

$$\int_{\Omega}u_t^2(x,T)dx\le\frac{\delta^2}{1-\beta^2}\int_Q u_t^2dxdt+\frac{1}{\delta^2(1-\beta^2)}\int_Q f^2dxdt.$$

As a result, we get

$$\sum_i I_i\le K_1\delta^2\int_Q u_t^2dxdt+\frac{K_2}{\delta^2}\int_Q f^2dxdt,$$

where K_1, K_2 - independent of ε.

We choose, in particular, $K_1\delta^2=\frac{1}{4}$, $\delta^2=\frac{1}{4K_1}$. Then we get

$$\frac{1}{4}\int_Q u_t^2dxdt+\frac{1}{2}\sum_{i=1}^{n}u_{x_i}^2dxdt+\varepsilon\sum_{i=1}^{n}\int_Q u_{x_it}^2dxdt\le 4K_1K_2\int_Q f^2dxdt.$$

Hence, in particular, we have

$$\int_Q u_t^2dxdt\le 16K_1K_2\int_Q f^2dxdt,\ \sum_{i=1}^{n}\int_Q u_{x_i}^2dxdt\le 8K_1K_2\int_Q f^2dxdt,\ \varepsilon\sum_{i=1}^{n}\int_Q u_{x_it}^2dxdt\le 4K_1K_2\int_Q f^2dxdt.$$

Adding all the terms, we get

$$\int_Q u_t^2dxdt+\sum_{i=1}^{n}\int_Q u_{x_i}^2dxdt+\varepsilon\sum_{i=1}^{n}\int_Q u_{x_it}^2dxdt\le K_0\int_Q f^2dxdt,$$

where K_0 - is independent of ε. As required.

Thus, the following theorem is proved.

Theorem. Let the conditions be satisfied: $c(x,T)-\lambda^2\alpha^2c(x,0)\ge 0$, $c_t(x,T)\le 0$, $f,f_t\in L_2(Q)$. Then the boundary value problem (1) - (4) cannot have more than one solution in space $W_2^{2,2}(Q)$.

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КӨПӨЛШЕМДІ ШЕКТЕЛГЕН ОБЛЫСТА ГИПЕРБОЛАЛЫҚ ТЕҢДЕУЛЕР ҮШІН ЛОКАЛЬДЫ ЕМЕС ШЕТТІК ЕСЕПТЕРДІҢ ШЕШІМДІЛІГІ

Аннотация Бұл жұмыста шектелген көпөлшемді цилиндрлі облыста $Q = \Omega \times (0,T)$, $\Omega \subset R^n$, $S = \Gamma \times (0,T), \Gamma = \partial\Omega$ $0 < t < T < +\infty$ келесі ғиперболалық теңдеу үшін

$$u_{tt} - \Delta u + c(x,t)u = f(x,t), \qquad (x,t) \in Q \tag{1}$$

келесі түрдегі локалды емес шеттік есеп

$$u(x,t)\big|_S = 0, \tag{2}$$

$$u(x,0) = \alpha u(x,T), \tag{3}$$

$$u_t(x,0) = \beta u_t(x,T) \tag{4}$$

қарастырылады, мұндағы $f(x,t)$ – $x \in \overline{\Omega}$, $t \in [0,T]$ берілген функциялар $\alpha, \beta \in R$ - сандар.

Азғындалған квазиғиперболалық теңдеулер үшін шеттік есептердің шешімділігін зерттеу В.Н. Врағовтың жұмыстарынан бастау алады.

Сызықты параболалық теңдеулер үшін интегралдық шарты бар, тақ ретті дифференциалдық теңдеулер үшін, стационарлы емес кейбір теңдеулердің класында локальды емес шеттік есептердің шешімділігінің мәселесі, соңғы жылдары, А.И. Кожановтың жұмыстарында зерттелген.

$$u_{tt} - \Delta u + c(x,t)u - \varepsilon\Delta u_t = f(x,t) \in L_2(Q) \tag{5}$$

$$u(x,t)\big|_S = 0, \tag{6}$$

$$u(x,0) = \lambda\alpha u(x,T), \tag{7}$$

$$u_t(x,0) = \lambda\beta u_t(x,T) \tag{8}$$

(1)-(4) есебінің регулярлы шешімін зерттеу үшін қосалқы (5)-(8) регулярланған есебін қарастырамыз. Осы алынған қосалқы (5)-(8) есеп үшін: λ параметрі бойынша жалғастыру әдісін, сәйкес функционалдық кеңістіктерде априорлы бағалаулар әдістерін қолданамыз. ε-нан бірқалыпты тәуелді бағалауларда $\varepsilon \to 0$ ұмтылдырыу арқылы, зерттеліп отырған есептің регулярлы шешімінің бар болуы және жалғыздығы туралы теорема дәлелденеді.

Түйін сөздер: ғиперболалық теңдеу, шеттік есептер, регулярлы шешімдер, бар болуы, жалғыздығы.

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РАЗРЕШИМОСТЬ КРАЕВЫХ ЗАДАЧ С НЕЛОКАЛЬНЫМИ УСЛОВИЯМИ ДЛЯ МНОГОМЕРНЫХ ГИПЕРБОЛИЧЕСКИХ УРАВНЕНИЙ

Аннотация. В данной работе исследуется разрешимость новых нелокальных краевых задач для гиперболических уравнений в многомерной ограниченной области.

В ограниченной цилиндрической области $Q = \Omega \times (0,T)$, $\Omega \subset R^n$, $S = \Gamma \times (0,T), \Gamma = \partial\Omega$ $0 < t < T < +\infty$ рассмотрим гиперболическое уравнение

$$u_{tt} - \Delta u + c(x,t)u = f(x,t), \qquad (x,t) \in Q \tag{1}$$

со следующими нелокальными по времени краевыми условиями

$$u(x,t)\big|_S = 0, \tag{2}$$

$$u(x,0) = \alpha u(x,T), \tag{3}$$

$$u_t(x,0) = \beta u_t(x,T) \tag{4}$$

где $c(x,t), f(x,t)$ – заданные функции, определенные при $x \in \overline{\Omega}$, $t \in [0,T], \alpha, \beta \in R$ -некоторые числа.

Исследование разрешимости краевых задач для вырожденных квазигиперболических уравнений (по аналогии с квазиэллиптическими уравнениями) началось с работ В.Н. Врагова.

Исследования нелокальных задач с интегральными условиями для линейных параболических уравнений, для дифференциальных уравнений нечетных порядков и для некоторых классов нестационарных уравнений в последние время активно исследуются в работах А.И. Кожанова.

Для исследования нелокальной краевой задачи (1)-(4) рассмотрим следующую регуляризованную задач

$$u_{tt} - \Delta u + c(x,t)u - \varepsilon \Delta u_t = f(x,t) \in L_2(Q) \tag{5}$$

$$u|_S = 0 \tag{6}$$

$$u(x,0) = \lambda \alpha u(x,T), \tag{7}$$

$$u_t(x,0) = \lambda \beta u_t(x,T). \tag{8}$$

Для этой регуляризованной задачи (5)-(8) мы применим метод продолжения по параметру. Для задачи (5)-(8) получим априорные оценки в соответствующих функциональных пространствах. И в конце переходим к пределу $\varepsilon \to 0$.

Таким образом, для изучаемой задачи доказывается теорема существования и единственности регулярного решения.

Ключевые слова: гиперболическое уравнение, краевые задачи, регулярные решения, существование, единственность.

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